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Optimal Dynamic Allocation and Space Reservation for Electric Vehicles at Charging Stations ⋆

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Abstract: We propose an optimal allocation and reservation system for Electric Vehicles (EVs) at charging stations distributed in an urban environment. The system assigns and reserves an optimal space at a charging station based on the user's cost function that combines proximity to current location (or destination) and charging cost. Our approach is motivated by a similar system we have developed for "smart parking", where resources are parking spaces rather than EV charging station spaces. We solve a Mixed Integer Linear Program (MILP) problem at each assignment decision point over time. The solution of each MILP is an optimal allocation based on current state information, and is updated at the next decision point. Formal guarantees are included that there is no resource reservation conflict and that no user is ever assigned a resource with a higher than this user's current cost function value. Simulation results are included to illustrate how our system, compared to uncontrolled processes or guidance-based approaches, reduces the average time to find a charging space and the associated user cost, while the overall charging space capacity is more efficiently utilized.

1. INTRODUCTION

The increasing popularity of battery-powered vehicles (BPVs) has set the stage for a variety of research problems stemming from four major BPV characteristics: limited cruising range, long charge times, sparse coverage of charging stations, and their energy recuperation ability [Artmeier et al. (2010)] which can be exploited. Focusing on the problem of recharging Electric Vehicles (EVs), the sparsity of charging stations makes it critical for an EV to identify an optimal station given its current location, destination, and charge state. Moreover, the limited capacity of charging spaces at a station creates an additional difficulty similar to the classic parking problem: it is estimated that, on a daily basis, 30% of vehicles on the road in downtown areas of major cities are cruising for a parking spot and it takes an average of 7.8 minutes to find one [Arnott et al. (2005)]. This causes not only a waste of time and fuel (or battery energy) for drivers looking for parking, but it also contributes to additional waste of time and fuel for other drivers as a result of traffic congestion. For example, it has been reported by Shoup (2005) that over one year in a small Los Angeles business district, cars cruising for parking created the equivalent of 38 trips around the world, burning $47,000$ gallons of gasoline and producing 730 tons of carbon dioxide. This problem is exacerbated by the presence of EVs requiring "charging spaces" which in fact are sometimes dedicated on-street parking spaces in urban areas and may be incorporated into the overall parking problem to be dealt with.

The parking problem has been addressed over the past decade through so-called Parking Guidance and Information (PGI) systems. PGI systems present drivers with dynamic information on parking within controlled areas and direct them to vacant parking spots. Parking information may be displayed on variable-message signs (VMS) at major roads, streets, and intersections, or it may be disseminated through the Internet [Streetline (2012); Griffith (2000); Teodorovic and Lucic (2006)]. PGI systems are based on the development of autonomous vehicle detection and parking space monitoring, typically through the use of sensors placed in the vicinity of parking spaces for vehicle detection and surveillance, e.g., see Parkhelp (2012); Cheung and Varaiya (2007); Mimbela and Klein (2000). However, it has been found that using PGI systems, system-wide reductions in travel time and vehicle benefits may be relatively small [Thompson and Bonsall (1997)]. As pointed out in [Geng and Cassandras (2013)], current guidance-based systems have several shortcomings. For example, drivers may not find vacant parking spots by merely following a VMS; drivers may miss a better parking spot; and parking space utilization becomes imbalanced. Most importantly, these systems may in fact cause added traffic congestion by guiding drivers to an area with a few attractive spaces which only a subset of these drivers can occupy while the rest contribute to congestion in the area. In the case of EV charging stations, the same problem arises if several vehicles are guided to a specific station with a limited numbers of spaces.

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In this paper, we propose an approach which parallels a "smart parking" system developed in [Geng and Cassandras (2013)] and which is specifically designed for EV charging stations. This system explicitly *allocates and reserves optimal EV charging spaces* for its users, as opposed to simply guiding them to a space that may not be available by the time it is reached. The allocation is based on the user's objective function that combines proximity to the EV's current location (or its ultimate destination) and the charging cost, while also ensuring that the overall charging space capacity is efficiently utilized with a built-in "fairness" guarantee. Similar to the parking problem [Teodorovic and Lucic (2006)], it is also possible to dynamically control the price of charging spaces so as to further alleviate congestion.

