

Cooperative Adaptive Cruise Control of Vehicles with Sensor Failures

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ABSTRACT This paper investigates nonlinear control of cooperative adaptive cruise control (CACC) system with sensor failures. A nonlinear vehicular model involving sensor failure is established. Based on the nonlinear model, a switching controller design method is proposed. It is shown that the obtained control scheme can achieve the objective of individual vehicle stability and string stability. The effectiveness of the proposed method is demonstrated by a numerical simulation.

Key words: Cooperative adaptive cruise control; switching control; sensor failure; individual vehicle stability; string stability.

1. INTRODUCTION

Traffic congestion is one of today's most serious social, economical, and environmental problems in the world. In China alone, traffic congestion costs billions of dollars each year with hundreds of thousands of persons killed or injured. The problem is intractable since continue to build additional highway capacity is becoming increasingly difficult, for both financial and environmental reasons. As a result, the solution to the problem must lie in other approaches that can make better use of the existing highway infrastructure. Cooperative adaptive cruise control (CACC) is one such strategy regarded as the most promising in intelligent transportation system applications [1-3]. CACC is an extension of the existing longitudinal control function known as adaptive cruise control (ACC), which relieves the driver from adjusting the speed to the vehicle in front.

The synthesis of a CACC system consists of designing a spacing policy and a controller to regulate the speed of the vehicle [4]. Generally, there are two types of spacing policies that are widely used for vehicular cooperative control, i.e., the constant-spacing policy and the constant time headway spacing policy, depending on whether the required spacing of a vehicle is free of its speed. The constant time headway spacing policy applies mainly to ACC of a single car driving control, which has been equipped in many luxury cars [5]. The constant-spacing policy is widely used for autonomous platoon control. Here, as in [6], we will investigate a combined spacing policy.

It is worth noting that most existing results on CACC are limited in at least the following two aspects. First, linearization is frequently used to simplify the model [7]-[9], which has clear shortcomings in practice, since it is usually very difficult for implementation, especially when treated jointly with the effect of sensor failures. Sensor failure is another factor that increases the difficulty of CACC. The

problem of sensor failure has been investigated by researchers in different circumstances, see, e.g., in [10] a sensor data fusion technology was used to estimate the dynamics of the front target which can realized tracking control for autonomous vehicle with sensor failures, and in [11] a guaranteed cost method dealing with limited sensing capability was proposed. However, these results are based on a simplified vehicle model and hence are not adequate for achieving more stringent performance requirement for CACC systems that are nonlinear inessential. To the authors' knowledge, strategies systematically taking into account the desired system performance, nonlinear vehicle dynamics and sensor failures have not yet been reported.

The rest of this paper is organized as follows. In Section 2, a nonlinear CACC model is built by taking into account the sensor failures. In Section 3, a switching controller is designed for the nonlinear CACC system to deal with sensor failures. The issue of string stability is investigated in section 4. Numerical simulations are presented in Section 5, showing the usefulness and effectiveness of the proposed method. The conclusions are given in Section 6.

1. PROBLEM FORMULATION

Consider a CACC system composed by n vehicles (see Fig.1) running in a horizontal environment. All followers are equipped with on-board sensors to measure the distance and relative velocity between it and its preceding vehicle. Each vehicle transmits its acceleration to its follower via a wireless communication channel. In what follows, we will describe the nonlinear CACC vehicle model, sensor failures, and our objective in detail one by one.

1.1. Nonlinear CACC system modeling

Denote by z_i, v_i and a_i the i th ($i=0, \dots, n-1$) vehicle's position, velocity and acceleration, with $i=0$ standing for the lead vehicle and the others being followers.

