Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Image-based Consensus of Networked Robotic Manipulators without Visual Velocity Measurements ⋆

Lijiao Wang * **Bin Meng** **

* *Beijing Institute of Control Engineering, Beijing 100190, China, (Tel: 010-6837-9607; e-mail: lijiaowang@126.com)*
** *Science and Technology on Space Intelligent Control Laboratory, Beijing Institute of Control Engineering, Beijing 100190, China, (e-mail: mengb@amss.ac.cn)*

Abstract: In this paper, we study the fixed-camera visual servoing consensus problem of multiple robotic manipulators with uncertain robotic dynamics, kinematics and camera parameters. Our control objective is to achieve image-space consensus without the measurements of visual velocity. The communication topology is assumed to be directed graphs containing a spanning tree. A novel decentralized image-space position observer with online parameter updating is presented to avoid the reliance on visual velocity and to handle the uncertain robotic kinematics and camera parameters. Based on the observed visual information, we perform the distributed adaptive controller design in a cascade framework. The asymptotic convergence of consensus error is proved by use of Lyapunov analysis tool and input-output stability analysis tool. Finally, simulations with networked robotic manipulators are performed to validate the effectiveness of the proposed strategy.

Keywords: Visual Servoing Consensus, Robotic Manipulators, Observer, Adaptive Controller

1. INTRODUCTION

In recent years, synchronization and distributed control of multi-robot systems has gained great attention due to its promising applications in cooperative tasks, such as coordinated grasping of objects, parallel execution of tasks in manufacturing, teleoperation and planetary exploration. The distinct challenge for control of networked robotic agents lies in the nonlinearity and the uncertainties, which makes it difficult to apply directly the distributed control strategies developed for linear systems.

There has been a great deal of literature on adaptive consensus control of multiple robotic systems, where the objective is to reach an agreement on certain variables of interest among the agents with parametric uncertainties through local interaction. The joint-space consensus problem with dynamic uncertainties has been studied in (Chopra, Spong and Lozano, 2004; Cheng, Hou and Tan, 2008; Nuno, Ortega, Basanez and Hill, 2011) and the references therein. Passivity-based adaptive scheme is utilized in (Chopra et al., 2004) to achieve asymptotic consensus of bilateral teleoperators in the presence of dynamic uncertainties and time delays. Cheng et al. (2008) employs the backstepping-based adaptive technique to solve consensus problem of multiple robotic manipulators interacting on undirected graphs. An extended Slotine and Li controller is designed in (Nuno et al., 2011) to achieve consensus of networked Euler-Lagrange systems with dynamic uncertainties, where the cascade framework is first introduced and provides a simple and direct solution for consensus problem under directed graphs containing a spanning tree.

In fact, in most implementations, the robots are required to perform tasks in task space, such as Cartesian space or image space, inevitably adding uncertainties to the kinematic models. To broaden the applications of the joint-space adaptive consensus schemes, many efforts have been made on task-space adaptive consensus of multiple robots for the case where the unknown parameters of the Jacobian matrix from the joint space to the task space can be linearly parameterized. Cheng, Hou, Tan, Liu and Zou (2008) tackles task-space consensus problem of networked robotic agents with uncertain kinematics on undirected graphs. Wang, (2013) further extends the joint-space cascade framework in (Nuno et al., 2011) to task space and solves task-space consensus problem with dynamic and kinematic uncertainties on strongly connected graphs. Based on the cascade framework in (Wang, 2013), Wang, Meng and Wang (2013) proposes an observer-based consensus scheme such that task-space velocity-free consensus on directed graphs containing a spanning tree can be achieved.

Yet, in visual servoing problems where cameras are used to obtain position measurements, the depth information of the feature point appears nonlinearly in the interaction matrix and generally keeps changing during the robot motion (Cheah, Liu and Slotine, 2010). Hence, the depth-related parameters cannot be updated together with

⋆ This work is supported by the National Basic Research Program(973) of China under grant 2013CB733100, and the National Natural Science Foundation of China under grant 61333008, 61004058 and 61273153.

unknown kinematic parameters of the robots. To solve this problem, various uncalibrated visual servoing schemes with depth estimation have been developed for a single robotic manipulator. Liu, Wang, Wang and Lam (2006) proposes a depth independent interaction matrix such that the unknown parameters appear linearly in the closed-loop dynamics and then, a new adaptive controller is devised for image-based regulation of a robot manipulator with unknown intrinsic and extrinsic camera parameters. The scheme in (Liu et al., 2006) is further extended to solve the image-based tracking problem in (Wang, Liu and Zhou, 2007). Yet, the work in (Liu et al., 2006) and (Wang et al., 2007) only consider the case when robot kinematics and dynamics are exactly known. Taking uncertain robot kinematics, dynamics and camera parameters into account, (Cheah et. al, 2010) and (Cheah, Liu and Slotine, 2007) address adaptive regulation and tracking problems, respectively, with novel parameter updating laws for depth information. However, the aforementioned control schemes depend on the measurements of visual velocity, which is subject to big noises due to low sampling rate of the vision loop (Wang, Liu and Chen, 2010). To remove this undesired factor, a new uncalibrated visual tracking controller with an estimator of visual velocity is developed in (Wang et al., 2010) and asymptotic tracking is achieved based on the exact knowledge of robot dynamics and kinematics. Characteristic-model based all-coefficient adaptive control strategy proposed by Wu (Wu, Hu and Xie, 2007) has also been successfully used in the control of robotic manipulators with advantages in the simplicity of design, convenience of adjustment and strong robustness (Wu, Hu and Xie, 2009). This scheme can be easily extended to the visual servoing tracking of a single robot by providing a feasible low-order intelligent controller independent of the robotic dynamics. To the best of our efforts, no work has been found on uncalibrated image-space synchronization of multiple robotic manipulators without the measurements of visual velocity. The asymmetric Laplacian matrix, the uncertain time-varying depth information appearing nonlinearly in the overall Jacobian matrix as well as the uncertain robotic kinematics and dynamics make it not a straightforward extension of the existing methods.

