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Boundary Control of an Industrial Tubular Photobioreactor Using Sliding Mode Control

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Abstract: In this paper, the pH boundary control problem of a tubular photobioreactor is treated. The method of characteristics is used to transform the hyperbolic system of equations into a set of first order ordinary differential equations without approximation. The proposed approach is geometric in nature. The method is shown to be effective in controlling pH concentration of a tubular photobioreactor through simulations under real data disturbances.

Keywords: Distributed Sliding Mode Control; Nonlinear Control; Sliding Mode Control; Method of Characteristics; Microalgae; Photobioreactor

1. INTRODUCTION

Distributed Parameter Systems (DPSs) represent many processes in engineering when more than one independent variable exists. Under such circumstances the governing equations will contain temporally dependent terms as well spatial gradients. Examples can be found in many fields ranging from chemical and metallurgical processes to physiological systems and ground water modelling. Although the study of DPSs has originated more than 50 years ago [Tzafestas and Stavroulakis, 1982], a definitive control methodology is still under research for the general class of nonlinear DPSs.

On the other hand, the theory of Variable Structure Systems (VSS), and their associated sliding regimes, has been extensively developed to lumped systems during the last 60 years. The study of VSS for DPSs is not much developed. The reason seems to be twofold. First, there is no general theory on Partial Differential Equations (PDEs) with discontinuous right hand side. Secondly, application of VSS techniques to DPSs must exploit boundary conditions in order to develop the control law. Once the boundary conditions vary, the designed sliding mode may not always exist.

Most available results in literature are based on approximating the DPS by a finite dimensional lumped parameter system, as can be seen in [Boubaker and Babary, 2003]. But discretization and order reduction often may result in loss of relevant dynamics and always raises the question on the discretization efficiency.

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The approach shown in [Godasi et al., 2002], utilizes symmetry group theory and allows controller design for a general nonlinear DPS. However no rigorous proofs are offered since the solution of sets of arbitrary PDEs, via group theory, still remains an open area of research in mathematics.

The complete theory just can be found for Nonlinear First Order PDEs (NFOPDEs) by the Method of Characteristics (MoC). Applying this method, the NFOPDEs are transformed into a finite set of characteristic Ordinary Differential Equations (ODEs) which, along with their Cauchy data (initial conditions), exactly describe the original PDE. Thus, control design may be subsequently performed on a set of nonlinear ODEs in place of the NFOPDEs without approximation.

The use of Sliding Mode Control (SMC) design for DPSs based on the MoC was firstly proposed by [Sira-Ramirez, 1989, 1990] and, subsequently, developed in [Hanczyc and Palazoglu, 1994, 1995]. The MoC is used to exploit features of the flows associated to the characteristic direction field of the closed-loop system. Thus, the fundamental properties and the characterization of sliding regimes can be proven by means of a geometric approach.

The theory proposed in [Sira-Ramirez, 1989, 1990] and [Hanczyc and Palazoglu, 1994, 1995] is used in this paper to develop a control law for the boundary control of pH concentrations of a tubular photobioreactor. Due to the actuator characteristics, the control law can be implemented with the discontinuous control law or with a continuous control law.

The contributions of this paper are twofold. On one hand, a nonlinear control law to pH control of a tubular photobioreactor is developed. On the other hand, it is shown an effective control approach for SISO (Single-

Input Single-Output) NFOPDEs using the exact solution obtained by the MoC.

The paper is organized as follows: Section 2 presents the background material of the MoC. Section 3 discusses the application of distributed sliding mode control to nonlinear DPSs by exploiting the set of characteristic ODEs. Section 4 briefly presents the tubular photobioreactor process and its dynamic model. Section 5 describes the proposed control design for the tubular photobioreactor. Section 6 shows the simulation results of an on-off control standard strategy compared with the proposed boundary SMC strategy. Finally, the main conclusions are presented in Section 7.

2. METHOD OF CHARACTERISTICS

In this section, it is briefly reviewed the MoC used to reduce the NFOPDEs into nonlinear ODEs. For more details about the MoC, see [Arnold, 1983].