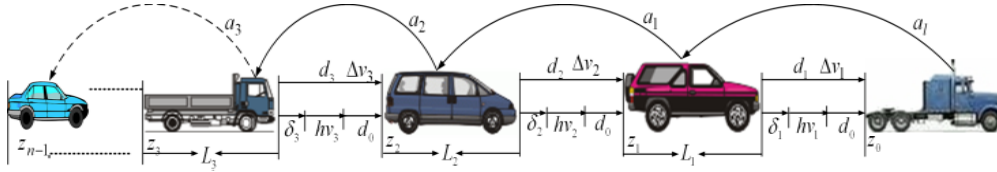


Fig. 1. CACC system

Define the spacing error of the i th following vehicle as:

$$\delta_i = z_{i-1} - z_i - L_i - hv_i - d_0 \quad (z_0 = 0 \text{ in } \delta_1), \quad (1)$$

where h is the time gap, d_0 is a given minimum distance, L_i is the length of the vehicle. Then the dynamics of the i th following vehicle can be modeled by the following nonlinear differential equations (see e.g., [12] [13] for details):

$$\dot{\delta}_i = v_{i-1} - v_i - hv_i, \quad (2)$$

$$\dot{e}_i = a_{i-1} - a_i, \quad (3)$$

$$\dot{a}_i = f_i(v_i) + g_i(v_i)u_i, \quad (4)$$

where u_i is the control input of the i th vehicle's engine/brake, with $u_i \geq 0$ and $u_i < 0$ representing the throttle input and the brake input, respectively, $f_i(v_i, a_i)$ and $g_i(v_i)$ are given by:

$$f_i(v_i) = -\frac{1}{\zeta_i} \left(\dot{v}_i + \frac{\sigma A_i c_{di}}{2m_i} v_i^2 + \frac{d_{mi}}{m_i} \right) - \frac{\sigma A_i c_{di} v_i \dot{v}_i}{m_i}, \quad (5)$$

$$g_i(v_i) = \frac{1}{\zeta_i m_i}, \quad (6)$$

with σ , A_i , c_{di} , d_{mi} , m_i and ζ_i being the specific mass of the air, the cross-sectional area, drag coefficient, mechanical drag, mass and engine time constant of the i th vehicle, respectively. Here, $\sigma A_i c_{di} / 2m_i$ stands for the air resistance. Note that the vehicles considered here can be different (in size and weight, etc.), while most existing results consider identical vehicles.

By combining the dynamics of the vehicular system (2)-(4) and equation (1), and setting $w_i(t) = a_{i-1}(t)$ as a measurable disturbance from the preceding vehicle, we end up with the following nonlinear state space equation for the CACC system

$$\dot{x}_i(t) = F_i(x_i(t)) + G_i u_i(t), \quad y_i(t) = [x_i^T(t), w_i(t)]^T, \quad (7)$$

where $x_i(t) = [\delta_i(t) \quad e_i(t) \quad a_i(t)]^T$ ($i=1, \dots, n-1$) is the state of the CACC system, $y_i(t)$ is the measurement output,

$$F_i(x_i(t)) = [v_{i-1} - v_i - ha_i \quad a_{i-1} - a_i \quad f_i(v_i, a_i)]^T, \quad \text{and}$$

$$G_i = \begin{bmatrix} 0 & 0 & 1/m_i \zeta_i \end{bmatrix}^T \text{ is a nonlinear time-varying term.}$$

Since $x_i=0$ is an equilibrium of the CACC system (7), i.e., $F_i(0)=0$, and according to [17], we can design the following switching control $u_i(k)$ to stabilize each following vehicle:

$$u_i(k) = C_i u_i(k-1) + D_i y_i(k-1), \quad (8)$$

where $k \in N$ and $u_i(k)$ represents the control between $[kT, (k+1)T]$, T is the fixed period of the switching time, $u_i(k)$ switches values at every fixed time kT , and C_i is a proper matrix, $D_i = [p_p \quad p_e \quad p_a \quad p_{ac}]$ are the controller gain vector to be designed.

Remark 1. A). Note that the switching time T is different from the sampling period of digital implementation of continuous-time control systems. Usually, the switching time is much longer than the sampling period. **B).** The control law (8) is based on the spacing error and the relative velocity error between vehicle i and its preceding vehicle, the acceleration of vehicle i , and the acceleration of vehicle $i-1$. The first two quantities are measured by on-board sensors while the preceding vehicle's acceleration is transmitted through a wireless communication channel.