In this paper, we consider the uncertainties of robotic dynamics, kinematics and camera parameters and address fixed-camera visual servoing consensus problem of networked robotic agents with time-varying depth information. We present a novel decentralized image-space position observer with adaptive parameter updating such that the measurements of visual velocity can be removed and the uncertainties of robotic kinematics and camera parameters can be handled in the observer loop. Based on the observed visual information, we construct the distributed adaptive controller in the dynamic loop. The major contribution is represented in the elimination of visual velocity measurements in the image-space adaptive consensus problem.

2. PRELIMINARIES

2.1 Modeling

We consider m robotic agents labeled as agents 1 to m. The dynamic model of the ith agent in joint space can be written as the following Euler-Lagrange equation (Spong and Vidyasagar, 1989)

$$H_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \tag{1}$$

where $q_i \in \Re^n$ is the joint variable, $H_i(q_i) \in \Re^{n\times n}$ is the symmetric and positive definite inertia matrix, $C_i(q_i, \dot{q}_i) \in \Re^{n\times n}$ is the coupled centripetal and Coriolis matrix, $g_i(q_i) \in \Re^n$ is the gravitational force and $\tau_i \in \Re^n$ is the exerted joint torque.

The dynamic model (1) bears the following properties (Spong and Vidyasagar, 1989).

Property 1: There exist positive constants k_{m1} and k_{m2} such that $0 < k_{m1}I_n \le H_i(q_i) < k_{m2}I_n$, where I_n denotes the $n \times n$ identity matrix.

Property 2: The dynamics of the agent is linearly parametric with respect to a group of unknown dynamic parameters $a_{di} \in \Re^{r_1}$, i.e.

$$H_i(q_i)z_1 + C_i(q_i, \dot{q}_i)z_2 + g_i(q_i) = Y_{di}(q_i, \dot{q}_i, z_2, z_1)a_{di}$$

$\forall z_1, z_2 \in \Re^n$, where $Y_{di} \in \Re^{n\times r_1}$ is referred to as the dynamic regressor matrix.

Property 3: The matrix $\dot{H}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric with an appropriate choice of $C_i(q_i, \dot{q}_i)$.

For the ith agent, a fixed pin-hole camera labeled as camera i is utilized to monitor the motion of a feature point attached to the robot end-effector and hence, the task space is defined as camera image space in pixels.

Denote $y_i = [u_i(t), v_i(t)]^T \in \Re^2$ as the coordinate of the feature point's projection on the ith camera image plane. Then, based on the perspective projection model of the camera, we get (Forsyth and Ponce, 2003; Liu et al., 2006)

$$\begin{bmatrix} y_i(t) \\ 1 \end{bmatrix} = \frac{1}{c_{zi}(t)} M_i x_i(t) \tag{2}$$

where M_i is a 3×4 matrix referred to as the perspective projection matrix determined by the uncertain intrinsic and extrinsic parameters of the ith camera, and $x_i \in \Re^4$ is the coordinate of the feature point in the robot base frame expressed as (Craig, 2005)

$$x_i(t) = h_i(q_i) \tag{3}$$

where $h_i(\cdot) \in \Re^n \to \Re^4$ is a nonlinear mapping from joint space to Catesian space, and $c_{z_i}(t)$ is the depth of the feature point with respect to the camera frame given by

$$c_{z_i}(t) = m_i^{3T} x_i(t) \tag{4}$$

where m_i^{jT} represents the jth row vector of M_i and there exists a constant $\bar{c}_{z_i} > 0$ such that

$$c_{z_i}(t) \ge \bar{c}_{z_i} > 0 \tag{5}$$

From the forward kinematics of the robotic manipulator, we have (Craig, 2005)

$$\dot{x}_i = J_i(q_i)\dot{q}_i \tag{6}$$

where $J_i(q_i)$ is referred to as the Jacobian matrix of the ith robot manipulator.

Now, differentiating (2) with respect to time and utilizing (4) and (6) yields (Liu et al., 2006)

$$\dot{y}_i = \frac{1}{c_{z_i}(t)} A_i(q_i, y_i, \theta_i)\dot{q}_i \tag{7}$$

where

$$A_i(q_i, y_i, \theta_i) = \begin{bmatrix} m_i^{1T} - u_i(t) m_i^{3T} \\ m_i^{2T} - v_i(t) m_i^{3T} \end{bmatrix} J_i(q_i)$$

and $\theta_i \in \Re^{r_2}$ is a constant vector determined by the products of the camera parameters and robot kinematic parameters, $A_i \in \Re^{2\times n}$ is called the depth-independent image Jacobian matrix determined by q_i, y_i and θ_i. Now, we summarize the property for A_i and c_{z_i}.

