Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Torque Overlay Based Robust Steering Wheel Angle Control for Lateral Control Using Backstepping Design

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Abstract: We propose a torque overlay based robust steering wheel angle control of electric power steering (EPS) for lateral control using backstepping design. The main contribution of this paper is that the proposed method is designed based on torque overlay and that the global uniform ultimate boundedness of the steering wheel angle tracking error is guaranteed using only steering wheel angle feedback with external disturbances. The key idea is to make the EPS dynamics be simplified. Then, the external disturbances, system function, and input gain uncertainty are regarded as a disturbance. An augmented observer is designed to estimate the full state and the disturbance. A nonlinear damping controller is developed via backstepping to suppress a position tracking error using input-to-state stability property. The proposed method uses only steering wheel angle feedback and nominal value of the input gain. The proposed method is simple to implement in real-time control and robust to the parameter uncertainties and the external disturbances. Since the proposed method is designed based on torque overlay as add-on type, it can be integrated with the conventional EPS system facilitating driver's intervention.

Keywords: Lateral control, Backstepping control, Automotive control, Electric power steering, Stability

NOMENCLATURE

- θ_h: Steering wheel angular position [rad]
- ω_h: Steering wheel angular velocity [rad/s]
- θ_{h_d}: Desired steering wheel angular position [rad]
- θ_m: Motor angular position [rad]
- ω_m: Motor angular velocity [rad/s]
- i: Current input of the motor [A]
- T: Input torque of EPS system ($T = K_t i$) [N·m]
- i: Motor current input [A]
- K_t: Motor torque constant [N·m/A]
- T_d: Driver's torque [N·m]
- T_f: Friction torque [N·m]
- T_r: Road reaction torque on the rack and pinion [N·m]
- J_c: Steering column moment of inertia [Kg·m^2]
- B_c: Steering column viscous damping [N·m/(rad/s)]
- K_c: Steering column stiffness [N·m/rad]
- M_r: Mass of the rack [kg]
- B_r: Viscous damping of the rack [N·m/(rad/s)]
- R_p: Steering column pinion radius [m]
- K_r: Tire spring rate [N/m]
- J_m: Motor moment of inertia [kg·m^2]
- B_m: Motor shaft viscous damping [N·m/(rad/s)]
- K_t: Motor torque constant [N·m/A]

1. INTRODUCTION

In the autonomous vehicles, the aim of the lateral control is a lane-keeping control to keep the vehicle between lanes (Antony [2003]). Various lane keeping control methods have been studied for the lane-keeping. A lane-keeping control method based on lead-lag control was presented in (Taylor et al. [1999]). In (Chaib et al. [2004]), performances of four lane keeping control methods were compared. A Lyapunov based lateral control method was proposed for lane-keeping system in (Rossetter et al. [2006]). In (Wu et al. [2008]), a fuzzy gain scheduling lane-keeping system was proposed and implemented on their vehicle platform. A lane-keeping control method using a potential field was designed to solve stability and robustness issues for a simple look-ahead control scheme in (Talvala et al. [2011]). For lateral control, the desired steering wheel angle is derived by the lane-keeping control method considering vehicle lateral dynamics in the lane keeping systems (LKSs). Then, the steering wheel angle is controlled by the power steering system.

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This work was supported by the Industrial Source Technology Development Program(10044620, Automatic lane change system for novice drivers) funded by the Ministry of Trade, Industry and Energy (MOTIE, Korea).

Nowadays electric power steering (EPS) system is substituted for hydraulic power steering (HPS) system since the EPS system is superior in several aspects including safety, cost, energy efficiency, environmental protection, and assembly compared with the traditional HPS system (Antony [2003]). The schematic diagram of a column-mounted EPS system is depicted in (Antony [2003]). When the driver manually handles the steering wheel, the main role of the EPS system is the torque control of the motor to make the assistant torque. Then the assistant and driver torques are combined to make the tires turn. Various methods for the torque control in EPS have been studied (Chen et al. [2008], Marouf et al. [2012]).

For lateral control in the autonomous vehicles, the main role of the EPS system is to make the steering wheel angle track the desired steering wheel angle derived by the LKS. Therefore, in the previous lateral control methods, the steering wheel angle controllers based on an angle overlay were used for lane-keeping. The angle overlay based methods do not allow the torque combination so that it is difficult to use all of the basic EPS functions. Thus, it is difficult for driver to smoothly take over the steering wheel control. Furthermore, the angle overlay requires the modification of the EPS system. In the torque overlay based method, a torque integration with basic EPS functions is available for the driver's convenience (Nicolas. [2012]). Thus, the driver can smoothly take over the steering wheel control from the LKS without uncomfortable feeling (Beecham. [2010]). However, in the torque overlay approach, since the torque imposed by the driver is activated as disturbance input in the lateral control of LKS, it can cause the performance degradation of the steering wheel angle control. Furthermore, the unsymmetrical hysteresis behavior occurred due to the structure and friction of the EPS system can also make the torque overlay based steering wheel angle control more difficult. Thus the steering wheel angle control method of EPS for lateral control should be designed based on torque overlay with the consideration of both the steering wheel angle tracking and the compensation of the model uncertainty and external disturbances (the steering torque imposed by the driver, the unsymmetrical hysteresis behavior, and the friction so on.)