2.2. Effect of sensor failures

In this subsection, we consider the problem of sensor failures, and adopt the general failure model in [14] to describe the failure phenomena in the distance and relative velocity sensors, namely, $[\delta_i^f(k) \quad e_i^f(k)] = \rho_i [\delta_i(k) \quad e_i(k)]$, where the failure status ρ_i is a Bernoulli process with probabilities $\Pr[\rho_i = 0] = p_i$ and $\Pr[\rho_i = 1] = 1 - p_i$. Thus p_i represents the sensor failure probability of the i th vehicle.

Taking sensor failure effects into consideration, the measurements output vector for vehicle i can be written as:

$$y_i^f(k) = \bar{\rho}_i y_i(k), \quad (9)$$

where $y_i^f(k)$ is the output from the sensor that failed, $\bar{\rho}_i = \text{diag}\{\rho_i, \rho_i, 1, 1\}$ is the failure status matrix of the i th vehicle.

We now proceed to show how the sensor failures affect the nonlinear CACC system. To this end, we rewrite the controller in (8) as

$$u_i(k) = C_i u_i(k-1) + D_{iw} w_i(k-1) + D_{ix\bar{\rho}_i} x_i(k-1), \quad (10)$$

where $D_{ix\bar{\rho}_i} = [\rho_i p_p \quad \rho_i p_e \quad p_a]$, $D_{iw} = p_{ac}$.

Replacing $F_i(x_i(t))$ in (7) by its Taylor series expansion and according to (10), the CACC system (7) can be rewritten as:

$$\dot{x}_i(t) = A_i x_i(t) + G_i u_i(k) + h_i(x_i), \quad (11)$$

$$u_i(k+1) = C_i u_i(k) + D_{iw}(k) w_i(k) + D_{ix\bar{\rho}_i} x_i(k). \quad (12)$$

where $h_i(x_i)$ contains all the high-order terms of the Taylor series of $F_i(x_i(t))$, and $A_i = (\partial F_i(x_i)/\partial x_i)|_{x_i=0}$

From (11), we then have

$$x_i(k+1) = e^{A_i T} x_i(k) + \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} h_i(x_i) d\tau + G_i \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} d\tau u_i(k). \quad (13)$$

Let $\hat{x}_i(k+1) = [x_i(k+1), u_i(k+1)]^T$. Then, by (12) and (13), we can obtain that

$$\hat{x}_i(k+1) = H_i \hat{x}_i(k) + M_i(k, \hat{x}_i(k)) \quad (14)$$

where

$$H_i = \begin{bmatrix} e^{A_i T} & \Delta_{i0}(k) \\ D_{i\alpha} e^{A_i T} & C_i + \Delta_{i0}(k) \end{bmatrix},$$

$$M_i(k, \hat{x}_i(k)) = [\Delta_{i1}(k) \quad D_{i\alpha} w_i(k)]^T,$$

and

$$\Delta_{i0}(k) = G_i \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} d\tau, \quad \Delta_{i1}(k) = \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} h_i(x_i) d\tau.$$

Since $h_i(x_i)$ contains all high-order terms of the Taylor series expansion of $F_i(x_i(t))$ at the equilibrium configuration, $\lim_{\|x_i\| \rightarrow 0} (\|h_i(x_i)\|/\|x_i\|) = 0$. It follows that there exists a $\varepsilon_i > 0$ such that

$$\|h_i(x_i)\| \leq \|x_i\| \quad (15)$$

whenever $\|x_i\| \leq \varepsilon_i$.

2.3. The objective

Our objective of this research is to design a switching controller for the CACC system to maintain a safety inter-vehicle spacing and to meet the following criteria:

(i) Individual vehicle stability: the entire closed-loop CACC system is exponentially stable.

(ii) Steady state performance: the relative velocity errors $\Delta v_i(z)$ approach to zero for all vehicles.

(iii) String stability: the oscillations are not amplifying with vehicle index due to any maneuver of the lead vehicle, namely, $\|G(e^{w})\| \leq 1$ for any w , where $G(z) = \delta_i(z)/\delta_{i-1}(z)$ with $\delta_i(z)$ and $\delta_{i-1}(z)$ denotes the z-transforms of the spacing $\delta_i(t)$ and $\delta_{i-1}(t)$, respectively.

