

Non-Constant Prediction-Step MPC for Processes with Multi-Scale Dynamics

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Abstract: Motivated by applications in minerals processing, a novel model predictive control scheme is presented with non-constant prediction step size. In the proposed scheme, the sampling rate of the prediction and control horizons changes, with nearer steps having a shorter sampling period than ones in the more distant future. This approach allows for fine tuning of the control trajectory over the near horizon, whilst still allowing for long prediction and control horizons to account for slow dynamics. This may find application in many chemical and minerals processing plants where process units have multiple time-scale dynamics. Extensions to decentralized control of process networks are presented. The approach is illustrated using an industrial case study.

Keywords: Model Predictive Control, Minerals Processing, Multi-Scale Dynamics

1. INTRODUCTION

Model predictive control (MPC), is one of the most successful advanced control methods applied in the process industries. Key drivers of this success are MPCs ability to handle multi-variable control problems, operational constraints and to generate an optimal control sequence Qin and Badgwell [2003]. As such, there is a large body of research studying the stability and optimality of MPC, a seminal paper on which is Mayne et al. [2000].

A common feature of chemical and minerals processing systems is dynamics operating on multiple time-scales, these are due to chemical and physical phenomena occurring at different rates. Examples of such systems may include thickeners (Bürger et al. [2005]), bioreactors and reactor networks (Christofides and Daoutidis [1996], Kumar et al. [1998]) and reactive distillation columns Taylor and Krishna [2000]. A naïve approach of applying standard MPC algorithms to such systems would require a short sampling time and a long prediction horizon to capture all time scales. Due to the high computational cost associated with this approach, different methods have been developed to reduce the computational complexity.

The use of singular perturbations is a classical approach to control design for such systems, Kokotović et al. [1999]; and has been used in MPC design e.g. Wogrin and Glielmo [2010], Chen et al. [2011, 2012], and Baldea and Touretzky [2013]. In this approach separate controllers are designed to control the fast and slow dynamics. This allows for a simplification of the control algorithms and prevents ill-conditioning of the controllers. In addition, it naturally lends itself to both linear and nonlinear design approaches. A disadvantage, however, is that its stability can only be

ensured if the time-scale separation is sufficiently large, a condition which may be difficult to test for nonlinear systems.

As the computational complexity of MPC depends heavily on the number of decision variables (essentially the number of control actions in the generated trajectory), an approach known as ‘move blocking’ has also been applied. In this approach constraints of the form $u(t+i) = u(t)$, for some values of the t and i are implemented, thus decreasing the number of independent inputs. A survey of this approach is presented in Cagienard et al. [2007]. A potential disadvantage of this approach is that the feasible set may be decreased in size, least restrictive approaches have been developed to address this, e.g. Gondhalekar and Imura [2010].

In the current paper we propose an MPC algorithm for systems exhibiting multiple time-scale dynamics based on a non-constant sampling period of the prediction and control horizons. Whereby, fine tuning the control in the near future is obtained with relatively coarse predictions of the distant future. This approach is similar to that of move-blocking, however, in addition to blocking the inputs, the prediction of the state is also ‘blocked’. The proposed approach has the advantage of decreasing the number of decision variables (thus decreasing computational complexity), and is shown by simulation studies to exhibit high levels of performance. Stability of the approach is demonstrated by applying Lyapunov arguments and, alternatively, dissipative systems theory. This latter approach has the advantage of being applicable to the large-scale systems and decentralized and distributed control. As dissipativity offers a scalable approach to ensuring stability and performance bounds on such systems. It should be noted that the proposed approach is not ‘multi-rate’ MPC whereby different measurements and/or actuators are available with different sampling rates i.e.

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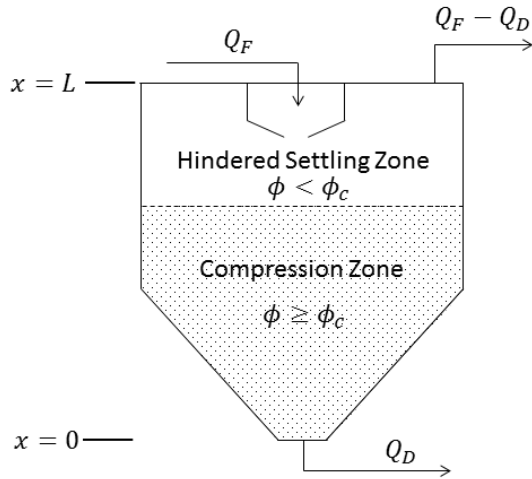


Fig. 1. A continuous operating paste thickener

Heidarinejad et al. [2011], in the proposed approach, all measurements/actuators have the same sampling rate, it is only the predictions which has multiple sampling rates.