The rest of the paper is organized as follows. In Section 2, we introduce the framework for our system by adopting the key features of the "smart parking" system developed in [Geng and Cassandras (2013)]. In Section 3, we describe the dynamic resource allocation model and formulate the MILP problem which is solved at every decision point with certain performance guarantees. Simulation results are presented in Section 4.

2. SYSTEM FRAMEWORK

Our proposed system for optimal EV charging space allocation and reservation adopts the basic structure of a "smart parking" system in [Geng and Cassandras (2013)]. The system includes a Request Processing Center (RPC) and a Space Allocation Center (SAC). A central Resource Management Center (RMC) collects and updates all real-time charging station information, and disseminates it via VMS or Internet. The RPC collects EV charging requests from users and real-time information (e.g., EV location), keeps track of user allocation status, and sends back assignment results to users. The SAC communicates with both the RPC and RMC: based on the user requests from the RPC and the resource states (i.e., the availability of spaces at charging stations) from the RMC, the SAC makes assignment decisions and *allocates and reserves* charging station spaces to users.

The basic allocation process (to be precisely modeled and analyzed in the next section) is described as follows. Users (i.e., EV drivers) who are looking for charging station spaces send requests to the RPC. A request is accompanied by two requirements: (*i*) a constraint (upper bound) on the charging rate cost and (*ii*) a constraint (upper bound) on the distance between a charging space and the EV's current location (or its eventual destination). It also contains the user's basic information such as license number, current location, car size, etc. The SAC collects all user requests in the PRC over a certain time window and makes an overall allocation at decision points in time, seeking to optimize a combination of user-specific and system-wide objectives. An assigned charging space is sent back to each user through the RPC. If a user is satisfied with the assignment, he has the choice to reserve that space. Once a reservation is made, the user still has opportunities to obtain a better space (with a guarantee that it can never be worse than the current one) before the current assigned space is reached. The RMC then updates the corresponding space state from *vacant* to *reserved*, and provides the guarantee that other users have no permission to take that space. If a user is not satisfied with the assignment (either because of limited systemwide resources or his own overly restrictive requirements) or if he fails to accept it for any other reason, he has to wait until the next decision point. During intervals between allocation decisions made by the SAC, users with no charging space assignment have the opportunity to change their cost or distance requirements, possibly to increase the chance to be allocated if the system is highly utilized (it is of course possible that no space is ever assigned to a user).

In order to physically realize this system, there are four main requirements.

(1) Charging Space Detection. First, the SAC has to know the state of all charging spaces. Current sensing technologies make monitoring parking or charging station spaces implementable; for example, in the "smart parking" system currently implemented at Boston University (see [Geng and Cassandras (2013)]), both induction loop sensors placed on the ground at individual parking spaces and cameras are used. In addition, standard GPS technology provides accurate localization and speed estimates of vehicles [Mimbela and Klein (2000)].

(2) Vehicle-Allocation Center Communication. The second requirement involves effective wireless communication between EVs and the SAC. This is also achievable through existing wireless networks that may be proprietary or part of cellular telephone service providers. Once a user sends out a request, the system will send back space allocation results based on his preferences and the state of the system. There are two possible allocation results: (*i*) If the system fails to find a charging space, a notification asks the user to wait for the next allocation time with an appropriate justification, e.g., there are no vacant spaces or the user's requirements are too restrictive. The user may then either release his request by changing his preferences to increase the chance to be allocated, or simply wait. (*ii*) If a space is allocated, the user may accept it or reject it and adjust his requirements. If the user is satisfied with the allocation, then the system reserves that space for him and driving directions to the reserved space are given through standard guidance methods (usually through a smartphone application as in the "smart parking" system in [Geng and Cassandras (2013)]. Note that while driving towards the assigned space the system may notify the user about a better space for him based on his real-time position. The driver needs to respond and tell the system whether he/she accepts it or not.