Property 4: $\forall \xi \in \Re^n$, the product $A_i(q_i, y_i, \theta_i)\xi$ can be linearly parameterized as (Liu et al., 2006)

$$A_i(q_i, y_i, \theta_i)\xi = Y_{Ai}(q_i, y_i, \xi)\theta_i \tag{8}$$

and the depth information c_{z_i} can be linearly parameterized as (Cheah et al., 2007)

$$c_{z_i} = Y_{zi}(q_i)\theta_i \tag{9}$$

where $Y_{Ai}(q_i, y_i, \xi) \in \Re^{2\times r_2}$ and $Y_{zi}(q_i) \in \Re^{1\times r_2}$ are, respectively, called the generalized kinematic regressor matrix and the depth regressor matrix independent of unknown parameter vector θ_i.

According to the customary rule, the derivative of c_{z_i} is denoted by $c_{\dot{z}_i}$. By (9), we can obtain

$$|c_{\dot{z}_i}| = |\dot{Y}_{zi}(q_i)\theta_i| \le \beta_i \|\dot{q}_i\| \tag{10}$$

where β_i is a positive constant determined by the upper bound of $\|\theta_i\|$.

2.2 Graph theory

A directed graph is used in this paper to describe network communication topology among the robotic agents. Denote the vertex set as $\mathcal{V} = \{1, \cdots, m\}$. The edge (j, i) denotes the information flow between agents j and i. An edge $(j, i) \in \mathcal{E}$ denotes that agent i can obtain information from agent j. The neighbors of agent i constitute a set $\mathcal{N}_i = \{j | j \in \mathcal{V}, (j, i) \in \mathcal{E}\}$. Then, we make the following assumption to characterize the directed graph utilized in this paper.

Assumption 1: The communication topology among the m robotic agents is directed and has a spanning tree.

The adjacency matrix $\mathcal{W} = [w_{ij}] \in \Re^{m\times m}$ associated with the graph is defined as $w_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $w_{ij} = 0$ otherwise. We assume the self edges do not exist, i.e., $w_{ii} = 0$. Then, we define the Laplacian matrix $L = [\ell_{ij}] \in \Re^{m\times m}$ as

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{m} w_{ik} & i = j \\ -w_{ij} & i \ne j \end{cases}$$

The property of L is represented in the following Lemma.

Lemma 1. (Olfati-Saber, Fax and Murray, 2007) If Assumption 1 holds, L has a single zero-eigenvalue and the rest of the spectrum of L has positive real parts. Moreover, $L1_m = 0_m$, where 1_m and 0_m denote $m \times 1$ column vector of all ones and all zeros, respectively.

2.3 Problem Formulation

In this paper, for the m robotic manipulators in image space represented by (1)-(3) with the uncertain camera parameters and robot kinematics reflected in θ_i and uncertain dynamics reflected in a_{di}, our control objective is to drive the image-space position of the end-effectors to reach consensus, i.e., $y_i - y_j \to 0$ and $\dot{y}_i \to 0$ as $t \to \infty$, provided that the visual velocity is not measurable and the information topology is directed graphs containing a spanning tree.

3. OBSERVER-BASED ADAPTIVE IMAGE-SPACE CONSENSUS

We employ the cascade structure and separate our design into two cascaded modules, i.e., the kinematic module and the dynamic module. The kinematic module design is comprised of two steps, where the observer loop is first constructed to estimate image-space position and to handle the uncertainties of robotic kinematics and camera parameters, and then, the kinematic loop design is performed based on the observed information, which aims at obtaining the appropriate task-space reference velocity such that consensus can be achieved. Finally, based on the task-space reference velocity, the distributed adaptive controller is proposed in the dynamic module so as to satisfy the design conditions for the kinematic module.

3.1 Kinematic Module Design

In this subsection, we first construct the observer loop such that the reliance on visual velocity $\dot{y}_i$ can be removed in the subsequent controller design.

The decentralized image-space position observer is proposed as

$$\dot{y}_{oi} = \hat{\dot{y}}_i - (\Lambda_{oi} + K_{1i}\|\dot{q}_i\|)(y_{oi} - y_i) - \frac{\hat{A}_{oi} - \hat{A}_i}{c_{\hat{z}_i}}\dot{q}_i \tag{11}$$

where

$$\hat{\dot{y}}_i = \frac{1}{c_{\hat{z}_i}}\hat{A}_{oi}\dot{q}_i \tag{12}$$

$$c_{\hat{z}_i} = Y_{zi}(q_i)\hat{\theta}_i \tag{13}$$

$$\hat{A}_i = A_i(q_i, y_i, \hat{\theta}_i) \tag{14}$$

$$\hat{A}_{oi} = A_i(q_i, y_{oi}, \hat{\theta}_i) \tag{15}$$

Here, $y_{oi} = [u_{oi}, v_{oi}]^T$ is the observed image-space position, $\hat{\theta}_i$ is the estimation of θ_i, the estimated depth $c_{\hat{z}_i}$ is assumed to be nonsingular, Λ_{oi} and K_{1i} are adjustable symmetric and positive definite gain matrices.

