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# Numerical optimal control as a method to evaluate the benefit of thermal management in hybrid electric vehicles

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**Abstract:** In this paper, a numerical solution of the optimal thermal management problem for a parallel hybrid electric vehicle is presented by taking into account the engine temperature. This temperature influences the fuel consumption and has a dynamical behavior which is controlled by the engine torque. Simulation results are presented to evaluate the benefit of adding this new state variable to the optimization problem, by comparison to the simplified problem where only the State Of Charge (SOC) is taken into account.

Keywords: Energy management system, parallel hybrid electric vehicle, thermal management, Pontryagin's Minimum Principle.

## 1. INTRODUCTION

Spurred by environmental requirements, economic factors and energy-saving interests, energy management systems (EMS) or supervisory control of hybrid electric vehicles (HEV) has attracted much attention from the scientific community in the last ten years (Chasse and Sciarretta [2011], Kessels et al. [2008], Sciarretta et al. [2004], Sciarretta and Guzzella [2007]). The task of such controllers is to coordinate the components of the power-train in an efficient manner. Their objectives are to satisfy the power request from the driver while reducing the overall energy use and emissions.

Common EMS are based on heuristics inspired by well established driving best practices (Chen and Salman [2005], Delprat et al. [2003], Michel et al. [2012], Gaoua et al. [2013]). On the other hand, optimized strategies have been studied. They are based on the definition of a cost function to be minimized for a dynamical system representing the vehicle. Usually, the fuel consumption or engine emissions, or a combination of both over a fixed time window corresponding to a given driving cycle is the objective to minimize while the main constraints bear on battery charge-sustaining operation (Rousseau et al. [2007], Rousseau et al. [2008], Michel et al. [2012]). In numerous studies, one frequent hidden assumption is that HEV system is under thermal equilibrium. However, in practice thermal transients are not negligible for several reason: i) the engine is subject to stop-start phases, ii)engine temperature impacts emission and fuel consumption rates, and the efficiency of after treatments systems is relatively poor at low catalyst temperatures. Few research programs have recently been conducted with the aim of including thermal states in the optimization problem formulation : there are referred to as thermal management system (Lescot et al. [2010], Merz et al. [2012], Serrao et al. [2011b]). The study in (Lescot et al. [2010]) represents a first attempt to include engine temperature dynamic in the optimization problem and to quantify the corresponding gain in fuel consumption. In (Merz et al. [2012]), a more general framework for Pontryagin Minimum Principle (PMP) based optimization including thermal dynamics was presented with a numerical comparison to quantify the benefit in fuel consumption and pollutant emissions. This paper follows this approach and considers one representative problem of EMS for a parallel hybrid electric vehicle. In a deterministic context, and assuming the model of the vehicle and the driving conditions are known, we address the following question: determine the optimal policy for the torque split between the engine and the electric motor.

This general question is studied from the viewpoint of model complexity. In this paper, we wish to select the right level of modeling to optimize the accuracy/complexity trade-off. A main motivation is that the number of state variables greatly impacts the resolution numerical methods. Considering additional state variable increases the level of complexity and the computational burden. It may also jeopardize the robustness of numerical methods employed to compute the optimal trajectories.

This observation holds for dynamic programming, and methods using PMP or direct formulations (e.g. collocation methods) (Hargraves and Paris [1987]). In this article, a special focus is on the quantification of the gain in fuel consumption of including engine temperature in the optimization for a parallel hybrid electric vehicle. The resulting improvement (reduction) is an upper bound on what can be achieved by any real time implementable method.

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The paper is organized as follows: in section 2, a mathematical control-oriented model which takes into account the influence of engine temperature on fuel consumption is presented. On this basis, a PMP solution is derived in section 3 to include the new state (engine temperature) through the introduction of a second adjoint state besides that adjoint to the battery State Of Charge (SOC). In section 4, the numerical method is presented. Numerical results are presented in section 5. In light of these findings, we reach a conclusion on the relevance of thermal management approach in terms of minimizing fuel consumption.

## 2. MATHEMATICAL CONTROL MODEL

The system considered here is a parallel hybrid-electric vehicle (HEV). We assume that the vehicle follows a prescribed driving cycle and we neglect all fast dynamics taking place in the power-train.