2. SWITCHING CONTROLLER DESIGN

In this subsection, we give a switching control method for the nonlinear CACC system to ensure that all the vehicles in the string are asymptotically stable under the effect of sensor failures. We first present the following two propositions which play a key role in the main results.

Proposition 1: Consider the CACC system composed by (11) and (12). For any $k \in N$, it is true that $\|x_i(r)\| \leq \chi_i$ for all

$r \in [kT, (k+1)T]$, whenever $\|x_i(k)\| \leq \chi_{i1}$ and $\|u_i(k)\| \leq \chi_{i1}$, where $\chi_{i1} = e^{-T(\|A_i\|+1)}(1+1/m_i \zeta_i) \chi_i$.

Proof. Suppose this is not true, then there must exist a $r_0 \in [kT, (k+1)T]$, such that $\|x_i(r_0)\| = \chi_i$ and $\|x_i(r)\| \leq \chi_i$ for all $r \in [kT, r_0]$. By (11), $\forall r \in [kT, r_0]$, we have that

$$\|x_i(r)\| \leq \|x_i(k)\| + \|u_i(k)\|/m_i \zeta_i + \int_{kT}^r [\|A_i\| + 1] \|x_i(\tau)\| d\tau, \quad \text{where}$$

we have used the fact $\|h_i(x_i(\tau))\| \leq \|x_i(\tau)\|$. Since $\|x_i(k)\| \leq \chi_{i1}$ and $\|u_i(k)\| \leq \chi_{i1}$, by the Gronwall- Bellman inequality, the following holds true

$$\|x_i(r)\| \leq (\|x_i(k)\| + \|u_i(k)\|/m_i \zeta_i) e^{\|A_i\| (r-kT)} \quad (16)$$

for all $r \in [kT, r_0]$.

Hence, $\|x_i(r_0)\| \leq \chi_i e^{\|A_i\| (r_0-kT-T)} \leq \chi_i$, where $r_0 < kT + T$.

This contradicts with the assumption that $\|x_i(r_0)\| = \chi_i$. This completes the proof.

It follows from (17) that

$$\|x_i(r)\| \leq (\|x_i(k)\| + \|u_i(k)\|)(1+1/m_i \zeta_i) e^{\|A_i\| (r-kT)}, \quad (17)$$

for all $r \in [kT, (k+1)T]$, whenever $\|x_i(k)\| \leq \chi_{i1}$ and $\|u_i(k)\| \leq \chi_{i1}$.

Proposition 2: Consider the CACC system composed by (11) and (12). For any given $\varepsilon_{i1} > 0$, there exists a $\chi_{i4} > 0$ and $\chi_{i4} \leq \chi_{i1}$, such that for any given $k \in N$ it is true that $\|\Delta_{i1}(k)\| \leq \varepsilon_{i1} \|\hat{x}_i\|$, whenever $\|x_i(k)\| \leq \varepsilon_{i4}$ and $\|u_i(k)\| \leq \varepsilon_{i4}$.

Proof. For any given $\varepsilon_{i1} > 0$, choose $\varepsilon_{i3} > 0$ such that $\varepsilon_{i1} = \sqrt{2} \varepsilon_{i3} \cdot e^{2(\|A_i\|+1)}(1+1/m_i \zeta_i)$. According to (15), there exists a χ_{i3} such that $\|h_i(x_i)\| \leq \varepsilon_{i3} \|x_i\|$ whenever $\|x_i\| \leq \chi_{i3}$.

Define $\chi_{i4} = \min\{\chi_{i1}/2, (\chi_{i3}/(2(1+1/m_i \zeta_i) e^{\|A_i\| (T-kT)}))\}$. Then, whenever $\|x_i(k)\| \leq \chi_{i4}$ and $\|u_i(k)\| \leq \chi_{i4}$, it is true by (17) that

$$\|x_i(r)\| \leq 2\chi_{i4}(1+1/m_i \zeta_i) e^{\|A_i\| (r-kT)} \leq \chi_{i3} \quad (18)$$

for all $r \in [kT, (k+1)T]$. So, from (17), we have that

$$\|h_i(x_i(r))\| \leq \varepsilon_{i3} (\|x_i(k)\| + \|u_i(k)\|) \cdot (1+1/m_i \zeta_i) e^{\|A_i\| (r-kT)} \quad (19)$$

for all $r \in [kT, (k+1)T]$, whenever $\|x_i(k)\| \leq \chi_{i4}$ and $\|u_i(k)\| \leq \chi_{i4}$.