Some notation used in the remainder of this paper is briefly introduced; $\|v\|_{2e}$ refers to the extended \mathcal{L}_2 norm of the vector signal $v(t)$. For a symmetric matrix A , $A > 0$ means A is positive definite. The eigenvalues of the matrix A are denoted by $\lambda(A)$, and the singular values by $\sigma(A)$. The 2-norm of a matrix A is denoted $\|A\|$.

2. MOTIVATING EXAMPLE

Thickening is a method used to separate water from solids in a slurry. It is commonly applied in minerals and coal preparation processes as well as in wastewater treatment operations. Figure 1 shows a continuous paste thickener fed with a slurry of flow rate Q_F , with product thickened slurry removed at a rate of Q_D from the bottom of the thickener. The overflow from the thickener $Q_F - Q_D$ is to be recycled back to the plant, assumed to contain water only. The height above the base of the thickener is denoted by x , with $\phi(x)$ the concentration of solids as volume fraction. The critical concentration, ϕ_c , divides the process into hindered settling and compression zones. At this point, also known as the gel point, particles start to coalesce due to close proximity between each other, forming a bed layer.

In modelling the process, Bürger et al. [2004] proposed the nonlinear sedimentation-consolidation model which accounts for the continuous process as well as the varying cross sectional area of the paste thickener:

$$\frac{\partial \phi(x, t)}{\partial t} + \frac{1}{S(x)} \frac{\partial}{\partial x} (Q_D(t)\phi + S(x)f_{bk}(\phi)) = \frac{1}{S(x)} \frac{\partial}{\partial x} (S(x) \frac{\partial A(\phi)}{\partial x}), \quad (1)$$

The solids concentration is assumed to be a function of the local concentration only Coe and Clevenger [1916]. The cross sectional area is $S(x)$, $Q_D(t)$ is the underflow volumetric flow rate at the bottom of the thickener, $f_{bk}(\phi)$ is the Kynch batch flux density function and $A(\phi)$ is

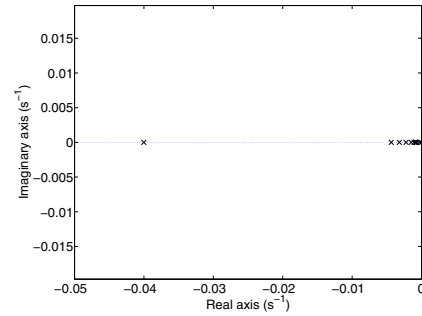


Fig. 2. Poles of the linearized system

the consolidation function. For $f_{bk}(\phi)$, the sedimentation function proposed by Michaels and Bolger [1962] is used:

$$f_{bk}(\phi) = \begin{cases} v_\infty \phi (1 - \frac{\phi}{\phi_{max}})^N & \text{for } 0 \leq \phi \leq \phi_{max}, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where v_∞ is the terminal settling velocity of a single particle in a liquid. The parameter N is related to the shape of particle Moreland [1963]. The consolidation function $A(\phi)$ is described as:

$$A(\phi) = \int_0^\phi a(s) ds, a(\phi) = \frac{f_{bk}(\phi)\sigma'_e(\phi)}{\Delta\rho g\phi}, \quad (3)$$

where $\Delta\rho$ is the difference in solid-liquid density and g is the gravity acceleration constant, taken to be $9.81ms^{-2}$. The solid stress function $\sigma_e(\phi)$, which accounts for the interactive effect between particles in the compression zone, can be represented as Tiller and Leu [1980].

$$\sigma_e(\phi) = \begin{cases} 0 & \text{for } \phi \leq \phi_c, \\ \sigma_0 [(\frac{\phi}{\phi_c})^k - 1] & \text{for } \phi > \phi_c, \end{cases} \quad (4)$$

where k and σ_0 are particle properties, in this case, the coal properties. Note that the effective stress function does not exist when $\phi < \phi_c$ due to the relatively dilute solids concentration. The convergence and stability condition of the discretization approach appears in Bürger et al. [2004].