(3) Reservation Guarantee In order to implement this key system function, when a space is reserved by the user the system must guarantee that this will not be taken by other vehicles. This is achievable through wireless technology interfacing a vehicle with hardware that makes a space accessible only to the user who has reserved it (e.g., gates, "folding barriers," and obstacles that emerge from and retract to the ground). However, this approach incurs significant infrastructure costs. A simpler scheme is to use a light system placed at each space, where different colors indicate different states. In [Geng and Cassandras (2013)], a GREEN light indicates that a vacant space is available for any user, a RED light indicates that the space is reserved by another user, a blinking YELLOW light attracts a driver in the vicinity who has reserved that space, and a blinking RED light notifies a user who is accessing a space reserved by someone else. An LED light with a wireless sensor node is placed at each space. When a user is approaching the space reserved for him, this is automatically detected by the GPS data sent from the device used to make the request (typically, a smartphone) and the sensor node switches the light at his reserved spot from RED to blinking YELLOW. After accessing the space, the light goes off until the vehicle leaves and returns to its GREEN state or RED state if the space is reserved. Further details on the complete supervisory control mechanism for the light system are provided in [Geng and Cassandras (2013)].

(4) Optimal Allocation. This is carried out through an efficient allocation algorithm executed at the SAC. In what follows, we will concentrate on the methodology that enables us to make optimal space allocations and reservations and the specific algorithm developed for this purpose.

3. DYNAMIC RESOURCE ALLOCATION MODEL

For simplicity, we will employ the term "user" when referring to EVs or their drivers and the term "resource" when referring to charging spaces. We adopt a queueing model for the problem as shown in Fig. 1 (introduced in [Geng and Cassandras (2013)]), where there are N resources and every user arrives randomly and independently to join an infinite-capacity queue (labeled WAIT) and waits to be assigned. At the kth decision point occurring at time t_k, the system makes allocations for all users in both the waiting queue and the queue (labeled RESERVE) of users who have already been assigned and have reserved a resource from a prior decision point. If a user in WAIT is successfully assigned a resource, he joins the RESERVE queue, otherwise he remains in WAIT. A user in RESERVE may be assigned a different resource after a decision point and returns to the same queue until he can physically reach the resource and occupy it. A user leaves the system after occupying a resource for some amount of time at which point the resource becomes free again.

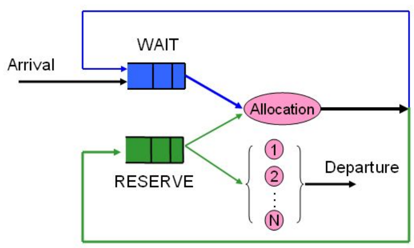

Fig. 1. Queueing Model for Dynamic Resource Allocation

At the kth decision point we define the state of the allocation system, $X(k)$, and the state of the ith user, $S_i(k)$ as explained next. First, we define

$$X(k) = \{W(k), R(k), P(k)\}$$

where $W(k) = \{i : \text{user } i \text{ is in the WAIT queue}\}$, $R(k) = \{i : \text{user } i \text{ is in the RESERVE queue}\}$, and $P(k) = \{p_1(k), ..., p_N(k)\}$ is a set describing the state of the jth resource with $p_j(k)$ denoting the number of free spaces at resource j, $j = 1, \ldots, N$. This allows the possibility that $p_j(k) > 1$ if a resource models a group of spaces, e.g., a whole charging station, rather than an individual space; when each resource is identical to a space, then $p_j(k) \in \{0, 1\}$.