By use of (7), (8), (9) , (12) and (13), we derive

$$\hat{\dot{y}}_i - \dot{y}_i = \frac{(\hat{A}_{oi} - \hat{A}_i)}{c_{\hat{z}_i}}\dot{q}_i + \frac{N_i(q_i, y_i, \dot{q}_i, \hat{\theta}_i)}{c_{\hat{z}_i}c_{z_i}}\tilde{\theta}_i \tag{16}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ and $N_i(q_i, y_i, \dot{q}_i, \hat{\theta}_i) \in \Re^{2\times r_2}$ is the regeressor matrix derived by

$$\begin{aligned} N_i(q_i, y_i, \dot{q}_i, \hat{\theta}_i) = & \ Y_{A_i}(q_i, y_i, \dot{q}_i)Y_{z_i}(q_i)\hat{\theta}_i \\ & - Y_{A_i}(q_i, y_i, \dot{q}_i)\hat{\theta}_i Y_{z_i}(q_i) \end{aligned}$$

The online updating law for $\hat{\theta}_i$ is then proposed as

$$\dot{\hat{\theta}}_i = -\frac{1}{c_{\hat{z}_i}}\Gamma_{oi}N_i^T(q_i, y_i, \dot{q}_i, \hat{\theta}_i)\tilde{y}_{oi} \tag{17}$$

where $\tilde{y}_{oi} = y_{oi} - y_i$ and Γ_{oi} is the adjustable symmetric and positive definite matrix.

Now, it is time for us to give the first result related to the observer loop.

Theorem 1. For the system constituted by (11) and (17), we have $\hat{\theta}_i \in \mathcal{L}_\infty$, $\dot{y}_{oi} \in L_\infty$, $\tilde{y}_{oi} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and $\tilde{y}_{oi} \to 0$ as $t \to \infty$, i, j=1, $\cdots$, m, provided that the following conditions hold

(C1) $K_{1i} \geq \frac{\beta_i}{2\bar{c}_{z_i}}$, where β_i and $\bar{c}_{z_i}$ are given in (10) and (5) respectively;

(C2) $\dot{q}_i \in \mathcal{L}_\infty$.

Proof We first substitute (16) into (11) and derive the closed-loop observer dynamics as

$$\dot{\tilde{y}}_{oi} = -(\Lambda_{oi} + K_{1i}\|\dot{q}_i\|)\tilde{y}_{oi} + \frac{N_i(q_i, y_i, \dot{q}_i, \hat{\theta}_i)}{c_{\hat{z}_i}c_{z_i}}\tilde{\theta}_i \tag{18}$$

Define the Lyapunov function

$$V_{1i} = \frac{1}{2}c_{z_i}\tilde{y}_{oi}^T\tilde{y}_{oi} + \frac{1}{2}\tilde{\theta}_i^T\Gamma_{oi}^{-1}\tilde{\theta}_i \tag{19}$$

Differentiating V_{1i} with respect to time and substitute (18) and (17) into $\dot{V}_{1i}$, we have

$$\begin{aligned}\dot{V}_{1i} &= \frac{1}{2}c_{\dot{z}_i}\tilde{y}_{oi}^T\tilde{y}_{oi} + c_{z_i}\tilde{y}_{oi}^T\dot{\tilde{y}}_{oi} + \tilde{\theta}_i^T\Gamma_{oi}^{-1}\dot{\hat{\theta}}_i \\ &= \frac{1}{2}c_{\dot{z}_i}\tilde{y}_{oi}^T\tilde{y}_{oi} - c_{z_i}\tilde{y}_{oi}^T(\Lambda_{oi} + K_{1i}\|\dot{q}_i\|)\tilde{y}_{oi} \\ &\quad + c_{z_i}\tilde{y}_{oi}^T\frac{N(q_i, y_i, \dot{q}_i, \hat{\theta}_i)}{c_{\hat{z}_i}c_{z_i}}\tilde{\theta}_i + \tilde{\theta}_i^T\Gamma_{oi}^{-1}\dot{\hat{\theta}}_i \\ &\leq -c_{z_i}\tilde{y}_{oi}^T\Lambda_{oi}\tilde{y}_{oi} - \tilde{y}_{oi}^T(c_{z_i}K_{1i} - \frac{\beta_i}{2})\|\dot{q}_i\|\tilde{y}_{oi}\end{aligned} \tag{20}$$

By (C1), it is obvious that

$$\dot{V}_{1i} \leq -c_{z_i}\tilde{y}_{oi}^T\Lambda_{oi}\tilde{y}_{oi} \leq 0 \tag{21}$$

Then, we can get the result that $\tilde{y}_{oi} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{\theta}_i \in \mathcal{L}_\infty$, which implies the boundedness of $\hat{\theta}_i$. Note that the above result is derived independent of Condition (C2). Based on (C2) and (18), we further obtain that $\dot{\tilde{y}}_{oi} \in \mathcal{L}_\infty$. Thus, $\tilde{y}_{oi} \to 0$, i.e., $y_{oi} - y_i \to 0$ as $t \to \infty$ can be derived from the input-output stability analysis tool (Lozano, Brogliato, Egeland and Maschke, 2000). In addition, from (7) and (C2), we get $\dot{y}_i \in \mathcal{L}_\infty$ and then, $\dot{y}_{oi} = \dot{\tilde{y}}_{oi} + \dot{y}_i \in \mathcal{L}_\infty$ is ensured.