A general NFOPDE is a mathematical expression given by

$$\frac{\partial z}{\partial t} + \phi\,(z,\,\mathbf{x},\,u,\,t,\,\mathbf{p}) = 0$$
$$y = \vartheta(z,\,\mathbf{x},\,t) \tag{1}$$

where z is the state of the system, t denotes time, $\mathbf{x}$ is the vector of n local spatial coordinate functions $x_i\,(i = 1,\,\dots,\,n)$, $\mathbf{p}$ is the n-dimensional vector with components p_i denoting the spatial partial derivatives of the state, $\partial z/\partial x_i$, $u = (z, \mathbf{x}, t)$ is a distributed time-varying feedback control law, ϕ is a smooth function of its arguments, and $y = \vartheta(z,\,\mathbf{x},\,t)$ is the system output defined by a smooth scalar function. Furthermore, it is introduced a new variable $q = \partial z/\partial t$ to represents the state time derivative.

All of the considerations and results are of local character on a given open set $\mathcal{N}$ of $\mathbb{R}^{n+2}$ described by the vector of local coordinate functions $(z, \mathbf{x}, t)$, denoted by χ. The projection of such an open set $\mathcal{N}$ onto $\mathbb{R}^{n+1}$, along the direction of z, is denominated as M with local coordinates $(\mathbf{x}, t)$. Also, it is denoted by η the vector of local coordinates $(z, \mathbf{x}, t, \mathbf{p}, q)$ in $\mathbb{R}^{2n+3}$, labelled as 1-jet of functions of $(z, \mathbf{x}, t, \mathbf{p}, q)$, which it is denoted as J^1.

The equation (1) can be interpreted as the expression of a $2n + 2$ dimensional hypersurface in J^1, denoted by $\mathbf{E}$ and defined as

$$\mathbf{E} = F^{-1}(0) := \left\{\eta \in J^{-1} \,|\, F(\eta) = q + \phi\,(z,\,\mathbf{x},\,u,\,t,\,\mathbf{p}) = 0\right\}.$$

It is generally assumed that the manifold $\mathbf{E}$ is noncharacteristic at all points η under consideration of the space J^1. In general, if the problem is well posed, the hypersurface $\mathbf{E}$ is considered to be noncharacteristic, requiring that $F(\eta)$ is smooth and the initial conditions for the system are consistent.

The jet characteristics of a PDE are the set of integral curves that are determined from the set of characteristic ODEs. The following expression is obtained for the components of the jet characteristic vector $\xi(\eta)$:

$$\begin{aligned} \dot{z} &= \mathbf{p}\frac{\partial F}{\partial \mathbf{p}} + q & \dot{\mathbf{p}} &= -\frac{\partial F}{\partial z}\mathbf{p} - \frac{\partial F}{\partial \mathbf{x}} \\ \dot{\mathbf{x}} &= \frac{\partial F}{\partial \mathbf{p}} & \dot{q} &= -\frac{\partial F}{\partial z}q - \frac{\partial F}{\partial t} \\ \dot{t} &= 1 \end{aligned} \tag{2}$$

To complete the problem definition, the initial condition for the equation (1) is needed. The state z must be assigned to some particular value on the points of a n-dimensional hypersurface defined in the space $n + 1$ dimensional, with coordinates $M = (\mathbf{x}, t)$. This is well known as Cauchy data. For more details about the calculation of it, see [Arnold, 1983].

3. DISTRIBUTED SLIDING MODE CONTROL

Sliding Mode Control is a classical nonlinear control method characterized by a switching control action. This control approach has, as the main idea, the definition of a surface on which the system has some desirable behavior. In this section, the classical concepts of SMC for lumped parameter systems to systems described by NFOPDEs is extended, labelled as Distributed Sliding Mode Control (DSMC).

The following feedback switching law determines the control action

$$u = \begin{cases} u^+(z,\,\mathbf{x},\,t) \text{ if } h(z,\,\mathbf{x},\,t) > 0 \\ u^-(z,\,\mathbf{x},\,t) \text{ if } h(z,\,\mathbf{x},\,t) < 0 \end{cases} \tag{3}$$

where $u^+(z, \mathbf{x}, t) > u^-(z, \mathbf{x}, t)$ and $h(z,\,\mathbf{x},\,t)$ is a scalar function of the state, being $h(z,\,\mathbf{x},\,t) = 0$ defined as the sliding surface or the switching boundary.

The condition $h(z,\,\mathbf{x},\,t) = 0$ is assumed to define a smooth local solution manifold $z = \varphi(\mathbf{x},\,t)$. Thus, the sliding manifold or sliding surface is defined as

$$S = \{(z,\,\mathbf{x},\,t) \in \mathbb{R}^{n+2} : z = \varphi(\mathbf{x}, t)\}. \tag{4}$$

The manifold S can be prolonged to the space of J^1 [Sira-Ramirez, 1990]. If the problem is well posed with consistent boundary and initial conditions, the sliding regime will exists for the NFOPDE.