In this paper, we propose a torque overlay based robust steering wheel angle control of electric power steering for lateral control using backstepping design. The main contribution of this paper is that the proposed method is designed based on torque overlay and that the global uniform ultimate boundedness of the steering wheel angle tracking error is guaranteed using only steering wheel angle feedback with external disturbances. The key idea is to make the EPS dynamics be simplified. Then, the external disturbances, system function, and input gain uncertainty are regarded as a disturbance. In order to estimate the full state and disturbance, an augmented observer is designed. Since the disturbance includes the external disturbances, system function, and input gain uncertainty, it is difficult to accurately estimate the disturbance. It may result in the performance degradation of the steering wheel angle control. A nonlinear damping controller is developed via backstepping to suppress a position tracking error using input-to-state stability (ISS) property (Krstic et al. [1995], Khalil [2002]). The proposed method uses only steering wheel angle feedback and nominal value of the input gain. This approach is robust to the parameter uncertainties and the disturbance, and simplifies the design process such that the control algorithm is suitable for real time control. Since the proposed method is designed based on torque overlay, a torque integration with basic EPS functions for the steering wheel angle control is available for the drivers convenience. The performance of the proposed method was validated via experiments.

2. ELECTRIC POWER STEERING SYSTEM MODEL

The EPS model can be represented in the state-space form as follows (Marouf et al. [2012])

$$\begin{aligned} \dot{\theta}_h &= \omega_h \\ \dot{\omega}_h &= \frac{1}{J_c}\left(-K_c\theta_h - B_c\omega_h + \frac{K_c}{N}\theta_m + T_d - T_f\right) \\ \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= \frac{1}{J_{eq}}\left(\frac{K_c}{N}\theta_h - \frac{K_c + K_r R_p^2}{N^2}\theta_m - B_{eq}\omega_m + T - \frac{R_p}{N}T_r\right) \end{aligned} \tag{1}$$

where $J_{eq} = J_m + \frac{R_p^2}{N^2}M_r$, and $B_{eq} = B_m + \frac{R_p^2}{N^2}B_r$. For simplification, (1) can be rewritten as

$$\begin{aligned} \dot{\theta}_h &= \omega_h \\ \dot{\omega}_h &= a_{21}\theta_h + a_{22}\omega_h + a_{23}\theta_m + d_1 \\ \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= a_{41}\theta_h + a_{43}\theta_m + a_{44}\omega_m + b_4 T + d_2 \end{aligned} \tag{2}$$

where $a_{21} = -\frac{K_c}{J_c}$, $a_{22} = -\frac{B_c}{J_c}$, $a_{23} = \frac{K_c}{J_c N}$, $d_1 = \frac{1}{J_c}(T_d - T_f)$, $a_{41} = \frac{K_c}{J_{eq}N}$, $a_{43} = -\frac{K_c + K_r R_p^2}{J_{eq}N^2}$, $a_{44} = -\frac{B_{eq}}{J_{eq}}$, $b_4 = \frac{1}{J_{eq}}$, and $d_2 = -\frac{R_p}{J_{eq}N}T_r$. Now we derive the normal form of (2). The output of the EPS system is $y = \theta_h$. The state x is defined as

$$x = [x_1\ x_2\ x_3\ x_4]^T = [\theta_h\ \omega_h\ \dot{\omega}_h\ \ddot{\omega}_h]^T. \tag{3}$$

The control input u is defined as $u = T$. From (2) and (3), we obtain the dynamic of x_4 as

$$\begin{aligned} \dot{x}_4 =& \omega_h^{(3)} \\ =& f(x) + gu + d_{ext}(d_1, \dot{d}_1, \ddot{d}_1, d_2) \end{aligned} \tag{4}$$

where f is the system function, g is the input gain, and d_{ext} is the external disturbance as

$$\begin{aligned} f =& (a_{23}a_{41} - a_{21}a_{43})x_1 + (a_{21}(a_{22} - a_{21} - a_{43}) - a_{22}a_{43})x_2 \\ &+ (a_{21} + a_{43} - a_{21}a_{22} - a_{22}a_{43})x_3 + (a_{22} + a_{44})x_4 \\ g =& a_{23}b_4 \\ d_{ext} =& -(a_{21} + a_{43})d_1 - (a_{22} + a_{44})\dot{d}_1 + \ddot{d}_1 + a_{23}d_2. \end{aligned}$$

The uncertainty Δg and nominal value g_0 of g are defined as

$$g = g_0 + \Delta g. \tag{5}$$

It is difficult to exactly know the parameters of the EPS. Furthermore, the parameters may vary in the operation. In this paper, only g_0 is known among the all of the parameters. Let define the disturbance d as

$$d = f(x) + \Delta g u + d_{ext}(d_1, \dot{d}_1, \ddot{d}_1, d_2). \tag{6}$$

The dynamics of x are obtained as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= g_0 u + d. \end{aligned} \tag{7}$$

The aim is to make the steering wheel angle $\theta_h = x_1$ track the desired steering wheel angle $\theta_{h_d} = x_{1_d}$ using only x_1 feedback and g_0 information.