Eq. (1) shows that the dynamic of the system is dependent on three mechanisms:

- (1) Convection. This is the transport of bulk flow of the suspension in the thickeners due to momentum transfer.
- (2) Sedimentation, which is the settling of particles by the force of gravity, and its dynamic can be represented in (2).
- (3) Consolidation which only occurs in the compression zone in which the particles interact, exerting a net pressure on each other and at the same time enhancing the dewatering ability.

These mechanisms respond at different time-scales due to different transport phenomena, leading to the different time-scales of the thickener. The fastest mechanism is convection, followed by sedimentation and consolidation at a comparable rate. Importantly, the consolidation mechanism is not present in conventional thickeners, yielding the need for a different control strategy. This can be seen in Figure 2, which plots the poles of the linearized system.

From Figure 2, it can be observed that there is a large range of dynamic response of the process, with one pole

located quite far apart from the others. From an analytical perspective, the fast dynamics is found to have a time constant of about 0.5 min while the slow dynamics with time constant of 265 hr. This shows the multiple time-scale nature of the process. As such, in order to implement a MPC, the prediction horizon of the controller must be long enough to cover the slowest time constant, which is rather impractical as the computational requirement will be extensive.

Hence, one way to tackle this issue is to have a prediction horizon in which the prediction in the nearer steps having more frequent and small sampling periods and larger sampling time for prediction horizon in the distant future. This will allow the prediction horizon to cover the dynamic of slow component in the process. This approach is outlined in the following section.

3. VARYING PREDICTION STEP SIZE MPC

The problem under consideration is to design a model predictive controller for the system

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t) \quad (5)$$

$$y(t) = Cx(t), \quad (6)$$

where $u(t) \in \mathbb{R}^p$, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^q$ and $d(t) \in \mathbb{R}^n$ are the manipulated variable, state, measured output and unmeasured disturbance/noise, respectively. In addition, the poles of the system are assumed to have different magnitudes. That is, $\left| \frac{\max \operatorname{Re}(\lambda(A))}{\min \operatorname{Re}(\lambda(A))} \right| \gg 1$. It is assumed that the continuous time system is controllable and observable. Using ZOH and exact discretization, it is well known that the discretized model is described by $A(\Delta t)$, $B(\Delta t)$ and C under these conditions, where

$$A(\Delta t) = e^{A\Delta t} \quad (7)$$

$$B(\Delta t) = \left(\int_0^{\Delta t} e^{At} dt \right) B. \quad (8)$$

The essential idea of the proposed approach is that the sampling rate used in the prediction and control horizons is non-constant, in particular, the sampling rate further into the future is longer than that at nearer steps. Ordering the sampling rates from the present into the future, the i th sampling rate is denoted Δt_i , as such $\Delta t_i \leq \Delta t_j$ for $i < j$. Due to the changing sampling rate in the prediction horizon, at any point in time the process may be seen as a linear parameter varying system from the controllers point of view, that is

$$x(t+1) = A(p(t))x(t) + B(p(t))u(t) + d(t) \quad (9)$$

$$y(t) = Cx(t), \quad (10)$$

where the parameter $p(t)$ is the sampling rate, with the state space matrices parameterized as in (7)-(8).

We present two MPC algorithms for this problem, the key difference being the way in which stability is ensured. In the first algorithm stability is ensured by showing the cost function acts as a Lyapunov function. This is applicable to the control of single system. Whereas in the second algorithm the controller is shown to yield the closed-loop system dissipative with finite \mathcal{L}_2 -gain from external disturbance to the controlled output. This approach may be applied to plant-wide control as the interaction effects

may be captured by the dissipativity properties of the closed-loop systems.

To improve readability, the first MPC algorithm is stated in the following section, for the case that the sampling period of the prediction changes only once from Δt_1 to $\Delta t_2 > \Delta t_1$.