To introduce a parametrization of the hypersurfaces representing the variable structure controlled system, a distributed switching function ν is defined, taking values in the discrete set $\{0, 1\}$. The switching function ν acts as a distributed control parameter and it is possible to rewrite the system give in equation (1) to

$$\frac{\partial z}{\partial t} + H(z,\,\mathbf{x},\,t,\,\mathbf{p}) + \nu G(z,\,\mathbf{x},\,t,\,\mathbf{p}) = 0, \tag{5}$$

with

$$H(z,\,\mathbf{x},\,t,\,\mathbf{p}) = \phi\left(z,\,\mathbf{x},\,u^-,\,t,\,\mathbf{p}\right)$$
$$G(z,\,\mathbf{x},\,t,\,\mathbf{p}) = \phi\left(z,\,\mathbf{x},\,u^+,\,t,\,\mathbf{p}\right) - \phi\left(z,\,\mathbf{x},\,u^-,\,t,\,\mathbf{p}\right)$$
$$\nu = \begin{cases} 1 \text{ if } h(z,\,\mathbf{x},\,t) > 0 \\ 0 \text{ if } h(z,\,\mathbf{x},\,t) < 0 \end{cases} \tag{6}$$

It follows, from equation (5), that the controlled vector field $\xi(\eta)$ is described by:

$$\dot{z} = \mathbf{p}\left(\frac{\partial H}{\partial \mathbf{p}} + \nu\frac{\partial G}{\partial \mathbf{p}}\right) + q$$
$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{p}} + \nu\frac{\partial G}{\partial \mathbf{p}}$$
$$\dot{t} = 1$$
$$\dot{\mathbf{p}} = -\left(\frac{\partial H}{\partial z} + \nu\frac{\partial G}{\partial z}\right)\mathbf{p} - \left(\frac{\partial H}{\partial \mathbf{x}} + \nu\frac{\partial G}{\partial \mathbf{x}}\right)$$
$$\dot{q} = -\left(\frac{\partial H}{\partial z} + \nu\frac{\partial G}{\partial z}\right)q - \left(\frac{\partial H}{\partial t} + \nu\frac{\partial G}{\partial t}\right) \quad (7)$$

Thus, the controlled system (5) has two hypersurfaces $\mathbf{E}^+$ and $\mathbf{E}^-$ defined in J^1, generated by the vector field $\xi^+(\eta)$, when $\nu = 1$, and $\xi^-(\eta)$, when $\nu = 0$. The components of the vector fields $\xi^+(\eta)$ and $\xi^-(\eta)$ can be directed obtained by set of ODEs given in equation (7).

The jet characteristics in J^1 uniquely define characteristics in the open set $\mathcal{N}$ by simple projection. Associated to such characteristics ones defines the characteristic vector fields $\kappa^+(\chi)$ and $\kappa^-(\chi)$ in $\mathcal{N}$ whose prolongations to the space of J^1 coincide with the jet characteristic vector fields $\xi^+(\eta)$ and $\xi^-(\eta)$, respectively.

The main idea in sliding control systems is to define the desired operating point in such a way that it is in the attractive sliding region.

The existence problem of a sliding mode resembles a generalized stability problem, therefore the second method of Lyapunov provides a useful setting for analysis. Specifically, stability to the switching surface requires selecting a candidate Lyapunov function $V(z, \mathbf{x}, t)$, which is positive definite and has a negative time derivative in the region of attraction. This approach will be used in Section 5.

4. TUBULAR PHOTOBIOREACTOR PLANT

The tubular photobioreactor plant studied in this paper is located at the Estación Experimental Las Palmerillas, property of Fundación CAJAMAR (Almería, Spain), located inside a greenhouse where the *Scenedesmus almeriensis* is cultivated (see http://aer.ual.es/MACROBIO for more information). A general scheme of the plant is depicted in Figure 1, showing its main components: the external loop and the bubble column.

The main objective of the external loop is to increase the surface exposed to the sun, allowing the microalgaes to capture a larger quantity of irradiance to perform the photosynthesis. The bubble column realizes several functions, mainly, desorption of O_2 produced during the photosynthesis at the external loop. The culture temperature control is made at the bubble column too. Moreover, the medium injection and the harvesting are located at the top of bubble column. The culture is continuously recirculated between the loop and the column by a pump located at the bottom of the column.