In addition, according to (15), $\Delta_{i1}(k)$ can be written as

$$\Delta_{i1}(k) = e^{\|A_i\| kT} \int_{kT}^{(k+1)T} \|h_i(x_i(\tau))\| d\tau \quad (20)$$

whenever $\|x_i\| \leq \chi_{i4}$ and $\|u_i\| \leq \chi_{i4}$. So, by (19), we have

$$\|\Delta_{i1}(k)\| \leq (\|x_i(k)\| + \|u_i(k)\|) \varepsilon_{i1} / \sqrt{2}, \quad \text{whenever the conditions}$$

$\|x_i\| \leq \chi_{i4}$ and $\|u_i\| \leq \chi_{i4}$ are satisfied. Furthermore, since $\|x_i(k)\| + \|u_i(k)\| \leq \sqrt{2} \|\hat{x}_i(k)\|$, we know that for any given

$\varepsilon_{i1} > 0$, there exists a $\varepsilon_{i4} > 0$, such that $\|\Delta_{i1}(k)\| \leq \varepsilon_{i1} \|\hat{x}_i(k)\|$, whenever $\|x_i(k)\| \leq \chi_{i4}$ and $\|u_i(k)\| \leq \chi_{i4}$ for any $k \in N$, where $\chi_{i4} \leq \chi_{i1}$. This completes the proof.

Theorem 1: The CACC system in the form of (11) can be locally asymptotically stabilized to the equilibrium state by the switching controller in (12), if C_i , D_{ix} and T can be chosen such that H_i has all eigenvalues within the unit circle, where

$$H_i = \begin{bmatrix} e^{A_i T} & G_i \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} d\tau \\ D_{ix} e^{A_i T} & C_i + G_i \int_{kT}^{(k+1)T} e^{A_i((k+1)T-\tau)} d\tau \end{bmatrix}. \quad (21)$$

Proof. Substitute the first term on the right-hand of (14) with its Taylor series expansion at the origin, and replace all high-order terms with Δ_{i2} , then, there must exist a χ_{i2} and ε_{i2} such that

$$\|\Delta_{i2}(\hat{x}_i)\| < \varepsilon_{i2} \|\hat{x}_i\| \quad (22)$$

By Proposition 2 and (22), we have $\|M_i(k, \hat{x}_i(k))\| \leq \|\Delta_{i1}(k)\| + \|\Delta_{i2}(\hat{x}_i)\| < (\varepsilon_{i1} + \varepsilon_{i2}) \|\hat{x}_i\|$, choosing $\varepsilon_i = \varepsilon_{i1} + \varepsilon_{i2}$, then $\|M_i(k)\| \leq \varepsilon_i \|\hat{x}_i(k)\|$ whenever $\|\hat{x}_i\| < \chi_{i0}$ and $\chi_{i0} = \min\{\chi_{i4}, \chi_{i2}\}$.

Assume all eigenvalues of H_i in (21) are within unit circle, then there must exist a positive-definite matrix P_i , such that $H_i^T P_i H_i - P_i = -2I$. Define the following Lyapunov function

$$V_i(\hat{x}_i, r) = \hat{x}_i^T(r) P_i \hat{x}_i(r). \quad (23)$$

For any $k \in N$, by (23), we have

$$\begin{aligned} & V_i(\hat{x}_i(k+1), k+1) - V_i(\hat{x}_i(k), k) \\ & \leq -2\|\hat{x}_i(k)\|^2 + \|M_i(k)\|^2 \cdot \|P_i\| + 2\|M_i(k)\| \|P_i H_i\| \|\hat{x}_i(k)\|. \end{aligned} \quad (24)$$