We assume that each resource j has a known location associated to it denoted by $y_j \in Z \subset \mathbb{R}^2$, and its capacity is n_j. We also define

$$S_i(k) = \{z_i(k), r_i(k), q_i(k), \Omega_i(k)\}$$

where $z_i(k) \in Z \subset \mathbb{R}^2$ is the location of user i, $r_i(k) \in \mathbb{R}^+$ is the total time that user i has spent in the RESERVE queue up to the kth decision point ($r_i(k) = 0$ if $i \in W(k)$), and $q_i(k)$ is the reservation status of user i:

$$q_i(k) = \begin{cases} 0 \text{ if } i \in W(k) \\ j \text{ if user } i \text{ is reserving resource } j \end{cases} \tag{1}$$

Finally, $\Omega_i(k)$ is a feasible resource set for user i, i.e., $\Omega_i(k) \subseteq \{1, \ldots, N\}$ depending on the requirements set forth by this user regarding the requested resource. We will define $\Omega_i(k)$ in terms of two *attributes* associated with user i. The first, denoted by D_i, is an upper bound on the distance between the resource that the user is assigned and his current location when making a request. This is a difference between this setting and that of "smart parking" where a user typically indicates his distance from a specific driving destination d_i. Although it is still possible that a user specifies such a criterion instead, we shall concentrate in the sequel on the former case. Thus, let $D_{ij}(k) = \|z_i(k) - y_j\|$ be the distance of the user at the kth decision point from resource j located at $y_j$where $\|\cdot\|$ is a suitable distance metric. Then, the constraint

$$D_{ij}(k) \leq D_i \tag{2}$$

defines a requirement that contributes to the determination of $\Omega_i(k)$ by limiting the set of feasible resources to those that satisfy (2). Alternatively, we can define the traveling time $t_{ij}(k) = f(D_{ij}(k), \omega)$, where ω denotes all random traffic conditions, and $\bar{t}_{ij}(k)$ the expected traveling time. We can then replace (2) by

$$\bar{t}_{ij}(k) \leq T_i \tag{3}$$

where T_i is an upper bound on the expected traveling time between the resource that the user is assigned and his current location when making a request. For simplicity, in what follows we will adopt (2) with the understanding that this can readily be replaced by (3).

The second attribute for user i, denoted by M_i, is an upper bound on the cost this user is willing to tolerate for the benefit of reserving and subsequently using a resource. The actual cost depends on the specific pricing scheme adopted by the allocation system and may include a fee dependent on the total reservation time and subsequently a fee for occupying the resource. Our approach does not depend on the specific pricing scheme used, but we will assume that each user cost is a function of the total reservation time $r_i(k)$ and the traveling time $t_{ij}(k)$ defined above. We use $M_{ij}(r_i(k), t_{ij}(k))$ to denote the total cost for using resource j, evaluated at the kth decision time, and $\bar{M}_{ij}(k)$ the associated expected value.. Comparing $\bar{M}_{ij}(k)$ to M_i, leads to the constraint

$$\bar{M}_{ij}(k) \leq M_i \tag{4}$$

This defines a second requirement that contributes to the determination of $\Omega_i(k)$ by limiting the set of feasible resources to those that satisfy (4). In order to fully specify $\Omega_i(k)$, we further define

$$\Gamma(k) = \{j : p_j(k) > 0,\ j = 1, \ldots, N\}$$

to be the set of free and reserved resources at the kth decision time and set

$$\Omega_i(k) = \{j : \bar{M}_{ij}(k) \leq M_i, D_{ij} \leq D_i,\ j \in \Gamma(k)\} \qquad (5)$$

Note that this set allows the system to allocate to user i any resource $j \in \Omega_i(k)$ which satisfies the user's requirements even if it is currently reserved by another user (i.e., if $p_j(k) = m \neq i$). If user i provides no requirements, then $\Omega_i(k) = \Gamma(k)$. It is worth pointing out that since $\bar{M}_{ij}(k)$ is generally an estimate of the cost a user incurs, it is subject to noise contributed by random traffic events and, therefore, so is the set $\Omega_i(k)$ as defined in (5). This implies that a resource $j \in \Omega_i(k)$ may in fact be such that $j \notin \Omega_i(k+l)$ for some $l > 0$. Indeed, it is possible that $\Omega_i(k) \neq \varnothing$ whereas $\Omega_i(k+l) = \varnothing$. In such cases, a user may perceive as unfair the fact that he is assigned a feasible resource which ultimately becomes infeasible subject to his requirements. We will assume that this happens as a result of uncontrollable random events, in which case the user must re-enter the allocation system with new D_i and M_i requirement parameters.