In order to maximize the microalgae production, it is required to operate several variables at the optimal value. Among them, the pH is one of the most critical variables that needs to be adequately regulated. On one hand, the supply of CO_2 as nutrient causes the transformation of carbon dioxide and consequently the pH declines. On the other hand, the microalgae realizes the photosynthesis when there is solar irradiance, consuming CO_2 and generating O_2, causing a gradual rise of the pH.

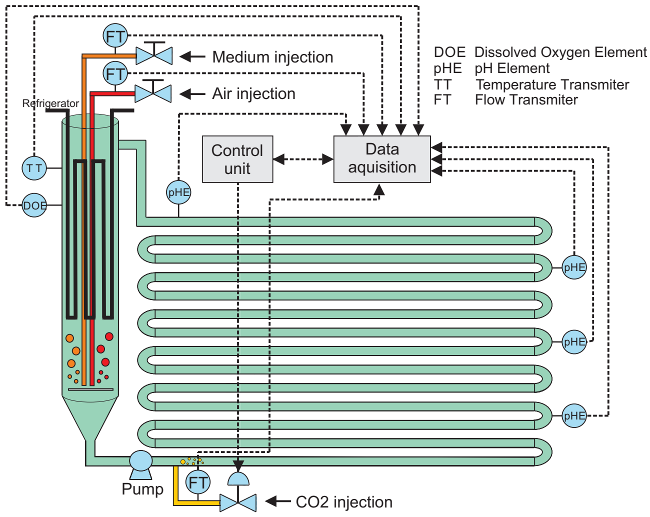

Fig. 1. Industrial tubular photobioreactor scheme.

In continuous operation mode, the manipulated variables can be the speed of the pump, the valve opening to supply CO_2 to the culture, the valve opening to inject air for O_2 desorption and the valve opening for injection of nutrients when the harvesting of biomass occurs. The main measured disturbance is the solar irradiance. The main measured variables are the pH at the end of the external loop and the dissolved oxygen at the bubble column.

Currently, the only controlled variable is the pH, since it is the most critical one. Regarding the control design purposes, the pH of the culture is influenced mainly by two phenomena: supplied CO_2, by means of an valve considered as the manipulated variable, and solar irradiance, as the main system disturbance. The other manipulated variables are operated at a constant value during the operation mode.

4.1 Dynamic Model

The dynamic model of microalgal production of the tubular photobioreactor was previously developed and described in [Fernández et al., 2012]. The model for a microalgae production system must consider the relationship between light availability and photosynthesis rate, the mixing and the gas-liquid mass transfer inside the system.

The mass balances for the liquid phase are given by

$$(1-\epsilon)A\frac{\partial [C_b]}{\partial t} + Q_l\frac{\partial [C_b]}{\partial x} = P_{O_2}(1-\epsilon)A[C_b]Y_{p/x}. \quad (8)$$
$$(1-\epsilon)A\frac{\partial [O_2]}{\partial t} + Q_l\frac{\partial [O_2]}{\partial x} = (1-\epsilon)A\frac{P_{O_2}[C_b]}{M_{O_2}} + (1-\epsilon)AK_{O_2}([O_2^*] - [O_2]). \quad (9)$$
$$(1-\epsilon)A\frac{\partial [C_T]}{\partial t} + Q_l\frac{\partial [C_T]}{\partial x} = (1-\epsilon)A\frac{P_{CO_2}[C_b]}{M_{CO_2}} + (1-\epsilon)AK_{CO_2}([CO_2^*] - [CO_2]). \quad (10)$$

In addition to the liquid phase, it is necessary to define the mass balances for the gas phase. The mass balance for oxygen and carbon dioxide molar fraction can be established as

$$\frac{\epsilon}{V_m}\frac{\partial yO_2}{\partial t} + \frac{Q_g}{V_m}\frac{\partial yO_2}{\partial x} = -(1-\epsilon)AK_{O_2}([O_2^*]-[O_2]). \quad (11)$$

$$\frac{\epsilon}{V_m}\frac{\partial yCO_2}{\partial t} + \frac{Q_g}{V_m}\frac{\partial yCO_2}{\partial x} = -(1-\epsilon)AK_{CO_2}([CO_2^*]-[CO_2]). \quad (12)$$