Choose $0 < \varepsilon_{i0} < (\sqrt{\|P_i H_i\|^2 + 2\|P_i\|} - \|P_i H_i\|) / \|P_i\|$, then $2\varepsilon_{i0} \|P_i H_i\| + \varepsilon_{i0}^2 < 2$. By Proposition 2, there must exist a $\chi_{i0}(\varepsilon_{i0})$, such that $\|M_i(k)\| \leq \varepsilon_{i0} \|\hat{x}_i(k)\|$, whenever $\|\hat{x}_i(k)\| < \chi_{i0}(\varepsilon_{i0})$. Then, from (24), we have

$$\begin{aligned} & V_i(\hat{x}_i(k+1), k+1) - V_i(\hat{x}_i(k), k) \\ & < \|\hat{x}_i(k)\|^2 (-2 + \varepsilon_{i0} \|P_i H_i\| + \varepsilon_{i0}^2 \|P_i\|) < 0 \end{aligned} \quad (25)$$

In what follows, we will show that (25) is true for any $k_0 \in N$ and all $k \geq k_0$ whenever

$$\|\hat{x}_i(k_0)\| < \sqrt{\lambda_{\min}(P_i) / \lambda_{\max}(P_i)} \chi_{i0}(\varepsilon_{i0}) \quad (26)$$

where $\lambda_{\min}(P_i)$ and $\lambda_{\max}(P_i)$ represent the minimum and maximum eigenvalues of P_i , respectively. From (22) and (26), we have

$$V_i(\hat{x}_i(k_0), k_0) \leq \lambda_{\max}(P_i) \|\hat{x}_i(k_0)\|^2 < \lambda_{\min}(P_i) \chi_i^2(\varepsilon_0).$$

Since $\|\hat{x}_i(k)\| \leq \chi_i(\varepsilon_0)$, we get from (25) that $V_i(\hat{x}_i(k_0+1), k_0+1) < V_i(\hat{x}_i(k_0), k_0)$. Therefore

$$V_i(\hat{x}_i(k_0+1), k_0+1) \leq \lambda_{\min}(P_i) \chi_i^2(\varepsilon_0), \quad (27)$$

which along with (22) implies that $\|\hat{x}_i(k_0+1)\| < \chi_i(\varepsilon_0)$. It then follows that $V_i(\hat{x}_i(k_0+2), k_0+2) < V_i(\hat{x}_i(k_0+1), k_0+1)$ and $\|\hat{x}_i(k_0+2)\| < \chi_i(\varepsilon_0)$. By mathematical induction, it is easy to obtain that $\|\hat{x}_i(k)\| < \chi_i(\varepsilon_0)$ for all $k \geq k_0$. Hence, (25) is true for all $k \geq k_0$ as long as (26) is true.

Choose $\mu_i = \max(e^{\|A_i\|} \cdot \pi_i e^{\|A_i\|})$. Then From (16), we know that $\|x_i(r)\| \leq \mu_i (\|x_i(k)\| + \|u_i(k)\|)$. Also, since $\|x_i(k)\| \leq \sqrt{2}\mu_i \|\hat{x}_i(k)\|$ whenever $\|x_i(r)\| \leq \chi_{i1}$, then

$$\begin{aligned} & \lambda_{\max}(P_i) (\|x_i(r)\|^2 + \|u_i(k)\|^2) \lambda_{\min}(P_i) \\ & \leq \lambda_{\min}(P_i) (\sqrt{2}\mu_i + 1)^2 \|\hat{x}_i(k)\|^2 \lambda_{\max}(P_i). \end{aligned}$$

Then, from (21), we have

$$\lambda_{\min}(P_i) \|\hat{x}_i(k)\|^2 \leq V_i(\hat{x}_i(k), r) \leq \lambda_{\max}(P_i) \|\hat{x}_i(r)\|^2$$

$$V_i(\hat{x}_i(r), r) \leq (\sqrt{2}\mu_i + 1)^2 \lambda_{\max}(P_i) V_i(\hat{x}_i(k), k) / \lambda_{\min}(P_i).$$

It follows from Theorem 4.1 in [15] that the equilibrium state of the system (11) and (12) is uniformly asymptotically stable. This completes the proof.