We can now concentrate on defining an objective function which we will seek to minimize at each decision point by allocating resources to users. We use a weighted sum to define user i's cost function, $J_{ij}(k)$, if he is assigned to resource j, as follows:

$$J_{ij}(k) = \lambda_i \frac{\bar{M}_{ij}(k)}{M_i} + (1 - \lambda_i) \frac{D_{ij}(k)}{D_i} \qquad (6)$$

where $\lambda_i \in [0, 1]$ is a weight that reflects the relative importance assigned by the user between *cost* and *resource quality*. In the case of charging station space allocation, resource quality is measured as the distance (or expected time) between the charging space the user is assigned and his current location.

From the system's point of view, the objective is to make allocations for as many users as possible and, at the same time, to achieve minimum user cost as measured by $J_{ij}(k)$. Define binary control variables:

$$x_{ij} = \begin{cases} 0 \text{ if user } i \text{ is not assigned to resource } j \\ 1 \text{ if user } i \text{ is assigned to resource } j \end{cases} \qquad (7)$$

and define the matrix $X = [x_{ij}]$. We can now formulate the allocation problem **(P)** at the kth decision point as follows:

$$\min_X \sum_{i \in W(k) \cup R(k)} \sum_{j \in \Omega_i(k)} x_{ij} \cdot J_{ij}(k) + \sum_{i \in W(k)} (1 - \sum_{j \in \Omega_i(k)} x_{ij}) \qquad (8)$$

s.t.

$$\sum_{j \in \Omega_i(k)} x_{ij} \leq 1, \quad \forall i \in W(k) \qquad (9)$$

$$\sum_{j \in \Omega_i(k)} x_{ij} = 1, \quad \forall i \in R(k) \qquad (10)$$

$$\sum_{i \in W(k) \cup R(k)} x_{ij} \leq p_j(k), \quad \forall j \in \Gamma(k) \qquad (11)$$

$$\sum_{j \in \Omega_i(k)} x_{ij} \cdot J_{ij}(k) \leq J_{iq_i(k-1)}(k), \quad \forall i \in R(k) \qquad (12)$$

$$\left[\sum_{n \in \Omega_i(k)} x_{in}\right] - x_{mj} \geq 0,\ \forall i, j, m \text{ s.t.} j \in \Gamma(k),\ j \in \Omega_i(k),\ m \in W(k),\ t_{mj}(k) > t_{ij}(k) \qquad (13)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in W(k) \cup R(k),\ \ j \in \Gamma(k) \qquad (14)$$

In this problem, the objective function focuses on user satisfaction. One can formulate alternative versions that incorporate system-centric objectives such as maximizing resource utilization or total revenue without affecting the essence of our approach. If the system fails to allocate a resource to some user i, i.e., $\sum_{j \in \Omega_i(k)} x_{ij} = 0$, a cost of 1 is added to the objective function. Therefore, the added term $\sum_{i \in W(k)}(1 - \sum_{j \in \Omega_i(k)} x_{ij})$ in (8) is the total cost contributed by the number of "unsatisfied" users. Since by its definition in (6) $J_{ij}(k) \leq 1$, the added cost of value 1 is sufficiently large to ensure that a user is assigned to a resource if there are free qualified resources left.