Now, we proceed to perform the kinematic loop design based on the observer constructed in Subsection 3.1. Our objective in this part is to design an appropriate joint-space reference velocity such that $y_i - y_j \to 0$ and $\dot{y}_i \to 0$ as $t \to \infty$, $\forall i \in \mathcal{V}$ and to give the design conditions, which will be satisfied by the dynamic loop design.

For $i \in \mathcal{V}$, we introduce the image-space reference velocity

$$\dot{y}_{ri} = -\sum_{j\in\mathcal{N}_i} w_{ij}(y_{oi} - y_{oj}) \tag{22}$$

and the estimated image-space sliding vector

$$\hat{s}_{yi} = \dot{\hat{y}}_i - \dot{y}_{ri} \tag{23}$$

Assume $\hat{A}_{oi}$ to be of full rank and then introduce the joint-space reference velocity

$$\dot{q}_{ri} = c_{\hat{z}_i}\hat{A}_{oi}^+\dot{y}_{ri} \tag{24}$$

and the joint-space sliding variable

$$s_i = \dot{q}_i - \dot{q}_{ri} \tag{25}$$

where $\hat{A}_{oi}^+ = \hat{A}_{oi}^T(\hat{A}_{oi}\hat{A}_{oi}^T)^{-1}$. Obviously, from (12), (23), (24) and (25), we have

$$s_{yi} = \frac{1}{c_{\hat{z}_i}}\hat{A}_{oi}\hat{s}_i \tag{26}$$

Here, $\dot{q}_{ri}$ and s_i will be used later in the dynamic loop design.

Then, we proceed to analyze the quality of the kinematic loop. From (7), (8), (14) and (15), we can get

$$\hat{A}_{oi}\dot{q}_i - \hat{A}_i\dot{q}_i = (y_{oi} - y_i)B_i(q_i, \dot{q}_i)\hat{\theta}_i \tag{27}$$

where $B(q_i, \dot{q}_i) \in \Re^{1\times r_2}$ is the regressor matrix independent of $\hat{\theta}_i$, y_i and y_{oi}.

Substituting (11), (22) and (27) into (23) and by a simple transform, we obtain the closed-loop kinematic equation

$$\dot{y}_{oi} = -\sum_{j\in\mathcal{N}_i} w_{ij}(y_{oi} - y_{oj}) + \sigma_i \tag{28}$$

where

$$\sigma_i = -(\Lambda_{oi} + K_{1i}\|\dot{q}_i\|)\tilde{y}_{oi} - \frac{1}{c_{\hat{z}_i}}\tilde{y}_{oi}B_i(q_i, \dot{q}_i)\hat{\theta}_i + \hat{s}_{yi}$$

Now, we give the result of the asymptotic convergence of image-space synchronization errors for the kinematic loop.

Theorem 2. For the system represented by (28) with the observer designed as (11) and the updating law (17), we can get $y_i - y_j \to 0$ as $t \to \infty$, if Assumption 1, conditions (C1), (C2) and the following condition hold

(C3) $\hat{s}_{yi} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$.

Proof. To perform the stability analysis, we first transform (28) in the matrix form as

$$\dot{y}_o = -(L \otimes I_2)y_o + \sigma \tag{29}$$

where $\otimes$ denotes the Kronecker product, y_o and σ are column stack vectors of y_{oi} and σ_i. Since (C1) and (C2) hold, $\tilde{y}_{oi} \to 0$, $\tilde{y}_{oi} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\hat{\theta}_i \in \mathcal{L}_\infty$ can be derived from Theorem 1. Then, $\sigma_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ is obtained based on (C2) and (C3).

Following (Scardovi, Arcak and Sontag, 2010; Nuno et al., 2011), we introduce the coordinate transform

$$z = (S \otimes I_2)y_o \tag{30}$$

where $S \in \Re^{(m-1)\times m}$ bears the following properties: (P1) $SS^T = I_{m-1}$; (P2) $S^TS = I_m - {}^1\!/_m 1_m1_m^T$; (P3) $S1_m = 0$; (P4) $(S \otimes I_2)y_o = 0_{2m} \Leftrightarrow y_o = 1_m \otimes y_{oc}$ for some $y_{oc} \in \Re^m$.

Based on Eq.(30), Lemma 1 and the above properties, Eq.(29) can be reformulated as

$$\dot{z} = -(SLS^T \otimes I_2)z + (S \otimes I_2)\sigma \tag{31}$$

with $-SLS^T$ being Hurwitz (Scardovi, 2010).

In Eq.(31), since $\sigma \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $-SLS^T$ is Hurwitz, we get $z \in \mathcal{L}_2 \cap \mathcal{L}_\infty, \dot{z} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $z \to 0$ from the input-output stability analysis (Lozano et al., 2000). Then, $y_{oi} - y_{oj} \to 0$ is obtained from (P4). Consequently, together with the result $\tilde{y}_{oi} \to 0$ in Theorem 1, we get that $y_i - y_j \to 0$ as $t \to \infty$.