Equations (8)-(12) describe the tubular photobioreactor model. However, a relationship between the total inorganic carbon concentration and the dissolved carbon dioxide in the culture is also needed. Since the total inorganic carbon is equal to the sum of inorganic carbon species, an equation can be obtained to represent the total inorganic carbon variation as

$$\frac{\partial [C_T]}{\partial t} = \left(1 + \frac{K_1}{[H^+]} + \frac{K_1K_2}{[H^+]^2}\right)\frac{\partial [CO_2]}{\partial t} - [CO_2]\left(\frac{K_1}{[H^+]^2 + \frac{2K_1K_2}{[H^+]^3}}\right)\frac{\partial [H^+]}{\partial t} \quad (13)$$

Taking into account the electroneutrality constraint, the following equation is derived

$$\frac{\partial [H^+]}{\partial t} = \frac{\frac{K_1}{[H^+]} + \frac{2K_1K_2}{[H^+]^3}}{1 + \frac{K_w}{[H^+]^2} + \frac{K_1[CO_2]}{[H^+]^2} + 2\frac{K_1K_2}{[H^+]^3}}\frac{\partial [CO_2]}{\partial t} \quad (14)$$

Equations (13) and (14) relate the three concentrations, $[H^+]$, $[C_T]$ and $[CO_2]$; knowing any one of these concentrations enable the calculation of the others.

All the parameters and variables are described in Table 1 [Fernández et al., 2012].

Table 1. Photobioreactor plant model variables and parameters

Symbol	Description
$[CO_2]$	carbon dioxide concentration in the liquid phase
$[C_T]$	total inorganic carbon concentration
$[O_2]$	oxygen concentration in the liquid phase
$[H^+]$	proton concentration in the liquid
$[C_b]$	biomass concentration
$[CO_2^*]$	CO_2 concentration in equilibrium with the gas phase
$[O_2^*]$	O_2 concentration in equilibrium with the gas phase
A	cross-sectional area
MO_2	molecular weight of O_2
MCO_2	molecular weight of CO_2
K_{CO_2}	mass transfer coefficient for CO_2
K_{O_2}	volumetric gas-liquid mass transfer coefficient for O_2
K_1	first equilibrium constant for bicarbonates buffer
K_2	second equilibrium constant for bicarbonates buffer
K_w	hydrolysis constant of the water
P_{CO_2}	carbon dioxide consumption rate
P_{O_2}	oxygen production rate per biomass mass unit
Q_g	gas flow rate supplied
Q_l	volumetric flow rate of liquid
V_m	molar volume at the conditions of the reactor
y_{CO_2}	molar fraction of CO_2 in the gas phase
y_{O_2}	oxygen molar fraction in the gas phase
$Y_{p/x}$	biomass yield coefficient produced by oxygen unit mass
P_t	total pressure
ϵ	gas holdup

5. CONTROLLER DESIGN

In this section, two approaches are developed in order to control the pH dynamics of the tubular photobioreactor previously described. The first one based on the classical on-off switching strategy and the second one is the proposed DSMC strategy. In this work, two case studies are made: (i) the valve is on-off and the control law is a SMC with a hysteresis to limit the frequency; (ii) the valve has a continuous characteristic and the control law used is DSMC equivalent control approach in order to smoothing the control action signal.

5.1 Standard on-off Control Law

The objective is to design a discontinuous control law in order to include a desired dynamic on the closed loop system. For this aim, the sliding manifold is chosen to be the surface determined by

$$h = pH(x,t) - pH_{ref}(x,t), \quad (15)$$

where $pH_{ref}(x,t)$ is the set-point profile over the spatial domain of the tubular photobioreactor.

The feedback switching control law based on the sliding surface of equation (15) requires a distributed input. Since the CO_2 injection of the tubular photobioreactor is temporally but not spatially dependent, the control input must be integrated over the spatial domain.

For practical implementation, the integral over the spatial domain of equation (15) is approximated by a weighted sum of the state error calculated at four different points along the pipe. This sum determines whether u^+ or u^- must to be applied.

Moreover, the switches of the CO_2 valve have a frequency limitation to operate safely. Forcing the switches to their maximum switching values can damage the valve. The most common solution is to limit the switching frequency with a hysteresis band as follows

$$u(t) = \begin{cases} u^+, & \text{if } h(t) > \delta \\ u^-, & \text{if } h(t) < -\delta \\ u_{prev}, & \text{if } -\delta \le h(t) \le \delta \end{cases} \quad (16)$$

where δ is a constant which defines the hysteresis band and u_{prev} the last value of $u(t)$. It is desirable that the closed loop system operates at a predetermined frequency. This can be achieved by means of the correct choice of the hysteresis band δ.