The constraints (9) indicate that any user in the WAIT queue may be assigned at most one resource but may also fail to get an assignment. On the other hand, (10) still guarantees that each user in the RESERVE queue maintains a resource assignment. The capacity constraints (11) ensure that every resource is occupied by no more than $p_j(k)$ users. The constraints (12) add a unique feature to our problem by guaranteeing that every user in the RESERVE queue is assigned a resource which is no worse than the one most recently reserved, i.e., $q_i(k-1)$. Together (10) and (12) ensure a reservation guarantee and improvement.

The final constraint (13) imposes a fairness requirement: as we can see from (9) and (10), a solution of **(P)** gives a higher assignment priority to users in the RESERVE queue. This is because these users are already incurring a positive cost (recall that the pricing scheme we assume does not impose a fee to unassigned users, i.e., users still in the WAIT queue). On the other hand, (9) makes no distinction among waiting users, regardless of where they are located. This introduces unfairness among waiting users. For example, a waiting user may be located right next to an available resource which, however, is assigned to another waiting user at a considerably larger distance from it. To explain (13), consider a resource j which is available for assignment (i.e., $j \in \Gamma(k)$) and qualified for user i (i.e., $j \in \Omega_i(k)$). If i fails to be allocated any resource, we have $\sum_{n \in \Omega_i(k)} x_{in} = 0$ and (13) requires that $x_{mj} = 0$, i.e., any other waiting user m located farther away from j than user i (i.e., $t_{mj}(k) > t_{ij}(k)$) is forbidden from being assigned to j. If, on the other hand, $\sum_{n \in \Omega_i(k)} x_{in} = 1$, i.e., user i is assigned some resource, then $x_{mj} \leq 1$, i.e., there is no constraint on allocating resource j to any user m as long as condition (11) is satisfied. We also note that there is no fairness issue related to users in the WAIT queue in terms of how long they have resided in it since this does not affect the cost objective unless a user is in the vicinity of his/her destination, a situation that we handle through the wandering ratio metric defined later in (15). Finally, if the traveling times t_{ij} are random, we replace $t_{mj}(k) > t_{ij}(k)$

by the corresponding expectations, i.e., $\bar{t}_{mj}(k) > \bar{t}_{ij}(k)$, as in (3).

Problem **(P)** is a Mixed-Integer Linear Programming (MILP) problem Bertsimas and Tsitsiklis (1997) that can be solved using any of several commercially available software packages (e.g., ILOG CPLEX). In this formulation, we can easily prove that the problem is always feasible. Indeed, letting the matrix $X^* \equiv [x^*_{ij}]$ denote a solution of (8), then the set

$$\{X^* : \sum_{j \in \Omega_i(k)} x^*_{ij} = 0,\ x^*_{mq_m(k)} = 1,\ i \in W(k),\ m \in R(k)\}$$

is always a feasible solution, since it implies that all users in $W(k)$ are not allocated and all users in $R(k)$ simply maintain their previous reservation (assuming that $R(k) \neq \varnothing$).

Regarding the computational complexity of obtaining a solution of **(P)**, the problem is obviously NP-hard and if the proposed dynamic resource allocation system is deployed in a large urban area the number of variables and constraints may become extremely large. Obtaining a solution at each decision point becomes time-consuming and during this time the system state changes and the solution may no longer be optimal. In such cases, there are several simple steps one can adopt to reduce the complexity of the problem as discussed in [Geng and Cassandras (2013)]. Briefly, one can partition the geographic area of interest into regions and solve problem **(P)** for user requests pertaining to each such region. One can also group resources which are located in close proximity, such as a single charging station with several spaces; in this case, we treat the station as a single resource with p_j denoting the number of vacant spaces. The system may randomly pick a vacant spot for the user upon arrival. Finally, we can adopt a "user discrimination" approach by restricting the number of users in the waiting queue who are assigned a resource according to certain criteria such as the urgency for recharging.