The cost function to be minimized is the fuel consumption over a fixed time window corresponding to a given driving cycle of duration $T$. The cost is given by:

$$J(u) = \int_0^T c(u,t)e(\theta)dt \tag{1}$$

where $u$ is the control variable (the engine torque), $\theta$ is the engine temperature state and $c(u,t)$ is the fuel consumption rate when the engine is warm. It is given by a quasi-steady map (as a function of the engine speed and the engine torque), which is derived from engine tests. The time variable accounts for the dependence of the consumption on the engine speed, which is a set path assumed to be tracked.

$e(\theta)$ is the correction factor of fuel consumption with respect to $\theta$. The correction factor is a decreasing function and is always greater or equal to one. It represents an extra-consumption factor that takes into account the increase of friction and, as a consequence, the increase of fuel fuel injected per cycle at low temperatures. In the case of warm engine, $e(\theta) = 1$ for all $\theta$.

The control $u$ is constrained to belong to $U^{ad} \subset L^\infty[0,T]$ defined by

$$u_{min}(t) \leq u(t) \leq u_{max}(t), \quad a.e. \quad t \in [0,T] \tag{2}$$

where the bounds are determined by the driving conditions, and physical limitations of the engine and the electric motor.

Two dynamics are taken into account in the problem formulation.

The SOC $\xi$ is governed by :

$$\frac{d\xi}{dt} = f(u,t), \quad \xi(0) = \xi_0 \tag{3}$$

where $f$ is a non linear function of its arguments. For more details, one can refers to (Serrao et al. [2011a]). One operational constraint for charge-sustaining HEVs requires that the final value of $\xi$ should be equal to its initial value

$$\xi(T) = \xi(0) \tag{4}$$

The final condition (4) is justified as a way to compare the results of different solutions by guaranteeing that they reach the same level of battery energy. In real vehicles, there is no need to have a fixed battery SOC at the end of each cycle.

Strictly speaking, the function $f$ in (3) also depends on $\xi$, but we neglect this dependency as is commonly done in the literature (Serrao et al. [2011a]).

The engine temperature satisfies the following first order non-linear differential equation:

$$C_e \frac{d\theta}{dt} = P_{th,e}(u,t,\theta) - G_e(\theta - \theta_0) - P_{th,aux} \tag{5}$$

where $C_e$ is an equivalent thermal capacity, $G_e$ is an equivalent thermal conductivity, $\theta_0$ is the ambient temperature, $P_{th,e}$ is the sum of friction power dissipated into heat and thermal power transferred from the engine to the coolant, given by a look-up table, and $P_{th,aux}$ is the thermal power drained by the cabin heater (It is considered constant). In what follows, we shall write (5) and consider the initial condition $\theta_0$ as

$$\frac{d\theta}{dt} = g(u,t,\theta), \quad \theta(0) = \theta_0 \tag{6}$$

One can see that equation (6) may be ignored if $e$ in (1) does not depend on $\theta$, because (3) does not depend on $\theta$.

More generally, the optimal control problem could include some instantaneous constraints on the state variables $\xi$ and $\theta$, but this would lead to a more complicated numerical solving. Moreover, it turns out that the optimal solution do not lead to violation of these state constraints on $\xi$ that we might impose. On the other hand, the constraints on $\theta$ are not important here, because the cost function will be independent from $\theta$ when $\theta \geq \theta_w$ which is generally an admissible value. Therefore, the state constraints are omitted here.

We can now define our optimal control problem (OCP)

$$(OCP) \quad \min_{u \in U^{ad}} J(u) \tag{7}$$

under the final constraint (4) and the dynamics (3,6).

## 3. MATHEMATICAL SOLVING

The OCP defined in (7) can be solved by numerous methods. Classically, the solution considered here is based on Pontryagin Minimum Principal (PMP) (Pontryagin et al. [1962]). We define the Hamiltonian $H$ by

$$H(u,t,\theta,\lambda,\mu) = c(u,t)e(\theta) + \lambda f(u,t) + \mu g(u,t,\theta), \tag{8}$$

where $\lambda$ and $\mu$ are the adjoint variables associated to $\xi$ and $\theta$, respectively.

For a given control $u^*$, the adjoint states $\lambda(t)$ and $\mu(t)$ are defined by

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial \xi} = 0 \tag{9}$$

$$\frac{d\mu}{dt} = -\frac{\partial H}{\partial \theta} = -c(u^*,t)\frac{\partial e}{\partial \theta} - \mu \frac{\partial g(u^*,t,\theta)}{\partial \theta} \tag{10}$$

with

$$\mu(T) = 0 \tag{11}$$

since the final temperature is free and the final time $T$ is fixed. On the other hand, we have no boundary condition on $\lambda(T)$ since the final SOC is constrained.