3.2 Dynamic Module Design

In this part, we aim at giving the control law τ_i for (1) such that not only the conditions (C2) and (C3) required for the observer loop design and kinematic loop design are satisfied but the consensus velocity $\dot{y}_i$ is driven to zero.

Based on the joint-space reference velocity $\dot{q}_{ri}$ derived in the kinematic loop in (24), we obtain the joint-space reference acceleration as

$$\ddot{q}_{ri} = c_{\dot{\hat{z}}_i}\hat{A}_{oi}^{+}\dot{y}_{ri} + c_{\hat{z}_i}\hat{A}_{oi}^{+}\ddot{y}_{ri} + c_{\hat{z}_i}\dot{\hat{A}}_{oi}^{+}\dot{y}_{ri} \qquad (32)$$

where

$$\ddot{y}_{ri} = -\sum_{j\in\mathcal{N}_i} w_{ij}(\dot{y}_{oi} - \dot{y}_{oj}) \qquad (33)$$

Now, we introduce the dynamic regressor matrix Y_{di} as

$$H_i(q_i)\ddot{q}_{ri} + C_i(q_i,\dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_{di}\left(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}\right)a_{di}$$

Then, we propose the following extended Slotine and Li controller

$$\tau_i = Y_{di}\left(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}\right)\hat{a}_{di} - \hat{A}_{oi}^{T}K_{di}c_{\hat{z}_i}\hat{s}_{yi} \qquad (34)$$

where K_{di} is the adjustable symmetric positive definite matrix and $\hat{a}_{di}$ is the estimated dynamic parameter updated by

$$\dot{\hat{a}}_{di} = -\Gamma_{di}Y_{di}^{T}\left(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}\right)s_i \qquad (35)$$

where Γ_{di} is the symmetric positive definite updating gain matrix.

Now, we present the result of asymptotic consensus in image space.

Theorem 3. Consider a network of m robotic manipulators described as (1)-(3) with uncertain dynamics parameters represented in a_{di} and uncertain kinematics and camera parameters represented in θ_i. Suppose the communication topology among the agents satisfies Assumption 1 and the measurements of visual velocity $\dot{y}_i$ is not available. If appropriate gains are selected to make Condition (C1) hold, then the controller (34), the observer (11) and the updating laws (35) and (17) give rise to $y_i - y_j \to 0$ and $\dot{y}_i \to 0$ as $t\to\infty$, i, j=1, $\cdots$, m.

Proof. We first substitute (34) into (1) to get the closed-loop dynamics as

$$H_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = Y_{di}\left(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}\right)\tilde{a}_{di} - \hat{A}_{oi}^{T}K_{di}c_{\hat{z}_i}\hat{s}_{yi} \qquad (36)$$

where $\tilde{a}_{di} = \hat{a}_{di} - a_{di}$.

Then, let us take the Lyapunov-like function as

$$V_{2i} = \frac{1}{2}s_i^T H_i s_i + \frac{1}{2}\tilde{a}_{di}^T\Gamma_{di}^{-1}\tilde{a}_{di} \qquad (37)$$

We differentiate V_{2i} with respect to time and substitute the updating law (35) into $\dot{V}_{2i}$. Due to the skew-symmetry property of $\dot{H}_i - 2C_i$, we can get

$$\dot{V}_{2i} = -(c_{\hat{z}_i}\hat{s}_{yi})^T K_{di}(c_{\hat{z}_i}\hat{s}_{yi}) \le 0 \qquad (38)$$

which implies that $s_i \in \mathcal{L}_\infty$, $c_{\hat{z}_i}\hat{s}_{yi} \in \mathcal{L}_2$ and $\tilde{a}_{di} \in \mathcal{L}_\infty$.

Now, we will prove that our overall design only requires conditon (C1) and the dynamic loop design can ensure (C2) and (C3) hold. Let us turn to the observer loop, where $\tilde{y}_{oi} \in \mathcal{L}_\infty$ and $\hat{\theta}_i \in \mathcal{L}_\infty$ are ensured under (C1). Then, we have $y_{oi} \in \mathcal{L}_\infty$, further indicating the boundedness of $\dot{y}_{ri}$ from (22). Since $c_{\hat{z}_i}$ is nonzero, we get $\dot{\hat{\theta}}_i \in \mathcal{L}_\infty$ from (17) and $\hat{s}_{yi} \in \mathcal{L}_2$. Meanwhile, we can get $\hat{s}_{yi} \in \mathcal{L}_\infty$ from (26). Thus, Condition (C3) holds. Since $\hat{\theta}_i \in L_\infty$, we have the boundedness of $c_{\hat{z}_i}$ from (13). Then, $\dot{q}_{ri} \in \mathcal{L}_\infty$ can be derived from (24) with $\hat{A}_{oi}$ being of full rank. Thus, we have $\dot{q}_i = s_i + \dot{q}_{ri} \in \mathcal{L}_\infty$, which indicates that (C2) holds. Together with the boundedness of $\dot{\hat{\theta}}_i$ and $\hat{\theta}_i$, the boundedness of $c_{\dot{\hat{z}}_i}$ is guaranteed.