An additional issue regarding problem **(P)** concerns the choice of decision points over time, or, equivalently, defining appropriate "decision intervals" $\tau(k)$, $k = 1, 2, \ldots$ Similar to the parking problem solution in [Geng and Cassandras (2013)], our approach is to follow a time-driven strategy for decision making. After the $(k-1)$th decision point, the system waits for some duration $\tau(k)$ and then makes a new allocation over all users that arrived during $\tau(k)$ and all previous users residing in either the WAIT or RESERVE queue. Clearly there is a tradeoff: a large $\tau(k)$ may eventually yield a lower cost for all users involved, but it also forces a large number of users to remain in the WAIT queue with no assignment, until it is either too late because a user has reached his destination or has lost patience and searches for resources by himself. This tradeoff was empirically explored in [Geng and Cassandras (2011)] for the closely related parking problem by varying $\tau(k)$ on the performance of the system. For simplicity, in practice we usually adopt a fixed interval τ between decision points.

A final issue, not directly addressed in problem **(P)**, arises when users in the waiting queue who are close to their intended destination reach it before having an opportunity to be assigned a charging space. To deal with this effect, we adopt the following *Immediate Allocation* (IA) policy. Let d_i be the user i destination which we will assume the user specifies even though, as mentioned earlier, this may not be used as part of the objective function (6). The purpose of this information is to assess if a user in the WAIT queue is approaching a critical point by which a charging space allocation needs to be made (if possible). Thus, whenever user i is in the WAIT queue and reaches a location z_i such that $\|z_i - d_i\| \leq v_i\tau$, the user is placed in an "immediate allocation" queue. Here τ is the interval between allocation decisions made by the system (as discussed above) and v_i is the average driving speed. If this queue is not empty, then, as soon as a user departure makes a resource available, the system immediately prioritizes user i over other users in $W(k)$ and assigns him this resource if it is feasible. This "immediate allocation" problem is easy to solve. We define an "urgent" user set

$$I(k) = \{i : i \in W(k),\ \|z_i - d_i\| \leq v_i\tau\}$$

and, as soon as a resource j becomes free, we allocate it to user i such that $J_{ij} = min_{n \in I(k), j \in \Omega_n(k)} J_{nj}$, if such i exists.

3.1 Performance Metrics

In solving problem **(P)** we aim to minimize user costs as defined by (6) at each decision point. In order to assess the overall system performance over some time interval $[0, T]$, we define several appropriate metrics evaluated over a total number of users N_T served over this interval (e.g., a simulation run length).

From the system's point of view, we consider resource *utilization* as a performance metric and break it down into two parts as in [Geng and Cassandras (2013)]: $u_r(T)$ is the utilization of resources by reservation (i.e., the fraction of resources that are reserved) and $u_p(T)$ is the utilization by occupancy (i.e., the fraction of resources that are physically occupied by a user).

From the users' point of view, we first define a satisfaction metric for those users that actually occupy a resource. Let $P(T)$ be the set of such users over $[0, T]$. Moreover, returning to (6), let $q^*_i \in \{1, \ldots, N\}$ be the resource ultimately assigned to user $i \in P(T)$. We then define

$$J_{iq^*_i} = \lambda_i \frac{\bar{M}_{iq^*_i}}{M_i} + (1 - \lambda_i)\frac{D_{iq^*_i}}{D_i}$$

and

$$\bar{J}(T) = \frac{1}{|P(T)|} \sum_{i \in P(T)} J_{iq^*_i}$$

measuring the *average cost* of users served. We point out that, unlike traditional queueing problems, waiting times are not a measure of user satisfaction, since users do not actually need a resource until they have physically reached it. Instead, another metric we will use is the *wandering ratio* $w(T)$ defined as follows. Let

$$A_W(k) = \{i : i \in W(k),\ \|z_i(k) - d_i\| \leq \epsilon\}$$

be the set of users who reach a destination but are still in the WAIT queue at the kth decision point, where $\epsilon \geq 0$ is a small real number used to indicate that a user is in the immediate vicinity of his destination d_i. Letting k_T denote the last decision point within the time interval of length T, we then define the fraction of unsatisfied users

$$w(T) = \frac{|A_W(k_T)|}{N_T} \tag{15}$$

Finally, we consider the *average time to a charging space* $t_c(T)$, which is the time from the instant a user sends a request to the instant he physically occupies a charging space.