3. NONLINEAR DAMPING POSITION CONTROLLER

In this Section, we design the nonlinear damping controller with following Assumption.

Assumption 1. x_1, x_2, x_3 and x_4 except for d are available. The estimated steering wheel angle $\hat{x}_1$ and the estimated disturbance $\hat{d}$ are bounded. ◇

In next Section, the observer will be designed to estimate full state and disturbance. We define the tracking error $e = [e_1\ e_2\ e_3\ e_4]^T$ as

$$e_i = x_i - x_{i_d} \tag{8}$$

where x_{1_d} is the desired steering wheel angle obtained by the LKS and x_{i_d}, $i \in [2,\ 4]$ will be designed. The estimated steering wheel angle tracking error $\hat{e}$ is defined as

$$\hat{e}_1 = \hat{x}_1 - x_{1_d}. \tag{9}$$

The tracking error dynamics are

$$\begin{aligned} \dot{e}_1 &= e_2 + x_{2_d} - \dot{x}_{1_d} \\ \dot{e}_2 &= e_3 + x_{3_d} - \dot{x}_{2_d} \\ \dot{e}_3 &= e_4 + x_{4_d} - \dot{x}_{3_d} \\ \dot{e}_4 &= g_0 u + d - \dot{x}_{4_d}. \end{aligned} \tag{10}$$

In order to guarantee the boundedness of e_1, we propose the following controller as

$$\begin{aligned} x_{2_d} &= -k_1 e_1 + \dot{x}_{1_d} \\ x_{3_d} &= -k_2 e_2 + \dot{x}_{2_d} \\ x_{4_d} &= -k_3 e_3 + \dot{x}_{3_d} \\ u &= \underbrace{\frac{1}{g_0}\left(-k_4 e_4 + \dot{x}_{4_d} - \hat{d}\right)}_{u_a} \\ &\quad + \underbrace{\frac{1}{g_0}\left(-\left(k_{d_1}\sqrt{\hat{e}_1^2 + \nu_1} + k_{d_2}\sqrt{\hat{d}^2 + \nu_2}\right) e_4\right)}_{u_b}. \end{aligned} \tag{11}$$

where control gains, k_1, k_2, k_3, k_4, k_{d_1}, k_{d_2}, ν_1 and ν_2 are positive.

Remark 2. In control input u of (11), u_a is the part for stabilization and the other part u_b is a nonlinear damping term to suppress e_1. Actually, since the disturbance includes the external disturbances, system function and input gain uncertainty, it may be difficult to exactly estimate d. As long as the disturbance estimation error $\tilde{d}$ increases, the steering wheel angle tracking error e_1 gets larger. Generally, when d relatively increases, $\tilde{d}$ relatively becomes larger. The nonlinear damping term u_b can enhance the damping effect to suppress indirectly the effect of $\tilde{d}$ to e_1 when $\hat{e}_1$ and $\hat{d}$ increase. ◇

For simplification, we define $k_d(\hat{e}_1, \hat{d})$ as

$$k_d(\hat{e}_1, \hat{d}) = \left(k_{d_1}\sqrt{\hat{e}_1^2 + \nu_1} + k_{d_2}\sqrt{\hat{d}^2 + \nu_2}\right). \tag{12}$$

With the control law (11), the tracking error dynamics (10) become

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + e_2 \\ \dot{e}_2 &= -k_2 e_2 + e_3 \\ \dot{e}_3 &= -k_3 e_3 + e_4 \\ \dot{e}_4 &= -k_4 e_4 - k_d(\hat{e}_1, \hat{d}) e_4 + d - \hat{d}. \end{aligned} \tag{13}$$

In order to prove the boundedness of e_1, we propose the following Theorem 3.

Theorem 3. The tracking error dynamics (13) are the serial interconnected system of the ISS system with the following property as

$$\begin{aligned} &|e_i(t)| \le \exp\left(-\frac{k_i}{2}t\right)|e_i(0)| + \frac{2}{k_i}\sup_{0\le\tau\le t}|e_{i+1}(\tau)| \\ &\text{for } i = 1,2,3 \text{ and} \\ &|e_4(t)| \le \exp\left(-\frac{k_4}{2}t\right)|e_4(0)| + \sup_{0\le\tau\le t}\sigma(\tau) \end{aligned} \tag{14}$$

where

$$\sigma = \frac{|d - \hat{d}|}{0.5k_4 + k_d(\hat{e}_1, \hat{d})}. \tag{15}$$

◇

Proof. From (13), the dynamics of e_i^2, $i \in [1,3]$ are obtained as

$$\begin{aligned} \frac{d}{dt}\left(\frac{e_i^2}{2}\right) &= -k_i e_i^2 + e_i e_{i+1} \\ &\le -\frac{k_i}{2}e_i^2 - \frac{k_i}{2}|e_i|\left(|e_i| - \frac{2}{k_i}|e_{i+1}|\right). \end{aligned} \tag{16}$$

Using Theorem C.2 in Krstic et al. [1995], we derive the following result as

$$|e_i(t)| \le \exp\left(-\frac{k_i}{2}t\right)|e_i(0)| + \frac{2}{k_i}\sup_{0\le\tau\le t}|e_{i+1}(\tau)|. \tag{17}$$