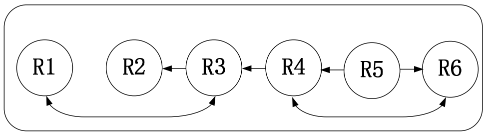

Fig. 1. Interaction graph of the robotic agents

Now that all the conditions required in Theorem 1 and Theorem 2 are satisfied, we can get $\dot{y}_{oi} \in \mathcal{L}_\infty$ and $y_i - y_j \to 0$ as $t\to\infty$. Then, we have $\ddot{y}_{ri} \in \mathcal{L}_\infty$ based on (33). Therfore, the boundedness of $\ddot{q}_{ri}$ is obtained from (32) with $\hat{A}_{oi}$ being of full rank. Since $\hat{a}_{di} \in \mathcal{L}_\infty$, $\dot{q}_i \in \mathcal{L}_\infty$, $s_i \in \mathcal{L}_\infty$ and $\hat{s}_{yi} \in \mathcal{L}_\infty$, it is obvious that $\dot{s}_i \in \mathcal{L}_\infty$ from the closed-loop dynamics (36). Then, we get $\ddot{q}_i = \dot{s}_i + \ddot{q}_{ri} \in \mathcal{L}_\infty$. Using the above results, we derive $\ddot{y}_i = \frac{\dot{A}_i\dot{q}_i + A_i\ddot{q}_i - c_{\dot{z}_i}\dot{y}_i}{c_{z_i}} \in \mathcal{L}_\infty$, which implies the uniform continuity of $\dot{y}_i$. Thus, $\dot{y}_i \to 0$ as $t\to\infty$ is derived from Barbalat's Lemma (Lozano et al., 2000). Combining this with the conclusion in Theorem 2, we obtain the synchronization of all the agents in the image space, i.e., $y_i - y_j \to 0$ and $\dot{y}_i \to 0$ as $t\to\infty$.

4. SIMULATION STUDY

In this part, we will validate the proposed scheme using six two-DOF planar robotic manipulators, each with a fixed camera to obtain the position information. The feature points are marked on the end-effectors of the robots. For the ith robotic agent, physical parameters of the two links are selected as $\bar{m}_i^1 = 1 + 0.3i$, $l_{Ci}^1 = 0.1 + 0.03i$, $r_i^1 = 0.1 + 0.03i$, $\bar{m}_i^2 = 1.5 + 0.3i$, $l_{Ci}^2 = 0.15 + 0.03i$, $r_i^2 = 0.15 + 0.03i$ where $m_i^{\bar{i}}$ is the mass of the $\bar{i}$th link, $l_{Ci}^{\bar{i}}$ is the position of the center of mass, $l_{ei}^{\bar{i}} = l_{Ci}^{\bar{i}} + r_i^{\bar{i}}$ is the length of the $\bar{i}$th link. For simplicity, the gravitational force is neglected in our simulation.

The intrinsic parameters for the ith camera are selected as $u_{0i} = 280$, v_{0i}=250, $k_{ui} = 63$, k_{vi}=63 and $\varphi_i = \pi/2$, where (u_{0i}, v_{0i}) is the position of the principal point of the camera, k_{ui} and k_{vi} are the scalar factors of the u_i and v_i axes, and φ_i is the angle between u_i and v_i. Denote T_{ci}^{bi} as the homogeneous transformation matrix of the ith camera frame with respect to the ith robot base frame derived by $T_{ci}^{bi} = \mathrm{rot}(\boldsymbol{x}_i, \pi - \alpha_i^1)\mathrm{rot}(\boldsymbol{y}_i, \alpha_i^2)\mathrm{rot}(\boldsymbol{z}_i, \alpha_i^3)$ $\mathrm{tran}(d_i^1, d_i^2, -d_i^3)$, where rot($\cdot$) and tran($\cdot$) denote the basic rotation and translation transformation (Spong and Vidyasagar, 1989), respectively. The true values are $\alpha_i^1 = \pi/18$, $\alpha_i^2 = \pi/4$, $\alpha_i^3 = \pi/4$, $d_i^1 = 1$, $d_i^2 = 1$ and $d_i^3 = 3$. Then, we get the extrinsic parameter as ${T_{bi}^{ci}}^{-1}$. Based on the intrinsic and extrinsic camera parameters, we can get the perspective matrix $M_i = m_i^{jk} \in \Re^{3\times 4}$ (Liu et al., 2006).

The information graph associated with the robotic agents is shown in Fig.1, where R1, $\cdots$, R6 denote the six agents, respectively. Fig.2 shows the response of the image-space position. Since agent 5 is the single rooted node in the information topology, the other agents are illustrated to asymptotically synchronize to a consensus position, i.e., position of R5. Fig.3 illustrates that the observed positions have asymptotically converged to the actual positions.