The PMP states that, if $u^*$ is an optimal control, then, for every $t$, $u^*(t)$ minimizes the Hamiltonian in the set defined by (2) along the optimal states and corresponding costates:

$$u^*(t) \in \arg\min_{u \in U^{ad}} H(u, t, \theta(t), \lambda(t), \mu(t)) \qquad (12)$$

Equations (3,4,9,10,11,12) constitute a two-point boundary value problem (TPBVP) that we shall denote by $(\Im)$, where the final condition $\lambda(T)$ is unknown. We can make two preliminary observations:

- from (9) we know that $\lambda$ is a constant, which is equal to one of the unknowns of the TPBVP.
- if $e$ does not depend on $\theta$, then clearly $\mu \equiv 0$ is a solution of (10,11). Under this assumption, the Hamiltonian condition on the optimal control becomes
$$u_1^*(t) \in \arg\min_{u \in U^{ad}} H_1(u, t, \lambda) \qquad (13)$$
with
$$H_1(u, t, \lambda) = c(u, t) + \lambda f(u, t) \qquad (14)$$

This second remark is quite intuitive: if the cost does not depend on the engine temperature, and if the dynamics of the SOC and its final value do not depend on this temperature either, then the temperature can be simply ignored and the OCP is reduced to an optimal management of the SOC. In this latter case, the TPBVP is reduced to (3,4,9,13), where the unknown boundary condition is the (final) value of the constant function $\lambda$. We shall denote this TPBVP by $(\Im_1)$.

By using the Minimum Principle, we have shifted our perspective: instead of trying to solve an OCP, we instead try to solve a two point boundary value problem. The original aim of this paper, as stated in the introduction, can be reformulated as follows:

- compute a good solution candidate [1] $u^*$ for the optimal control problem (7) by solving the TPBVP $(\Im)$
- compute an optimal control $u_1^*$ for the simplified problem where $e$ is a constant (typically $e \equiv 1$) by solving the TPBVP $(\Im_1)$
- compare $J(u^*)$ and $J(u_1^*)$. If $u^*$ is indeed a solution of (7), then we have $J(u^*) \leq J(u_1^*)$ because $u_1^* \in U^{ad}$. The question that we will study numerically on a few examples is: how great is the difference between the two costs? If it is small, this justifies further (including theoretical) studies of the use of the solution of the optimal SOC management for controlling a HEV.

## 4. NUMERICAL SOLVING

To assess the potential gain due to considering the thermal transients explicitly in the Hamiltonian, an offline optimization is performed to solve the TPBVP $(\Im)$. We assume that the driving cycle is known a priori.

Many numerical methods can be used. Among these are dynamic programming, direct methods (e.g. collocation ), and indirect methods based on PMP.

Dynamic programming is a very efficient method when applied to systems of low dimensions. Its main advantage is that is provides a feedback solution. On the other hand, its computational burden is relatively high. In our case, it leads to numerical difficulties and memory managing issues appear, due to the grid size for the state variables.

A critical aspect of the problem under consideration here is the discontinuity of the cost function (the instantaneous fuel consumption) about zero torque demand. Physically, this discontinuity models that the engine is turned-off when the torque demand is null. It discards other possible methods requiring continuous variations with respect to the decision variables e.g. Matlab routine `fmincon`. Interestingly, PMP based methods do not require the continuity with respect to the control variable and are thus not impacted.

As the PMP based methods are not impacted, one can employ collocation which is a popular method for solving TPBVP. It is implemented in Matlab through the routines `bvp4c` and `bvp5c`. It appears that solving problem $(\Im)$ with this routines leads to numerical instabilities that we suspect to originate in the lack of smoothness of some parts of the data.

We have thus chosen to use a specifically tailored shooting-related method.

Classically, the idea of this algorithm is to consider the initial conditions of the adjoint states $(\lambda_0, \mu_0)$ as unknown variables and the vector function which associates $[\xi(T) - \xi(0)]$ and $\mu(T)$ to $(\lambda_0, \mu_0)$, where $(\xi, \theta, \lambda, \mu)$ are the solution of the following system on $[0, T]$

$$\begin{cases} \dfrac{d\xi}{dt} = f(u^*, t), \quad \xi(0) = \xi_0 \\ \dfrac{d\theta}{dt} = g(u^*, t, \theta), \quad \theta(0) = \theta_0 \\ \dfrac{d\lambda}{dt} = 0, \quad \lambda(0) = \lambda_0 \\ \dfrac{d\mu}{dt} = -c(u^*, t)\dfrac{\partial e}{\partial \theta} - \mu \dfrac{\partial g(u^*, t, \theta)}{\partial \theta}, \quad \mu(0) = \mu_0 \end{cases} \qquad (15)$$