4. SIMULATION RESULTS

In this section, we seek to quantify the improvement resulting from our approach over an uncontrolled setting where users find charging stations without any guidance (NG) and with guidance (G). If there is guidance, users know exactly the location of free resources; otherwise, they always search for the closest free resource by themselves. We present results for a simulated small business district map shown in [Geng and Cassandras (2011)]. In the map, there are 30 charging stations and 5 destinations. In all simulations, user arrival times are Poisson distributed with rate λ, and uniformly located in the map. The user cost parameter M_i is uniformly distributed in the interval $[M_{min}, M_{max}]$, and the distance parameter D_i is also uniformly distributed in $[D_{min}, D_{max}]$. The resource occupancy time is exponentially distributed with rate μ.

We also adopt the same pricing scheme as in [Geng and Cassandras (2011)]: $M_{ij}(k) = e^{\alpha(r_i(k)+t_{ij}(k))} + CT_i$ where α is a positive constant, $r_i(k)$ is the time already spent at the RESERVE queue, $t_{ij}(k)$ is an estimate of the driving time for i to reach j, and T_i is the expected charging time of user i.

The distance cost is defined as $D_{ij} = \beta d_{ij}$ where β is a positive constant and d_{ij} measures the distance from resource j to user i's location.

In all simulations, we set $1/\mu = 220$ (time units), $M_{min} = 0$, $D_{min} = 0$, $M_{max} = 100$, $D_{max} = 200$, $\alpha = 0.025$, $\beta = 1$ and $C = 1$. We use a constant decision interval $\tau(k) = \tau$, $k = 1, 2, \ldots$ We set $\tau = 15$ since it was empirically found to produce better allocation results. Each simulation lasts for $T = 20000$, which is long enough for the simulation results to converge. We seek to compare results under different traffic intensities by changing the value of the interarrival interval $1/\lambda$. Our results are shown in Fig. 2 for fhe four performance metrics defined earlier. We find that as the traffic intensity increases, the improvement offered by our approach becomes more significant. For example, we see that charging space utilization may increase by 14% compared to a guidance-based system, the time to a charging space can be reduced by 9.5%, and the wandering ratio can be reduced by 30%.

5. CONCLUSIONS AND FUTURE WORK

We have proposed a system for optimal dynamic allocation of spaces for EVs at charging stations, including reserving these spaces. The system exploits technologies for space availability detection and for EV localization. It also allocates charging station spaces to users instead of only supplying guidance to them. We have focused on determining an efficient and optimal allocation strategy

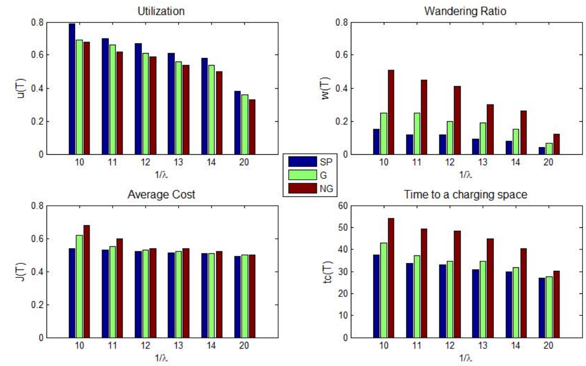

Fig. 2. Performance Metrics Under Different Traffic Intensities

for both users and the system by solving a sequence of MILP problems which are guaranteed to have a feasible solution and to satisfy some fairness constraints.

Current research focuses on selecting proper decision intervals and on the use of pricing control to adjust space prices for different classes of users or other bidding-type mechanisms that can enhance fairness.

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