3. STRING STABILITY ANALYSIS

In the previous section, considerations have been focused primarily on asymptotic stability of all the individual vehicles in the CACC system. This section is concerned with the issue of string stability, which is associated with objectives (ii) and (iii) given in subsection 2.3. Here, we give an additional set of constraint results on string stability, which are derived based on the switching controller (12).

Theorem 2. The closed-loop CACC system (11) is string stable if the following conditions are satisfied:

$$\begin{cases} n_2 = 0 \\ n_4 n_3 + n_1 n_0 \leq 0 \end{cases} \quad (27)$$

where $n_0 = \rho_i p_e T$, $n_1 = \rho_i p_p - p_{ac} T^2$, $n_2 = -\zeta_i m_i T + \rho_i p_p + \rho_i p_e T + p_v T + p_a T^2$, $n_3 = 2\zeta_i m_i T$, $n_4 = \zeta_i m_i$.

Proof: By Maclaurin series expansion of $f_i(v_i)$ in (4), we obtain

$$\dot{a}_i(t) = \frac{u_i(t) - d_{mi}}{m_i \zeta_i}, \quad (28)$$

The above equation can be represented by difference approximation,

$$u_i(k) = \frac{\zeta_i m_i (a_i(k) - a_i(k-1))}{T} + d_{mi}, \quad (29)$$

and combined with the switching controller (12), we can get $u_i(k) - u_i(k-1)$

$$\begin{aligned} & = \rho_i p_p \delta_i(k) + \rho_i p_e e_i(k) \\ & + p_v v_i(k) + p_a a_i(k) + p_{ac} a_{i-1}(k). \end{aligned} \quad (30)$$

From (29) and (30), the relation between $\delta_i(z)$ and $\delta_{i-1}(z)$ in the z-domain can be written as,

$$\frac{\delta_i(z)}{\delta_{i-1}(z)} = \frac{n_1 z^3 + n_0 z}{n_2 z^3 + n_3 z - n_4} \quad (31)$$

If we impose the condition $|\delta_i(e^{jwT})/\delta_{i-1}(e^{jwT})| \leq 1$ for any w , then, we can obtain the following inequality:

$$(n_2 n_3 - n_1 n_0) \cos(2Tw) + n_4 n_2 \cos(3Tw) - n_4 n_3 \cos(Tw) \geq 0, \quad (32)$$

If the conditions $n_2 = 0$ hold, then we get $-n_1 n_0 \cos(2Tw) - n_4 n_3 \cos(Tw) \geq 0$

Due to $n_4 n_3 > 0$, $\cos(Tw) \geq 1 - \frac{T^2 w^2}{2}$,

and $\cos(2Tw) \geq 1 - 2T^2 w^2$, we have for $w > 0$ that $n_4 n_3 + n_1 n_0 \leq 0$, which is identical with the second inequality in (27). This completes the proof.

Remark 2. It is important to emphasize that the string stability requirement does not impose serious constraints on the obtained switching controller gains in Theorem 1.

4. SIMULATIONS

In this section, we show how to apply the proposed control method to a three-vehicle CACC system, which runs in a virtual environment established using System Build software

package in MATLAB. Comparisons are made between the new method and the RBFNN in [16]. The parameters of the vehicles can be obtained from paper [13]. In the simulation, we suppose that all the following vehicles have the same sensor failure status, namely, $\rho_i = \rho$. We use a Bernoulli sequence to describe the sensor operating mode over time interval $[0, 60s]$, as shown in Fig. 2, the normal operation status $\rho = 1$ with probability 0.97 and failure status $\rho = 0$ with probability 0.03. (32)