where $u^*$ is defined by (12). In our case and due to the absence of analytical expressions of the cost function and some variables which are involved in the dynamics considered (3, 6), $u^*$ is determined as follow : $u$ is discretized in a finite number of possible values (a grid) satisfying the condition (2), the Hamiltonian $H$, defined by (8) is evaluated for each value and the one minimizing $H$ is taken as an optimal value for $u$. From theoretical viewpoint, this approach is consistent with the fact that the PMP is applicable in the case of discrete input control.

Then, the problem is recast into finding zeros of this function from $R^2$ into $R^2$. This is achieved using a Newton's method implemented in the popular `fsolve` Matlab function. The differential equations (15) are solved using a first order Euler method with a time step of 1 second.

Implementing such method requires special care in our situation. In particular, the discrete nature of the set into which the control is sought after makes it quite risky to rely on the automatic finite difference schemes that usually reveal very handy in others situation. If the finite difference parameter is set too small, the estimated derivatives are simply zero as no change is visible in the function. If it is set too high, then the estimate is biased by second order

[1] Note that the Minimum Principle is only a necessary condition.

terms. For these reasons, we employ the finite difference parameter as a tuning variable and we perform our own finite difference estimate of the gradient.

## 5. NUMERICAL RESULTS

The simulation results are obtained for a parallel hybrid electric vehicle (1932 kg curb weight) equipped with one electric motor and a 5 Ah Li-ion battery. Figure 1 describes $c(u,t)$, the fuel consumption map when the engine is warm. Figure 2 describes the electric power map of the motor.

The engine parameters, which have been identified from experimental data, are listed in table 1.

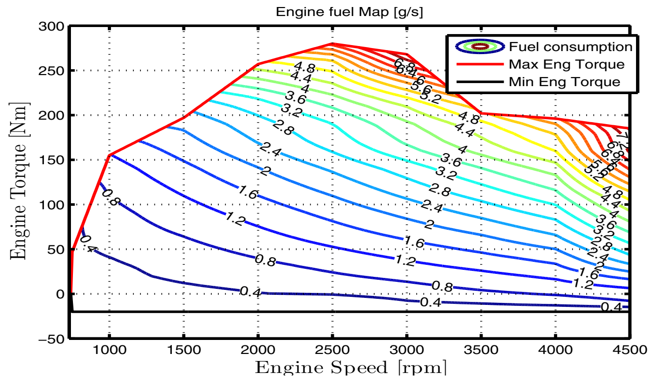


Fig. 1. Engine Fuel map $c(u,t)$ [g/s]

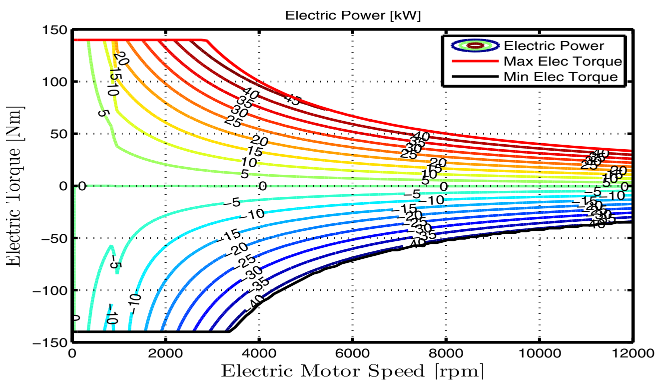


Fig. 2. Electric Power map [kW]

The correction factor $e(\theta)$ for our system can be approximated by

$$e(\theta) = \begin{cases} -a\theta + b, \ \theta_c \leq \theta \leq \theta_w \\ 1, \ \theta > \theta_w \end{cases} \tag{16}$$

where $a$ and $b$ are positive constants, which have been identified from experimental data.

Tables 2, 3 and 4 summarize the results obtained by solving the previous optimal control problems described in this paper for three cycles : NEDC, FUDS and FHDS in term of fuel consumption (Table 2), satisfaction of the final constraint (Table 3) and value of the costate associated to the SOC (Table 4).