Equation (17) guarantees that the relationship between e_i and e_{i+1} has ISS property. Under Assumption 1, $\hat{e}_1$ and $\hat{d}$ are bounded. The dynamics of e_4^2 are

$$\begin{aligned} \frac{d}{dt}\left(\frac{e_4^2}{2}\right) &= -k_4 e_4^2 - k_d(\hat{e}_1, \hat{d})e_4^2 + (d - \hat{d})e_4 \\ &\le -\frac{k_4}{2}e_4^2 - \left(\frac{k_4}{2} + k_d(\hat{e}_1, \hat{d})\right)|e_4|(|e_4| - \sigma). \end{aligned} \tag{18}$$

Then,

$$|e_4(t)| \le \exp\left(-\frac{k_4}{2}t\right)|e_4(0)| + \sup_{0\le\tau\le t}\sigma(\tau). \tag{19}$$

Equation (19) shows the relationship between e_4 and σ has ISS property. From (17) and (19), the ISS property of the overall tracking error system is (14). Thus the tracking error dynamics (13) are the serial interconnected system of the ISS system. ■

Remark 4. When $\hat{e}_1$ and $\hat{d}$ in the denominator of σ (15) get bigger, σ gets smaller simultaneously. That is, the nonlinear damping u_b in (11) grows so that the effects of $|\tilde{d}| = |d - \hat{d}|$ to e_4 can be sufficiently suppressed. In (19), we see that $k_d(\hat{e}_1, \hat{d})$ helps to suppress $|e_4|$. From the ISS property (14), as $t \to \infty$,

$$|e_1(\infty)| \le \frac{2}{k_1}\sup_{0\le\tau\le\infty}|e_2(\tau)| \cdots \le \frac{8}{k_1 k_2 k_3}\sup_{0\le\tau\le\infty}\sigma(\tau). \tag{20}$$

Consequently, the steering wheel angle tracking error e_1 can be sufficiently suppressed without small $|\tilde{d}|$. ◇

4. OUTPUT FEEDBACK CONTROLLER DESIGN

In this Section, the augmented observer will be designed to estimate full state and disturbance. Then the closed-loop stability will be studied. Let us define x_5 as $x_5 = d$. We define the augmented state x_a as

$$x_a = [x_1\ x_2\ x_3\ x_4\ x_5]^T. \tag{21}$$

The dynamics of d are defined as

$$\dot{d} = \delta. \tag{22}$$

The estimated state $\hat{x}$ and the estimated augmented state $\hat{x}_a$ are defined as

$$\begin{aligned} \hat{x} &= [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4]^T \\ \hat{x}_a &= [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4 \ \hat{x}_5]^T. \end{aligned} \tag{23}$$

The augmented observer is proposed as

$$\dot{\hat{x}}_a = A_o\hat{x}_a + B_o u + L(x_1 - \hat{x}_1) \tag{24}$$

where

$$A_o = \begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix}$$

$$B_o = [0 \ 0 \ 0 \ g_0 \ 0]^T$$

$$L = [l_1 \ l_2 \ l_3 \ l_4 \ l_5]^T$$

are the observer gain matrix. The estimation errors of the state and the augmented state, $\tilde{x}$ and $\tilde{x}_a$ are defined as

$$\begin{aligned} \tilde{x} &= x - \hat{x} \\ \tilde{x}_a &= x_a - \hat{x}_a. \end{aligned} \tag{25}$$

The dynamics of $\tilde{x}_a$ are

$$\dot{\tilde{x}}_a = (A_o - LC_a)\tilde{x}_a + B_d\delta \tag{26}$$

where $B_d = [0 \ 0 \ 0 \ 0 \ 1]^T$ and $C_a = [1 \ 0 \ 0 \ 0 \ 0]$.

Assumption 5. In EPS, $d_1^{(3)}$ and $\dot{d}_2$ exist and are bounded but unknown. ◇

Assumption 6. The state is bounded, i.e., $x \in B_x = \{x \in \mathbb{R}^4 | \|x\|_2 \le b_x\}$ where b_x is unknown positive constant. ◇

In most actual systems, all state variables are physically bounded (Kosut [1983]). Thus Assumption 6 is reasonable. Note that the information of b_x is not required. Under Assumptions 5 and 6, the upper boundedness $\delta_{\max}$ of $|\delta|$ exists such that $|\dot{d}| = |\dot{x}_5| = |\delta| \le \delta_{\max}$. $\delta_{\max}$ is unknown positive.