3.1 MPC for Single Systems

Algorithm 1.

$$\begin{aligned} \min_{u(t)} & \sum_{k=0}^{M-1} x^T(t+k\Delta t_i) Q x(t+k\Delta t_i) + u^T(t+k\Delta t_i) R u(t+k\Delta t_i) \\ & + \sum_{k=M}^{N-1} x^T(t+k\Delta t_i) Q x(t+k\Delta t_i) + u^T(t+k\Delta t_i) R u(t+k\Delta t_i) \\ & + x^T(t+M\Delta t_1 + (N-M)\Delta t_2) P x(t+M\Delta t_1 + (N-M)\Delta t_2) + \epsilon_x + \epsilon_u \end{aligned}$$

subject to:

$$x(t+1) = A(\Delta t_i)x(t) + B(\Delta t_i)u(t) \quad (11)$$

$$y(t) = Cx(t) \quad (12)$$

$$x^T(t) P_x x(t) \leq 1 + \epsilon_x, \quad \epsilon_x \geq 0 \quad (13)$$

$$u^T(t) P_u u(t) \leq 1 + \epsilon_u, \quad \epsilon_u \geq 0 \quad (14)$$

where the sampling period of the prediction changes at $t = M\Delta t_1$, Q , $R \geq 0$ are symmetric matrices and N is the prediction horizon.

Closed-loop stability may be ensured using the well known method of enforcing a terminal weight P . This is made precise below.

Theorem 1. The closed-loop system of system (5) with Algorithm 1 is asymptotically stable for vanishing disturbance if the terminal cost, $P > 0$, satisfies $P - \tilde{A}^T P \tilde{A} = \tilde{Q}$ and $(A(\Delta t_1), B(\Delta t_1))$ is stabilizable, where $\tilde{A} = A(\Delta t_1) + B(\Delta t_1)K$, $\tilde{Q} = Q + K^T R K$, and $(A(\Delta t_1) + B(\Delta t_1)K)$ is stable.

Proof. The proof follows using the same arguments as in Mayne et al. [2000] by showing that the conditions on the terminal cost, P , implies that the terminal cost acts as a Lyapunov function. \square

Remark 1. The conditions in Theorem 1 are based upon the first sampling rate in the cost function, Δt_1 . However, similar conditions can be developed for any of the sampling rates in the cost function. If Δ_i is used in the analysis, then it can be shown that the cost to go is decreasing every Δt_i time units. As such, it is recommended that the smallest Δt_i is used, so as to prevent poor transient response.

Remark 2. The above algorithm may be extended to the case where the sampling period of the prediction changes multiple times without significantly affecting the implementation or stability analysis.

3.2 MPC for Process Networks

The second, dissipativity based, MPC algorithm may be summarized as follows.

Algorithm 2.

$$\min_{K(t)} \sum_{k=0}^{N-1} x^T(t+k\Delta t_i) Q x(t+k\Delta t_i) + u^T(t+k\Delta t_i) R u(t+k\Delta t_i) + \epsilon_x + \epsilon_u$$

subject to:

$$x(t+1) = A(\Delta t)x(t) + B(\Delta t)u(t) + d(t) \quad (15)$$

$$y(t) = Cx(t) \quad (16)$$

$$u(t) = -K(t)x(t) \quad (17)$$

$$x^T(t)P_x x(t) \leq 1 + \epsilon_x, \quad \epsilon_x \geq 0 \quad (18)$$

$$u^T(t)P_u u(t) \leq 1 + \epsilon_u, \quad \epsilon_u \geq 0 \quad (19)$$

$$\begin{pmatrix} P^{-1} & (A-BK) & 0 \\ (A-BK)^T & P(I-\beta^2 P) & -(A-BK)^T P \\ 0 & -P(A-BK) & P(\alpha^2 P - I) \end{pmatrix} \geq 0, \quad (20)$$

where $P, Q, P_x, P_u > 0$ and $R \geq 0$ are symmetric matrices, and $\alpha > 0, \beta = \max(\sigma(C))$.

This algorithm is a convex quadratic program, and may be solved, for example, by converting it to a LMI problem and using an appropriate solver. The satisfaction of (20) ensures that the closed-loop system is dissipative, and in fact, stable with an upper bound of the closed-loop \mathcal{L}_2 -gain from $d(t)$ to $y(t)$ of $\gamma = \alpha\beta \frac{\max(\lambda(P))}{\min(\lambda(P))}$. This is proved in the following.

Theorem 2. Consider the online optimisation algorithm 2 as applied to the system (5). The closed-loop system is dissipative, with supply rate $(-P^T P, 0, \alpha^2 P^T P)$, asymptotically stable and satisfies

$$\|y(t)\|_{2e}^2 \leq \gamma^2 \|u(t)\|_{2e}^2 + x^T(0)Px(0), \quad (21)$$

if (20) is satisfied at every step.