5.2 Proposed DSMC Strategy

The objective of this section is to develop a smooth conrol law by finding the system dynamics on the sliding surface. For sake of simplicity it will be considered the total inorganic carbon $[C_T]$ instead of the pH concentration to simplify the mathematical development. This fact is due to the pH concentration is obtained by a conversion factor of the total inorganic carbon.

The sliding surface is expressed as

$$h = ([C_T](x,t) - [C_{T_{ref}}](x,t)) + \frac{1}{\tau_i}\int_0^t \langle [C_T] - [C_{T_{ref}}]\rangle dt. \quad (17)$$

where the first term denotes the error between the measurement distributed total carbon inorganic along the tube and the set-point profile. The second term corresponds to the integral of the average error and τ_i is the integral time constant. Notation $\langle\cdot\rangle$ denotes the average value of the respective variable.

Since the pH is obtained by a conversion factor of $[C_T]$, it can be observed that the system's relative degree is 2, i. e., the output must be differentiated twice to generate an explicit relationship between the output and the input (the control input appears explicitly on CO_2 molar fraction equation).

Thus, equations (10) and (12) are used to develop the control law. The extreme characteristic vector field of $[C_T]$ and y_{CO_2} are, respectively, given by

$$\kappa_1 = \dot{x}_1\frac{\partial}{\partial x_1} + \frac{\partial}{\partial t} + \dot{y_{CO_2}}\frac{\partial}{\partial y_{CO_2}} \quad (18)$$

$$\kappa_2 = \dot{x}_2\frac{\partial}{\partial x_2} + \frac{\partial}{\partial t} + \dot{[C_T]}\frac{\partial}{\partial [C_T]} \quad (19)$$

where the components $(\dot{x_1}, 1, \dot{y_{CO_2}})$ and $(\dot{x_2}, 1, \dot{[C_T]})$ of the jet characteristic vectors can be directly obtained by the MoC. For the sake of brevity, these equation are omitted here.

The directional derivative of the sliding surface h in the space of the extreme jet characteristics is expressed as follows:

$$L_{\kappa_1}h = \dot{x}_1\frac{\partial h}{\partial x_1} + \frac{\partial h}{\partial t} + \dot{y_{CO_2}}\frac{\partial h}{\partial y_{CO_2}} \quad (20)$$

$$L_{\kappa_2}h = \dot{x}_2\frac{\partial h}{\partial x_2} + \frac{\partial h}{\partial t} + \dot{[C_T]}\frac{\partial h}{\partial [C_T]} \quad (21)$$

where L_{κ_1} and L_{κ_2} is labeled as the directional (Lie) derivative. It can be noted that as the manipulated variable appears only in the CO_2 molar fraction balance, thus only $L_{\kappa_1}h$ must be defined for both values of the binary switching function.

The set of two hyperbolic PDEs can be combined and expressed as a single second-order PDE. The leading second-order derivatives can be expressed by the two Lie derivatives as follows

$$L_{\kappa_1}(L_{\kappa_2}[CT]) = \frac{\partial^2[C_T]}{\partial t^2} + \left[\frac{Q_g}{\epsilon} + \frac{Q_l}{(1-\epsilon)A}\right]\frac{\partial^2[C_T]}{\partial x\partial t} + \frac{Q_g}{\epsilon}\frac{Q_l}{(1-\epsilon)A}\frac{\partial^2[C_T]}{\partial x^2} = -\left[\frac{PCO_2}{MCO_2}\left(\frac{Q_g}{\epsilon}\frac{\partial[C_b]}{\partial x} + \frac{\partial[C_b]}{\partial t}\right) - K_{CO_2}H_tP_t\left((1-\epsilon)AK_{CO_2}([CO_2^*] - [CO_2])\frac{V_m}{\epsilon}\right) - K_{CO_2}\left(\frac{\partial[CO_2]}{\partial t} + \frac{Q_g}{\epsilon}\frac{\partial[CO_2]}{\partial x}\right)\right]. \quad (22)$$

As stated in Section 3, the existence problem of a sliding mode resembles a generalized stability problem. The Lyapunov stability problem for a set of hyperbolic PDE with relative degree 2 can be formulated in order to choose the switched feedback gains to guarantee

$$(L_{\kappa_1} + \lambda_1)(L_{\kappa_2} + \lambda_2)h < 0. \quad (23)$$

where λ_1 and λ_2 are two tuning parameters that guarantee the reachability of the systems trajectories to the sliding manifold in a finite time and, once on the sliding surface, the systems trajectories remain there.