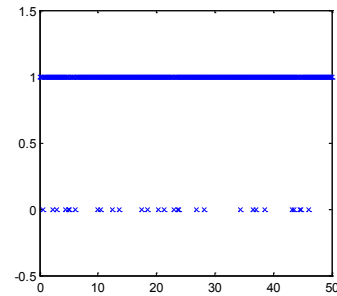


Fig. 2 sensor failure status

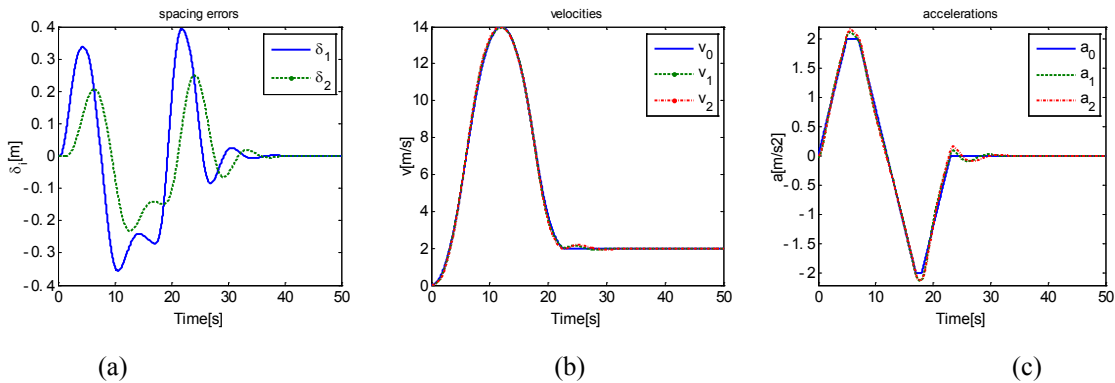


Fig. 3 Five-vehicle CACC system under proposed controller: (a) Spacing errors; (b) Velocities; (c) Accelerations.

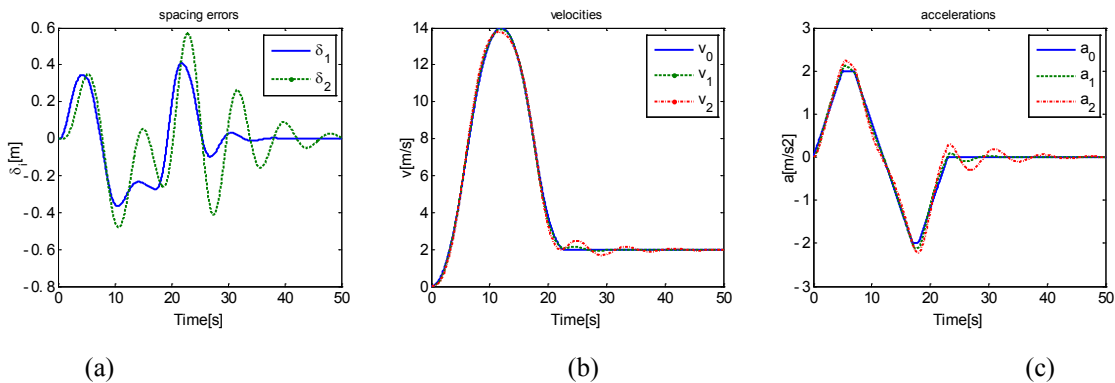


Fig. 4 Five-vehicle CACC system under controller [16]: (a) Spacing errors; (b) Velocities; (c) Accelerations.

By Theorem 1 and 2, choose $p_p = 28.84$, $p_e = 21.94$, $p_a = 2.13$, $p_{ac} = 1.56$, $p_i = 5\%$, $T = 0.1s$. Simple

calculations show that the eigenvalues of H_i are $0.075 \pm 0.043j$ and $0.065 \pm 0.168j$, which are within the unit circle. Choose sampling period of the digital system as

0.005s. Using the aforementioned parameters for the CACC system, Fig.3 was obtained, which has shown the obvious advantages over those given in [16]. The maximum absolute spacing error and acceleration are 0.39 m and 2.2 m/s^2 , respectively, showing that the whole vehicular string is tracking accurately. In the same case, when the method suggested in [16] is used, the system is string unstable (see Fig. 4 the spacing error is amplified as they propagate along the string of vehicles). The maximum absolute spacing error and acceleration are 0.6 m and 2.5 m/s^2 , respectively, which are much higher than in our case in Fig. 3.

5. CONCLUSIONS

This paper has developed a nonlinear CACC approach using a switching control scheme. By considering the sensor failure phenomena, a switching controller is designed. The effectiveness of the presented method was demonstrated by simulations.

In future research, we plan to study the integrated constraints of sensor and communication network and derive more effective and practical CACC methods.

6. ACKNOWLEDGEMENTS

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