Proof. Taking a Schur complement in the top left block, it is easily seen that (20) is equivalent to

$$\begin{pmatrix} P-\beta^2 P^T P - (A-BK)^T P(A-BK) & -(A-BK)^T P \\ -P(A-BK) & \alpha^2 P^T P - P \end{pmatrix} \geq 0, \quad (22)$$

Which is equivalent to

$$\begin{pmatrix} x(k) \\ d(k) \end{pmatrix}^T \begin{pmatrix} P-\beta^2 P^T P - (A-BK)^T P(A-BK) & -(A-BK)^T P \\ -P(A-BK) & \alpha^2 P^T P - P \end{pmatrix} \times \begin{pmatrix} x(k) \\ d(k) \end{pmatrix} \geq 0,$$

for any $x(k), d(k) \in \mathbb{R}^n$. Upon rearranging this yields

$$-\beta^2 x^T(k)P^T Px(k) + \alpha^2 d^T(k)P^T Pd(k) \geq V(x(k+1)) - V(x(k)),$$

for any $k \geq 0$ with $V(x(k)) = x^T(k)Px(k)$, acting as a Lyapunov function for vanishing $d(k)$, and thus implying asymptotic stability. This is a dissipation inequality (Willems [1972]) implying the dissipativity of the system from $d \rightarrow x$, using $Cx(k) \leq \beta x(k)$, we obtain

$$-y^T(k)P^T Py(k) + \alpha^2 d^T(k)P^T Pd(k) \geq V(x(k+1)) - V(x(k)), \quad (23)$$

which is the stated dissipativity condition. Summing (23) to any $T > 0$ yields (21). \square

In Theorem 2 it is assumed that P is known, indeed (20) is bilinear in the case that it is not. If the controller gain is parameterized as $K(t) = Y(t)P^{**}$, then pre- and post-multiplying (22) by $\text{diag}(P^{-1})$ and performing a Schur complement in the top left block yields

$$\begin{pmatrix} P^{-1} & (AP^{-1}-BY) & 0 \\ (AP^{-1}-BY)^T & P^{-1}-\beta^2 P^{-1} & -(AP^{-1}-BY)^T \\ 0 & -(AP^{-1}-BY) & \alpha^2 I - P^{-1} \end{pmatrix} \geq 0, \quad (24)$$

which is an LMI in P^{-1}, Y and α^2 . Note that due to the parameterization of $K(t)$ this condition will yield a non-convex condition if applied online. Thus, it is suggested that (24) be solved offline to minimise the closed-loop

**This is unrestrictive, as setting $Y(t) = \tilde{Y}(t)P^{-1}$ yields $K(t) = \tilde{Y}(t)$ for any $\tilde{Y}(t)$.

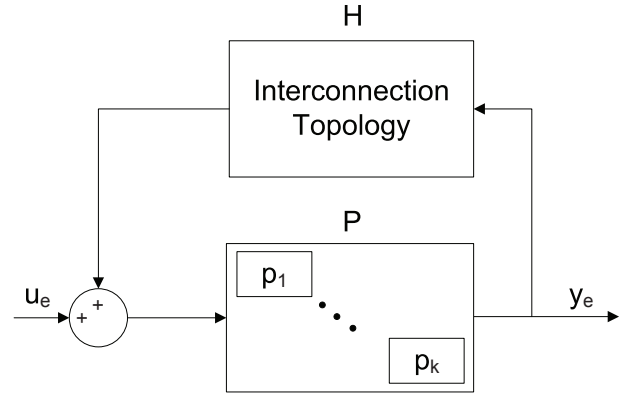


Fig. 3. Feedback view of process network

gain, γ , and to find a suitable P . Then, Algorithm 2 applied online. Note that, as an allowable $K = YP$ is determined in the offline step, and as soft constraints are used, Algorithm 2 is ensured to be globally feasible, and thus, the closed-loop system is globally asymptotically stable.

An advantage of Algorithm 2 over Algorithm 1 is that it provides an upper bound on the \mathcal{L}_2 -gain of the closed-loop system, thus providing worst case performance bounds. Additionally, the dissipativity of the closed-loop allows for an easy extension to decentralized control of large-scale systems, as dissipativity may be easily be used to show stability of such systems, see Moylan and Hill [1978] for a classical result in this area. A more recent application in the context of dissipativity-based decentralised and distributed MPC is Tippet and Bao [2013]. The application to decentralised control is developed further in the following.