Supposing that there exists a local attractive sliding regime to the distributed controlled system in an open set $\mathcal{N}$ of S. Thus, the total derivative of h in $S \in \mathbb{R}^{n+2}$ satisfies the condition (23). The directional derivative depends on the local of a given point in the sliding surface S, where for $h > 0$

$$(L_{\kappa_1^+} + \lambda_1)(L_{\kappa_2} + \lambda_2)h < 0 \quad (24)$$

and to $h < 0$

$$(L_{\kappa_1^-} + \lambda_1)(L_{\kappa_2} + \lambda_2)h > 0 \quad (25)$$

Thus, the directional characteristic flow satisfies the existence condition to sliding regime in S.

The system dynamic when is on the distributed sliding motion can be obtained imposing an invariance condition with respect to the manifold S in the controlled characteristic flow. This leads to restrict the controlled characteristic vector field to the null space of h:

$$(L_{\kappa_1^-} + \lambda_1)(L_{\kappa_2} + \lambda_2)h + \nu_{EQ}\left[(L_{\kappa_1^+} - L_{\kappa_1^-})(L_{\kappa_2} + \lambda_2)h\right] = 0. \quad (26)$$

Thus, the equivalent control function is given by

$$\nu_{EQ} = -\frac{(L_{\kappa_1^-} + \lambda_1)(L_{\kappa_2} + \lambda_2)h}{(L_{\kappa_1^+} - L_{\kappa_1^-})(L_{\kappa_2} + \lambda_2)h} \quad (27)$$

Using the equations (20)-(22) and (27) and integrating over the spatial domain, it is possible to find the equivalent system dynamics on the sliding surface. Thus, a smooth control law can be used in place of the discontinuous control law. The resulting control law is given by

$$u(t) = \epsilon\left\{K_{CO_2}H_tP_t\left[(1-\epsilon)AK_{CO_2}(\langle[CO_2^*] - [CO_2]\rangle)\frac{V_m}{\epsilon}\right] + K_{CO_2}\beta_2 + \frac{1}{\tau_i}\beta_1 + \lambda_1\left[\frac{PCO_2\langle[C_b]\rangle}{MCO_2} + K_{CO_2}(\langle[CO_2^*] - [CO_2]\rangle)\right] + \lambda_2\left[\beta_1 + \frac{1}{\tau_i}\langle[C_T] - [C_{T_{ref}}]\rangle\right] + \lambda_1\lambda_2\left(\langle[C_T] - [C_{T_{ref}}]\rangle + \frac{1}{\tau_i}\int_0^t\langle[C_T] - [C_{T_{ref}}]\rangle dt\right) - \frac{PCO_2}{MCO_2}\beta_3\right\} / \left\{\frac{PCO_2}{MCO_2} - K_{CO_2}[CO_2]|_{x=0}^{x=L} + \frac{Q_l}{(1-\epsilon)A}\left.\frac{\partial[C_{T_{ref}}]}{\partial x}\right|_{x=0}^{x=L} - \lambda_2\left[[C_T]|_{x=0}^{x=L} - [C_{T_{ref}}]|_{x=0}^{x=L}\right]\right\} \quad (28)$$

where

$$\beta_1 = -\frac{Q_l}{(1-\epsilon)A}[C_T]|_{x=0}^{x=L} + \frac{PCO_2\langle[C_b]\rangle}{MCO_2} + K_{CO_2}(\langle[CO_2^*] - [CO_2]\rangle)$$

$$\beta_2 = \frac{\beta_1}{P_2 - \langle[CO_2]\rangle P_1P_3}$$

$$\beta_3 = -\frac{Q_l}{(1-\epsilon)A}[C_b]|_{x=0}^{x=L} + PO_2\langle[C_b]\rangle Y_{p/x}$$

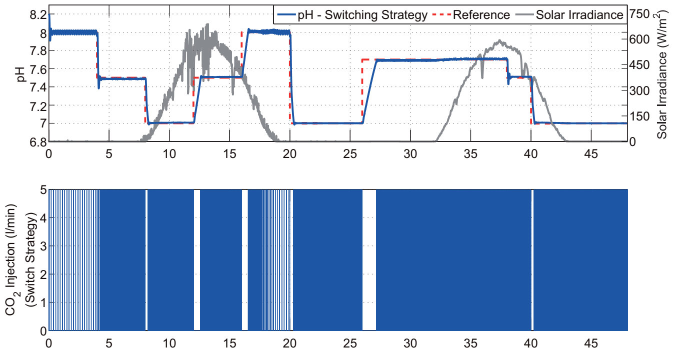

Fig. 2. Standard on-off SMC performance. Top: pH, reference and solar irradiance; Bottom: CO_2 injection.