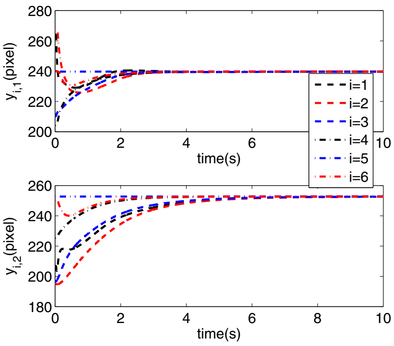

Fig. 2. Image-space position response of the robotic agents

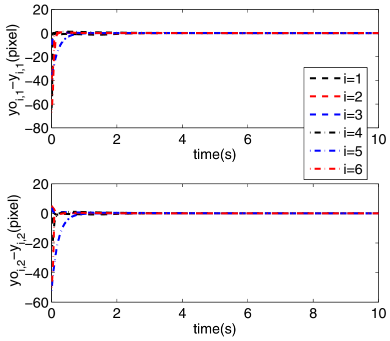

Fig. 3. The observation errors of the robotic agents

5. CONCLUSION

In this paper, we address image-space consensus problem for networked robotic agents under directed graphs containing a spanning tree. The agents are subject to uncertain robot kinematics, dynamics and camera parameters. A distributed cascaded observer-controller scheme is proposed to achieve image-space consensus without visual velocity measurements. The observer proposed in this paper gives a unified solution for both consensus and leader-following problems of multiple robotic agents. A new updating law for uncertain robot kinematic parameters and camera parameters is also proposed. Simulation results are presented to examine the performance of our strategy. Our future research will concentrate on the Characteristic-model based distributed adaptive control strategy, which might provide a new effective and simple solution to the consensus problem of multiple robotic agents.

REFERENCES

Cheah, C. C., Liu, C. and Slotine, J. J. E.(2010). Adaptive Jacobian vision based control for robots with uncertain depth information. *Automatica*, 46: 1228-1233.

Cheah, C. C., Liu, C. and Slotine, J. J. E.(2007). Adaptive vision based tracking control of robots with uncertainty in depth information. *IEEE International Conference on Robotics and Automation*, Roma, Italy, 2817-2822.

Cheng, L., Hou, Z. G. and Tan, M.(2008), Decentralized adaptive consensus control for multi-manipulator system with uncertain dynamics. *IEEE International Conference on Systems, Man and Cybernetics*, 2712-2717.

Cheng, L., Hou, Z. G., Tan, M., Liu, D. and Zou, A. M.(2008). Multi-agent based adaptive consensus control for multiple manipulators with kinematic uncertainties. *IEEE International Symposium on Intelligent Control Part of IEEE Multi-conference on Systems and Control*, San Antonio, Texas, USA, 189-194.

Chopra, N., Spong, M. W. and Lozano, R.(2004). Adaptive coordination control of bilateral teleoperators with time delay. *IEEE Conference on Decision and Control*. Atlantis, Pradise Island, Bahamas, 4540-4547.

Craig, J. J.(2005). *Introduction to robotics: Mechanics and control*. New York: Prentice Hall.

Forsyth, D. A. and Ponce, J.(2003). *Computer Vision: A Mordern Approach*. Englewood Cliffs, NJ: Prentice Hall, 2003

Liu, Y. H., Wang, H. S., Wang, C. Y. and Lam, K. K.(2006). Uncalibrated visual servoing of robots using a depth independent interaction matrix. *IEEE Transactions on Robotics*, 22(4): 804-817

Lozano, R., Brogliato, B., Egeland, O. and Maschke, B.(2000). *Dissipative systems analysis and control*. London:Springer-Verlag.

Nuno, E., Ortega, R., Basanez, L. and Hill, D.(2011). Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays. *IEEE Transactions on Automatic Control*, 56(4): 935-941.

Olfati-Saber, R., Fax, J. A. and Murry, R. M.(2007). Consensus and cooperation in networked multi-agent systems. *Proc. IEEE*, 95(1): 215-213.

Scardovi, L.(2010). Note on the reduced laplacian matrix, Private correspondance.

Scardovi, L., Arcak, M. and Sontag, E. D.(2010). Synchronization of interconnected systems with application to biochemical networks: An input-output approach. *IEEE Transactions on Automatic Control*, 55(6): 1367-1379.

Spong, M. W. and Vidyasagar, M.(1989). *Robot dynamics and control*. USA: John Willey & Sons, Inc..

Wang, H. L.(2013). Task-space synchronization of networked robotic systems with uncertain kinematics and dynamics. *IEEE Transactions on Automatic Control*, to be published.

Wang, H. S., Liu, Y. H. and Zhou, D. X.(2007). Dynamic visual tracking for manipulators using an uncalibrated fixed camera. *IEEE Transactions on Robotics*, 23(3): 610-617.

Wang, H. S., Liu, Y. H. and Chen, W. D.(2010). Uncalibrated visual tracking control without visual velocity, *IEEE Transactions on Robotics*, 18(6): 1359-1370.

Wang, L. J., Meng, B. and Wang, H. L.(2013). Adaptive task-space synchronization of networked robotic agents without task-sapce velocity measurements. *International Journal of Control*, DOI: 10.1080/00207179. 2013. 835173.

Wu, H. X., Hu, J. and Xie, Y. C.(2007). Characteristic model-based all-coefficient adaptive control method and its applications. *IEEE Transactions on systems, man and cybernetics-Part C: Applications and reviews*, 37(2): 213-221, 2007

Wu, H. X., Hu, J. and Xie, Y. C.(2009). *Characteristic model-based intelligent adaptive control*. Beijing: China Science and Technology Press.