Proposition 7. Consider the dynamics of $\tilde{x}_a$ (26). Under Assumptions 5 and 6 if the observer gains are chosen such that the roots of

$$s^5 + l_1 s^4 + l_2 s^3 + l_3 s^2 + l_4 s + l_5 = 0. \tag{27}$$

are in the left-half plane, then $\tilde{x}_a$ exponentially converges to the bounded ball $B_{\tilde{x}} = \{\tilde{x}_a \in \mathbb{R}^5 | \|\tilde{x}_a\|_2 \le 2\lambda_{\max}(P_o)\delta_{\max}\}$ where P_o is positive definite such that $(A_a - LC_a)^T P_o + P_o(A_a - LC_a) = -I$ and $\lambda_{\max}(P_o)$ is the maximum eigenvalue of P_o. And $\tilde{x}_a$ is globally uniformly ultimately bounded. ◇

Proof. We define the Lyapunov function V_o as

$$V_o = \tilde{x}_a^T P_o \tilde{x}_a. \tag{28}$$

The derivative of V_o with respect to time is

$$\begin{aligned} \dot{V}_o =& \tilde{x}_a^T [(A_a - LC_a)^T P_o + P_o(A_a - LC_a)]\tilde{x}_a + 2\tilde{x}_a^T P_o B_d \delta \\ \le& -\|\tilde{x}_a\|_2^2 + 2\delta_{\max}\|P_o\|_2\|\tilde{x}_a\|_2 \\ \le& -\|\tilde{x}_a\|_2(\|\tilde{x}_a\|_2 - 2\lambda_{\max}(P_o)\delta_{\max}). \end{aligned} \tag{29}$$

Thus $\tilde{x}_a$ exponentially converges to the bounded ball $B_{\tilde{x}}$. And $\tilde{x}_a$ is globally uniformly ultimately bounded. ■

Actually, only x_1 is available. In (11), $\hat{x}_i, \ i \in [2,4]$ is substituted for $x_i, \ i \in [2,4]$. Thus (11) becomes

$$\begin{aligned} x_{2_d} =& -k_1 e_1 + \dot{x}_{1_d} \\ \hat{x}_{3_d} =& -k_2 \hat{e}_2 + \dot{x}_{2_d} \\ \hat{x}_{4_d} =& -k_3 \hat{e}_3 + \dot{x}_{3_d} \\ \hat{u} =& \frac{1}{g_0}(-k_4\hat{e}_4 + \dot{\hat{x}}_{4_d} - \hat{d} - k_d(\hat{e}_1, \hat{d})\hat{e}_4) \end{aligned} \tag{30}$$

where $\hat{e}_2 = \hat{x}_2 - x_{2_d}$ and $\hat{e}_i = \hat{x}_i - \hat{x}_{i_d}, \ i \in [3,4]$. Equation (30) is implemented in (10) instead of (11). Thus tracking error dynamics (13) become

$$\dot{e} = A_e e + B_e \xi \tag{31}$$

where

$$A_e = \begin{bmatrix} -k_1 & 1 & 0 & 0 \\ 0 & -k_2 & 1 & 0 \\ 0 & 0 & -k_3 & 1 \\ 0 & 0 & 0 & -k_4 \end{bmatrix}$$

$$B_e = [0 \ 0 \ 0 \ 1]^T$$

$$\xi = -k_d(\hat{e}_1, \hat{d})e_4 + d - \hat{d} + g_0\hat{u} - g_0 u.$$

The closed-loop system is

$$\begin{aligned} \dot{e} =& A_e e + B_e \xi \\ \dot{\tilde{x}}_a =& A_o \tilde{x}_a + B_d \delta. \end{aligned} \tag{32}$$

In u (11) and $\hat{u}$ (30), the different desired velocities and the actual accelerations are used respectively. On the other hand, the same desired position x_{1_d} and the actual position x_1 are used in both u (11) and $\hat{u}$ (30). Thus, it is not difficult to show that there exists $k_{\tilde{x}} > 0$ such that

$$|d - \hat{d} + g_0\hat{u}(\hat{x}, x_{1_d}) - g_0 u(x, x_{1_d})| \le \gamma \|x_a - \hat{x}_a\|. \tag{33}$$

Theorem 8. Under Assumptions 5 and 6, the tracking error dynamics (31) have the following ISS property as

$$|e_i(t)| \le \exp\left(-\frac{k_i}{2}t\right)|e_i(0)| + \frac{2}{k_i}\sup_{0\le\tau\le t}|e_{i+1}(\tau)|$$

for $i = 1,2,3$ and

$$|e_4(t)| \le \exp\left(-\frac{k_4}{2}t\right)|e_4(0)| + \sup_{0\le\tau\le t}\sigma_1(\tau) \tag{34}$$

where

$$\sigma_1 \le \frac{\gamma\|\tilde{x}_a\|}{0.5k_4 + k_d(\hat{e}_1, \hat{d})}. \tag{35}$$

◇

Proof. In (14), we show that the tracking errors have the cascade nature. Since the control input u that uses the estimation state $\hat{x}$ is injected to e_4 subsystem of the slow system e, it is sufficient to investigate the behavior of the e_4 subsystem owing to the cascade nature. Since $\hat{u}$ (30) is substituted for u (11), the dynamics of e_4 become

$$\dot{e}_4 = -k_4 e_4 - k_d(\hat{e}_1, \hat{d})e_4 + d - \hat{d} + g_0\hat{u} - g_0 u. \tag{36}$$