If a process network has n process units, the i th closed-loop subsystem is denoted p_i , and they are connected as $u_i(t) = y_{e_j}(t) + u_{e_i}(t)$, where $u_{e_i}(t)$ is an external disturbance, then the interconnection relations may be represented as $\mathbf{u}(t) = H\mathbf{y}_e(t) + \mathbf{u}_e(t)$, where $\mathbf{u}^T(t) = (u_1^T(t), \dots, u_n^T(t))$, with $\mathbf{y}_e(t), \mathbf{u}_e(t)$ similarly defined. Then, the closed-loop process network may be represented as in Figure 3.

It is easy to see that if each closed-loop is dissipative with supply rate (Q_i, S_i, R_i) , then the system P is dissipative with supply rate (Q, S, R) with $Q = \text{diag}(Q_1, \dots, Q_k)$, $S = \text{diag}(S_1, \dots, S_k)$ and $R = \text{diag}(R_1, \dots, R_k)$. It is then well known that the overall system is dissipative with supply rate $(Q + SH + H^T S^T + H^T R H, S + H^T R, R)$. Using the results in Moylan and Hill [1978], the overall system is asymptotically stable if $Q + SH + H^T S^T + H^T R H < 0$. This is made precise below for this special case in Algorithm 2 that $S = 0$.

Proposition 1. A process network with n process units, with n decentralized model predictive controllers using Algorithm 2 is asymptotically stable if

$$Q + H^T R H < 0, \quad (25)$$

where $Q = \text{diag}(-P_1^T P_1, \dots, -P_k^T P_k)$ and $R = \text{diag}(\alpha_1^2 P_1^T P_1, \dots, \alpha_k^2 P_k^T P_k)$.

Proof. Omitted due to space constraints.

4. SIMULATION STUDIES

4.1 Academic Example

The following system is considered as an example

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 100 & -100 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 100 \end{pmatrix} u(t), \quad (26)$$

where the state is the output, and the time base is seconds. Note that $\lambda(A) = 0.99, -100.99$ implying a system with a fast stable, and slow unstable mode. Also, $\left| \frac{\max \operatorname{Re}(\lambda(A))}{\min \operatorname{Re}(\lambda(A))} \right| \approx 100$, showing the different time-scales inherent in the system. Three different prediction horizons are considered:

- (1) 7 steps of $\Delta t = 0.027$ by using a constant prediction horizon,
- (2) 7 steps of non-constant prediction horizon where $\Delta t_i = 0.0137 \exp(0.6933i)$, $i = 1, \dots, 7$,
- (3) 127 steps of $\Delta t = 0.027$ using a constant prediction horizon.

Note that the total number of prediction *steps* is equal in cases (1) and (2), and the total prediction length in *seconds* is equal in cases (2) and (3). In case (2), the sampling time follows an exponential function, similar to a study presented in Gondhalekar and Imura [2006]. In all cases, the weighting matrices are chosen to be $Q = I_2$ and $R = 0.1$. Simulation results are summarized in Table 1 and Figure 4 for initial state conditions of $(x_1, x_2) = (3, 2)$.

Table 1. Simulation Results for Academic Example

Case	Average computation time/step (s)	IAE
1	3.69×10^{-3}	25.99
2	3.22×10^{-3}	3.51
3	5.31×10^{-3}	3.16

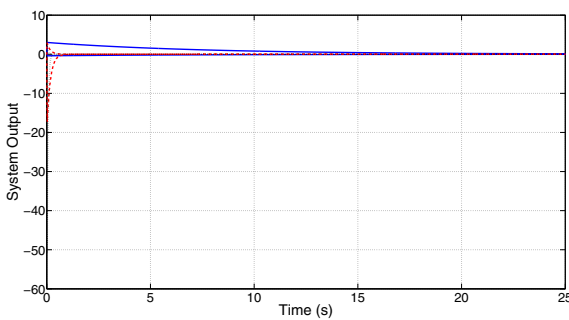


Fig. 4. System output for academic example (blue(—) = case (1), red(- -) = case (2), black(..) = case (3))