Table 1: Engine Parameters

| Parameter | Value |
|---|---|
| a | 0.0084 $[°C]^{-1}$ |
| b | 1.59 |
| $C_e$ | $10^5$ J/kg |
| $G_e$ | 14.3 $s^{-1}$ |
| $\theta_w$ | 70 °C |
| $\theta_c$ | -30 °C |

In what follows, the $S_i$, $i = 0 \ldots 2$, refer to a configuration defined by a TPBVP (which yields a control) and a fuel consumption formula (which yields a corresponding cost). $S_0$ refers to the solution of the TPBVP $(\Im_1)$ where $e(\theta)$ is constant, $e \equiv 1$; we denote the corresponding cost by $J_0$. We denote by $u_1^*$ the resulting control. $S_1$ results of the application of the control $u_1^*$ to the augmented model (5) (which includes thermal dynamics) and the cost $J$ which is influenced by the engine temperature. The obtained value can be fairly compared to the solution where the temperature is accounted for. $S$ corresponds to the solution of the TPBVP $(\Im)$ and to the cost $J$, we note $u^*$ its solution . Following (Merz et al. [2012]), $S_2$ is a pseudo solution of the TPBVP $(\Im)$ where we impose $\mu(t) = 0, \ \ \forall t$ in the Hamiltonian, and where the cost is $J$.

Table 2: Fuel consumption in **[L/100km]**.

| Cycle | $S_0$ | $S_1$ | $S_2$ | $S$ |
|---|---|---|---|---|
| NEDC | 4.25 | 4.80 | 5.20 | 4.79 |
| FUDS | 3.94 | 4.49 | 5.4 | 4.46 |
| FHDS | 4.98 | 5.34 | 5.71 | 5.32 |

Table 3: Constraint violation of the SOC $[\xi(T) - \xi(0)] \times 100$.

| Cycle | $S_{0,1}$ | $S_2$ | $S$ |
|---|---|---|---|
| NEDC | -0.027 | 1.1 | 0.095 |
| FUDS | 0.096 | 1 | 0.8 |
| FHDS | 0.091 | 0.08 | -0.1 |

Table 4: Constant adjoint state $\lambda$ associated to the SOC.

| Cycle | $S_{0,1}$ | $S_2$ | $S$ |
|---|---|---|---|
| NEDC | -0.4812 | -0.6066 | -0.4814 |
| FUDS | -0.5156 | -0.6174 | -0.5221 |
| FHDS | -0.4928 | -0.5294 | -0.4929 |

The optimal trajectories for the SOC, the temperature $\theta$ and the adjoint state $\lambda_2$ for the NEDC cycle are given by the Figures 3, 4 and 5 respectively. The indefinite integral of the instantaneous fuel consumption is given by Figure 6.

From Table 2, one can observe that the solution $S_2$ is far from the solutions $S_1$ and $S$ which are very close in term of fuel consumption.

From Figure 3 and Table 3, one can see that the final State Of Charge constraint is satisfied with an error below 1% for the NEDC cycle by using the control calculated in $S_1$ and $S$. This is also the case for the two other cycles under consideration with a maximum error bellow 0.8% for the FUDS cycle and less than 0.1% for the FHDS cycle.

From Table 4, one can see that the values of the adjoint state $\lambda$ for the two solution $S_1$ and $S$ are very close except for the FUDS cycle where the gap between the two values is more important. This is justified by the fact that the

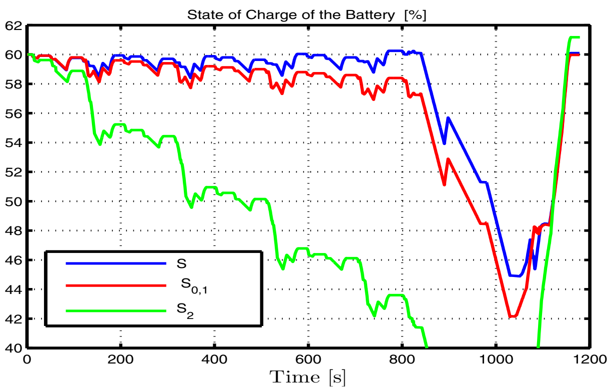


Fig. 3. State Of Charge of the battery SOC trajectories for NEDC driving cycle. For $S_1$, the SOC is the same as for $S_0$ because the control is the same and the SOC dynamics do not depend on the temperature.