Equation (18) is also changed into

$$\frac{d}{dt}\left(\frac{e_4^2}{2}\right) \le -\frac{k_4}{2}e_4^2 + \left(\frac{k_4}{2} + k_d(\hat{e}_1, \hat{d})\right)|e_4|(|e_4| - \sigma_1) \tag{37}$$

where $\sigma_1 = \frac{|d - \hat{d} + g_0\hat{u} - g_0 u|}{0.5k_4 + k_d(\hat{e}_1, \hat{d})}$. From (33),

$$\begin{aligned} \sigma_1 =& \frac{|d - \hat{d} + g_0\hat{u} - g_0 u|}{0.5k_4 + k_d(\hat{e}_1, \hat{d})} \\ \le& \frac{\gamma\|\tilde{x}_a\|}{0.5k_4 + k_d(\hat{e}_1, \hat{d})}. \end{aligned} \tag{38}$$

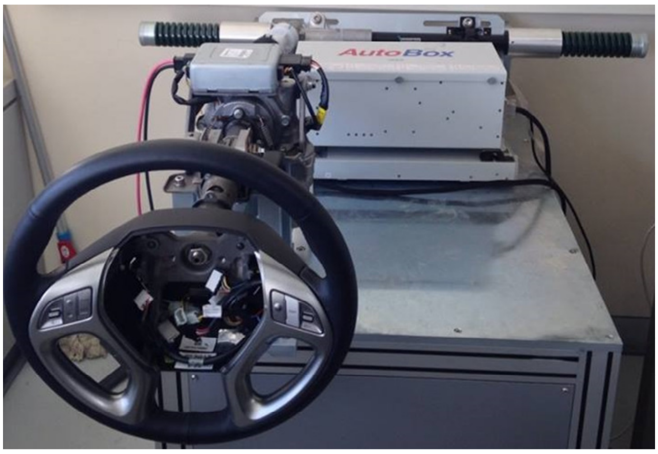


Fig. 1. Photo of the EPS HILS system

Then, (19) is rewritten as

$$|e_4(t)| \le \exp\left(-\frac{k_4}{2}t\right)|e_4(0)| + \sup_{0\le\tau\le t} \sigma_1(\tau). \qquad (39)$$

■

Remark 9. From (34) and (35), the upper bound of e_4 is determined by the that of σ_1 affected by the upper bound of estimation error. From Proposition 7, we see that the upper bound of estimation error is determined by b_x and L. Consequently, since we cannot know how big b_x is, we should suppress the upper bound of σ_1 to obtain the small e_1 using high gain L. However, actually, high observer gain L is not necessary for a small e_1. If the estimation error $\|\tilde{x}_a\|$ is relatively large due to the large b_x, then $\hat{e}_1$ and $\hat{d}$ increase so that σ_1 gets smaller simultaneously due to the nonlinear damping. Furthermore, it was proven that the overall tracking error system (12) is the serial interconnected system of the ISS system from (34). Thus,

$$|e_1(\infty)| \le \frac{2}{k_1}\sup_{0\le\tau\le\infty}|e_2(\tau)|\cdots \le \frac{8}{k_1k_2k_3}\sup_{0\le\tau\le\infty}\sigma_1(\tau) \qquad (40)$$

Consequently, small $\|\tilde{x}_a\|$ and high gain L are not required to obtain the precise steering wheel angle tracking. ◇

5. EXPERIMENTAL RESULTS

Experiments were executed to evaluate the performances of the proposed method. The EPS hardware in the loop simulation (HILS) system is shown in Fig. 1 was used. The EPS hardware in the loop simulation (HILS) system was used. The EPS HILS system consisted of the EPS system, the spring system and the dSPACE. In this system, the mounted spring was used to emulate the self-alignment torque. The torque angle sensor was used to measure the steering wheel angle θ_h and the driver torque T_d as torque sensor in Fig. 1. DS1501 manufactured by dSPACE Inc. was used as an embedded real-time controller. The control sampling rate was 100 Hz. Since the numerical value of used EPS parameters is proprietary information, it is omitted.

5.1 Performance Analysis of the Proposed Method

In order to evaluate the steering wheel angle tracking performance of the proposed method, the PI control method shown in Fig. 2 and the proposed method were tested. The used controller parameters were $k_1 = 200$, $k_2 = 35$, $k_3 = 11$, $k_4 = 10$, $k_{d_1} = 0.000005$, $k_{d_2} = 33$, $\nu_1 = 1$, $\nu_2 = 1$, $l_1 = 2.5133\times10^3$, $l_2 = 2.5266\times10^6$, $l_3 = 1.2700\times10^9$, $l_4 = 3.1919\times10^{11}$, and $l_5 = 3.2088\times10^{13}$. The observer gains were chosen for 4 Hz bandwidth of the augmented observer. In these experiments, $\theta_{h_d} = 0.3\sin(0.05\times2\pi t)$ was used. The experimental results of the PI control method without the driver's torque disturbance are shown in Fig. 3. Due to the structural vibration and quantization effect, the ripples were observed in the experimental results. Since EPS system has slow steering wheel angle response, the large lag in the steering wheel angle tracking was observed. The relatively large tracking errors near the zero velocity periods appeared due to the unsymmetrical hysteresis behaviors of EPS system. The high spring force in the experimental set up might be one of the main causes of the relatively large tracking errors near the zero velocity periods. To overcome the unsymmetrical hysteresis behaviors, the control input was also asymmetric. Since the driver's torque was not injected as the disturbance, the measured driver's torque was almost zero. In PI control method, the steering wheel angle became unstable or diverged due to