Case (2) offers a significant performance improvements over case (1), with comparable computation time. The slight decrease in average computation time for case (2) as compared to case (1) is due to the fact that in case (2) the system is driven to equilibrium much quicker, after which point the calculation of the optimal control is trivial. This is because, although the same number of steps used in both cases, due to the non-constant prediction step size, the total prediction length (in time domain) in case (2) is sufficient to cover the time-scales of all the dynamics of (26). Hence, for case (2), the controlled system is able to predict the more distant future, optimizing the control

action in advance, yielding improved performance. This result is promising, as the change in prediction step size enables the controller to predict a longer horizon without deteriorating the computational effort. On comparing cases (2) and (3), a significant, 65%, increase in computation time is observed for case (3) with similar performance levels. Thus showing that the proposed approach can yield considerable computational savings with minimal performance degradation. This is in contrast with the short horizon controller, case (1), where computational savings are achieved, but at the cost of significant performance degradation. Even with the simple system under study, these simulations demonstrate the computational savings that the proposed approach can achieve, in the following it is demonstrated that more pronounced savings can be made for complicated systems.

4.2 Industrial Paste Thickener

In this section, a simulation of the industrial paste thickener with 12 states modelled in Section 2 using both constant and non-constant prediction step MPC is presented. The cases considered are:

- (1) 60 steps of $\Delta t_1 = 1$ min and 7 steps of $\Delta t_2 = 1$ hour,
- (2) 480 steps of $\Delta t = 1$ min.

These cases both have a prediction horizon of 8 hours, with a different number of decision steps. Figure 6 shows the disturbances in the process, one being the inlet feed conditions and the other is the coal parameter which is estimated using actual measured plant data from an industrial paste thickener in NSW, Australia.

Figure 5 and Table 2 show that the performance of the controllers remains comparable (4% difference in IAE). This difference may be due to case (2) weighting the more distant future more heavily as the cost function has more steps in the distant future. Notably, however, there is a significant improvement in the computational time for the proposed approach. The computational time required to solve the optimization problem online for case (1) is approximately 40 times quicker as compared to case (2). This is as there are fewer decision variables, thus decreasing the computational effort required. The simulation shows that the non-constant prediction step can perform as well as a longer prediction horizon with a significant improvement in computational effort.

Table 2. Simulation Results for Thickener

Case	Average computation time/step (s)	IAE
1	0.14	330.74
2	5.79	346.44

5. DISCUSSION AND CONCLUSIONS

Motivated by applications in the minerals processing industry, an MPC scheme for systems exhibiting dynamics on multiple time-scales has been developed. The approach utilizes a shorter sampling period to predict the response of the system in the near future, and a longer period in the more distant future. This allows for the effects of both the fast and slow dynamics to be captured. The proposed approach provides similar levels of performance

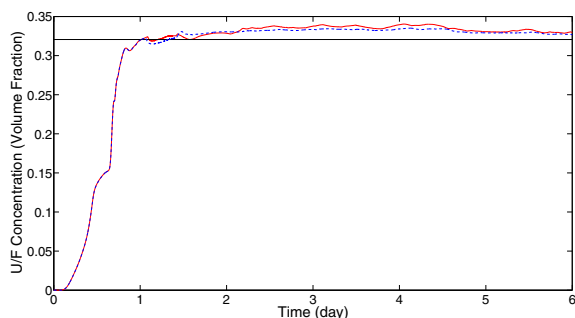


Fig. 5. Controlled variable using constant/non-constant prediction step (black(—) = set point, blue(- -) = case (1), red(—) = case (2))

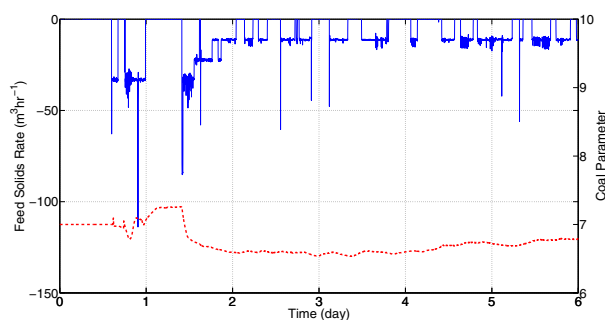


Fig. 6. Process disturbances (blue(—) = inlet feed solids, red(- -) = coal parameter)

as compared to classical MPC schemes, whilst decreasing the computational overhead. Further work may include the extension to nonlinear systems and an extension to distributed MPC for plant-wide control, potentially in a similar manner to Tippett and Bao [2013].

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