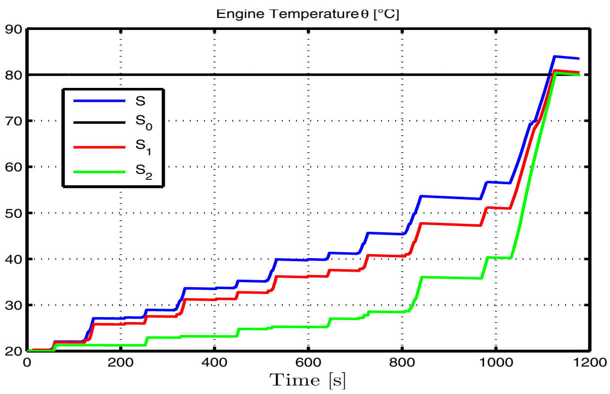


Fig. 4. Optimal Engine Temperature $\theta$ trajectories for NEDC driving cycle

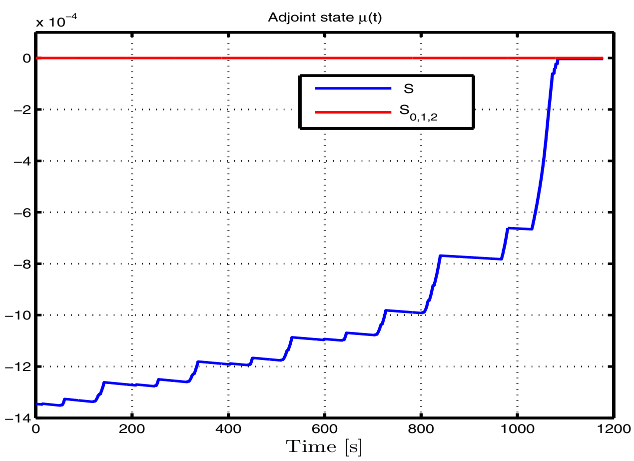


Fig. 5. Adjoint state variable $\mu$ for NEDC driving cycle

difference between the final SOC and its initial value for this cycle is more important than in the other cases.

The optimal temperature trajectory in Figure 4 shows that the optimal control of the solution $S$ improves the engine

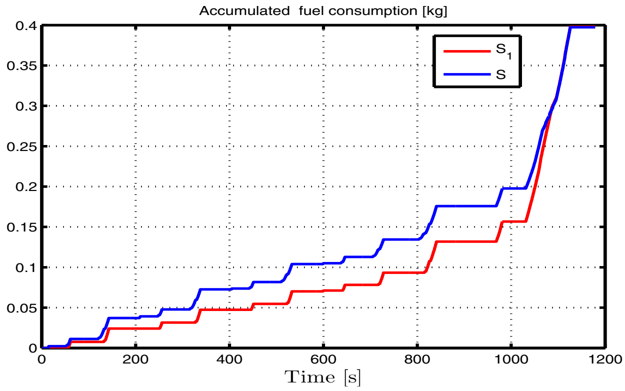


Fig. 6. Accumulated fuel consumption for NEDC driving cycle

efficiency by warming up the engine in the first phase of the driving cycle (corresponding to the time interval $[0, 800s]$). The price to pay for achieving this is an increased fuel consumption, as seen in Figure 6. In the second phase, the engine is warmer and more efficient and the fuel consumption obtained by using the control $u^*$ is less than when using the control $u_1^*$. This means that the control $u^*$ anticipates the influence of the engine temperature on the fuel consumption. However, the control cannot fully exploit the temperature dependent efficiency improvement of the engine. This is because it has to meet the final constraint $\xi(T) = \xi(0)$. Globally, this might explain why the thermal management does not significantly improve the fuel consumption in comparison with simply using the control $u_1^*$ obtained by solving the problem $(\Im_1)$ which ignores the influence of the engine temperature.

Finally, the adjoint state $\mu$ in Figure 5 becomes null when the engine is warm $(\theta \geq \theta_w)$. This is due to the thermal engine efficiency and fuel consumption being independent from $\theta$. This reveal the consistency of the numerical method.

## 6. CONCLUSION

The engine temperature is known to be one of the main factors influencing fuel consumption. The first intuition that we have is that, by considering the temperature in the optimization problem, we will reduce the fuel consumption. The numerical results we have obtained for several cycles (NEDC, FUDS, FHDS), with significantly different profiles, indicate that the gain is very small. These results suggest that, perhaps, it may be sufficient to use a temperature-free model to compute the optimal torque, even if there is an influence of the temperature on the fuel consumption. This conclusion is greatly influenced by the scenario under consideration which generates implicit upper bound on consumption saving. We are working on identifying these performance bottleneck. On the other hand, a general theoretical study of the influence of the model parameters on the presented results would prove under which general assumptions, the optimal control of $(\Im_1)$ is enough to perform thermal management.

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