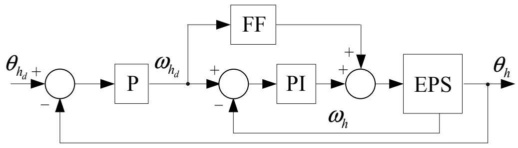
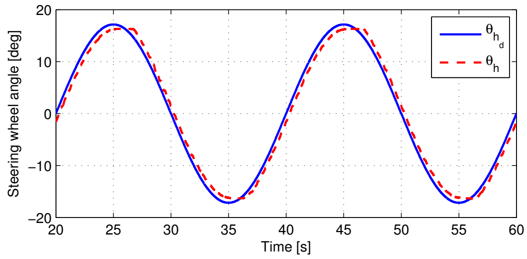
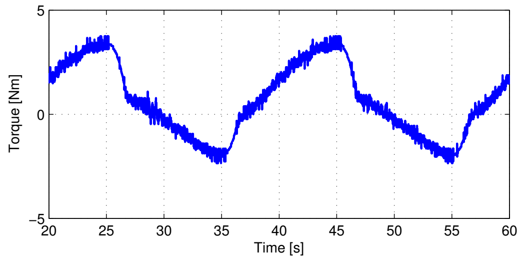


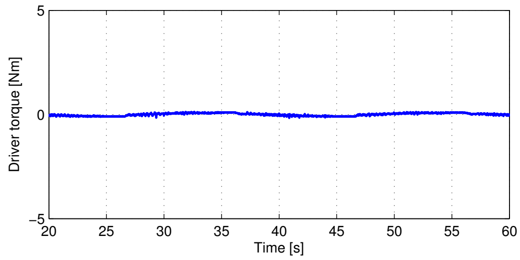
Fig. 2. Block diagram of PI control method



(a) Angle tracking

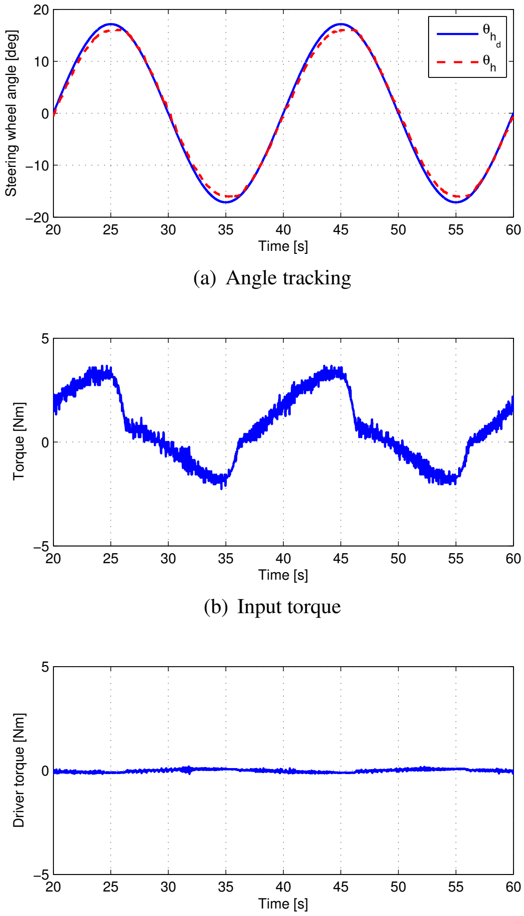


(b) Input torque



(c) Driver torque

Fig. 3. Steering wheel angle tracking performance of PI control method w/o the driver's torque disturbance