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\begin{aligned}
P_1 &= \frac{K_1}{\langle [H^+] \rangle^2} + \frac{2K_1K_2}{\langle [H^+] \rangle^3} \\
P_2 &= 1 + \frac{K_1}{\langle [H^+] \rangle} + \frac{K_1K_2}{\langle [H^+] \rangle^2} \\
P_3 &= \frac{\frac{K_1}{\langle [H^+] \rangle} + \frac{2K_1K_2}{\langle [H^+] \rangle^2}}{1 + \frac{K_w}{\langle [H^+] \rangle^2} + \frac{K_1 \langle [CO_2] \rangle}{\langle [H^+] \rangle^2} + 4\frac{2K_1K_2}{\langle [H^+] \rangle^3}}
\end{aligned}
$$

and the notation $[\cdot]|_{x=0}^{x=L}$ defines the difference of the variable $[\cdot]$ at the point (L, t) and $(0, t)$. The control parameters are λ_1, λ_2, and τ_i.

6. RESULTS

This section summarizes the simulation results obtained from the model described in section 4 in order to validate the proposed controller performance under real data disturbances.

The controller parameters were obtained by the minimization of the ITSE (Integral of the Time-weighted Square Error) index using a directional direct search optimization method [Kolda et al., 2003]. This was done since the objective was to improve performance rather than global optimality. The following set of parameters were obtained: $\lambda_1 = 126$, $\lambda_2 = 128$, $\tau_i = 2000$ s. On the other hand, for the switching strategy the switching frequency of 0.1 Hz was chosen in order to guarantee the safety operation conditions of the CO_2 valve.

In Fig. 2 and Fig. 3 it is shown the time response of the pH measured at the end of the loop of the tubular photobioreactor for the standard on-off switching controller and the proposed DSMC control strategy, respectively. As can be observed, the control system follows the desired reference during all the simulation. Moreover, the undesirable effects produced by the solar irradiance transients are reduced.

Obviously, the switching strategy has a faster response for tracking. However, it is important to highlight that, the chattering problem can be completely eliminated with the proposed control strategy, but with much lesser aggressive control effort (see the bottom graphics of Fig. 2 and 3). The controller parameters of the proposed approach just need to be chosen to reflect a trade-off between set-point tracking and disturbance rejection.

7. CONCLUSIONS

This paper discusses the pH control problem of a tubular photobioreactor, operating in continuous mode. The main

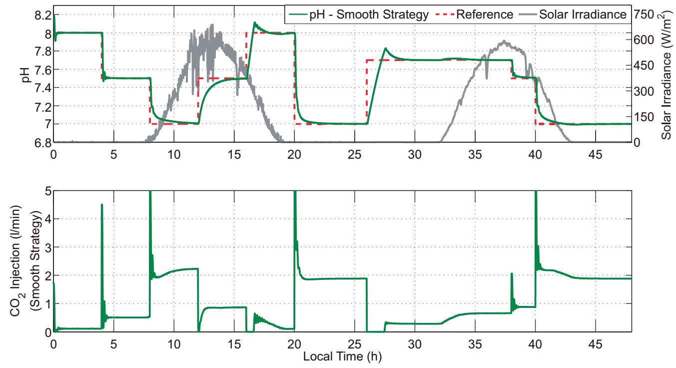

Fig. 3. Proposed DSMC performance. Top: pH, reference and solar irradiance; Bottom: CO_2 injection.

goal is to guarantee the set-point tracking and disturbance rejection. Due to the system distributed nature, a distributed sliding mode strategy is proposed to solve the control problems. It has been demonstrated that the tubular photobioreactor described by a system of hyperbolic PDEs can be effectively controlled using this methodology. Furthermore, the method of characteristics showed to be a powerful tool to analyse and obtain some system characteristics. As future work, experimental tests will be performed.

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