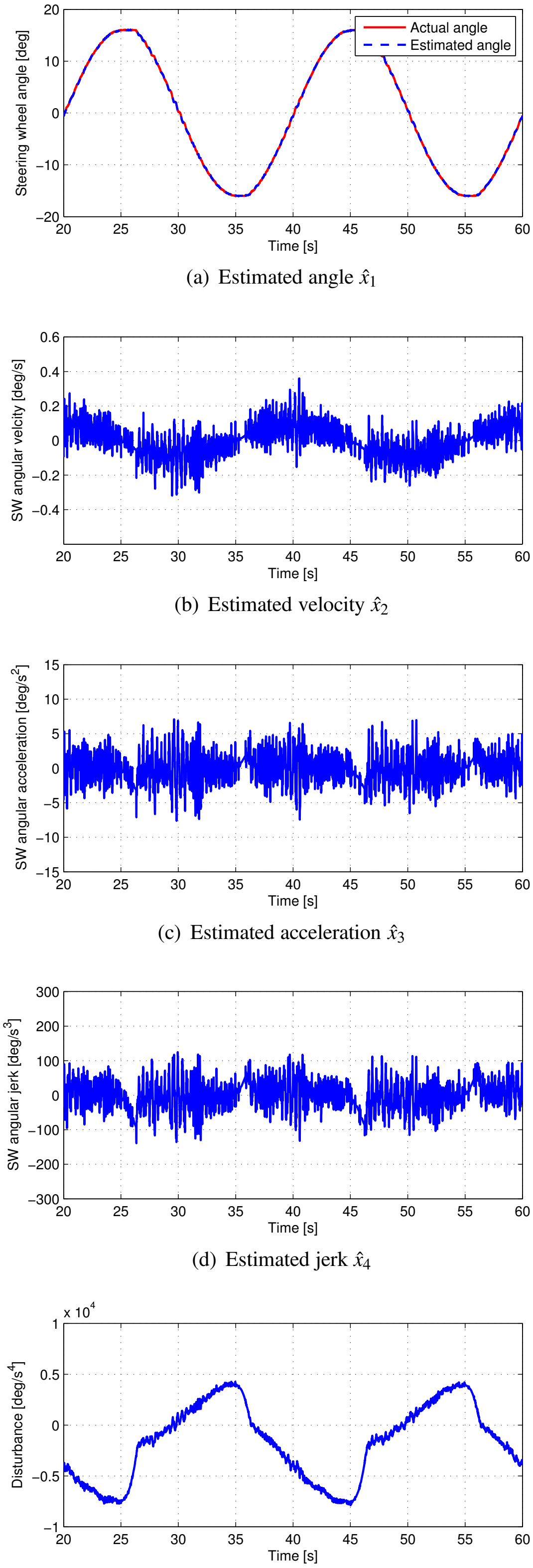


(a) Angle tracking

(b) Input torque

(c) Driver torque

Fig. 4. Steering wheel angle tracking performance of the proposed method w/o the driver's torque disturbance



(a) Estimated angle $\hat{x}_1$

(b) Estimated velocity $\hat{x}_2$

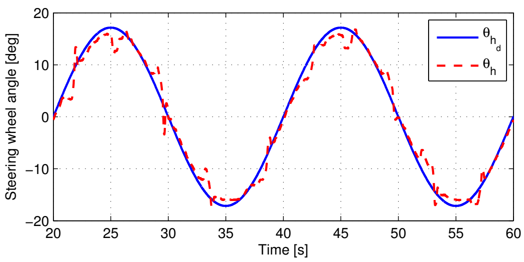
(c) Estimated acceleration $\hat{x}_3$

(d) Estimated jerk $\hat{x}_4$

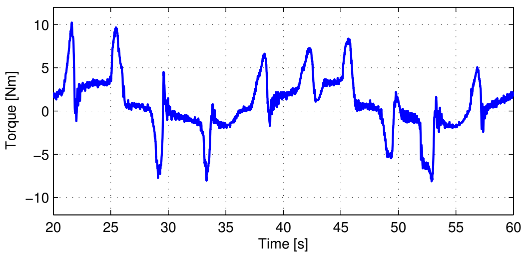
(e) Estimated disturbance $\hat{x}_5 = \hat{d}$

Fig. 5. Estimated state variables of the proposed method w/o the driver's torque disturbance

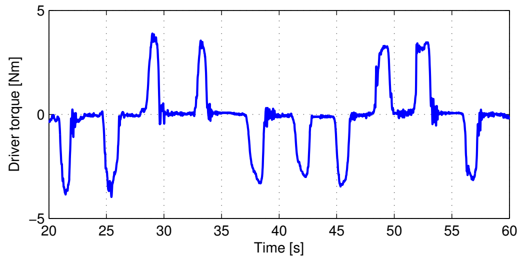
the absence of the disturbance compensation when the driver's torque was injected artificially. The experimental results of the proposed method without the driver's torque disturbance are shown in Fig. 4. The improved steering wheel angle tracking performance was observed compared to PI control method. The asymmetric control input was also observed. Despite the improved performance, the relatively large errors near the zero velocity periods still appeared due to the unsymmetrical hysteresis behaviors and high spring force of EPS system. The estimated state variables are shown in Fig. 5. In Fig. 5(a), it was observed that the estimated angle tracked the actual angle well. Due to the structural vibration, quantization effect, the ripples were also observed in the estimated state. Because of the unsymmetrical hysteresis behaviors, the estimated disturbance was also to the asymmetric in Fig. 5(e). The performances of the proposed method with driver's torque disturbance are shown in Fig. 6. In Fig. 6(c), when the driver tried to strongly hold the steering wheel, the measured driver's absolute torque went up to 4 Nm. To overcome driver's torque, the input torque of EPS also increased. Note that since the driver's torque to hold the steering wheel was activated as torque disturbance as well as angle disturbance in the torque overlay based steering wheel control, the steering wheel control cannot perfectly be free under the driver's torque although the driver's holding torque is compensated for in the torque overlay based steering wheel control. Thus the steering wheel tracking error was relatively



(a) Angle tracking



(b) Input torque



(c) Driver torque

Fig. 6. Steering wheel angle tracking performance of the proposed method w/ the driver's torque disturbance

larger, however, the performance was recovered after the driver released the steering wheel.

6. CONCLUSION

We proposed a torque overlay based robust steering wheel angle control of electric power steering for lateral control using backstepping design. In order to estimate the full state and the disturbance, the augmented observer was designed. The nonlinear damping controller was developed via backstepping to suppress a position tracking error. Via the experiments, it was observed that the steering wheel angle tracking performance was improved by the proposed method. Furthermore, the steering angle tracking performance was recovered due to the disturbance compensation without unstable status of the steering wheel control under the driver's torque disturbance.

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