

A Realistic Process Example for MIMO MPC based on Autoregressive Models

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Abstract: Advanced controllers such as model predictive control are in use for a wide range of application in the process industry. The potential utilization of such advanced predictive controllers is far from exhausted. One barrier for more widespread implementation is the lack of simple methodologies for advanced control design development which may be used by non experts in control theory. This paper presents and illustrates the use of a simple methodology to design an offset-free MPC based on ARX models. Hence a mechanistic process model is not required. The forced circulation evaporator by Newell and Lee is used to illustrate the offset-free MPC based on ARX models for a nonlinear multivariate process.

Keywords: Chemical Process Control, Model Predictive Control, Autoregressive Models

1. INTRODUCTION

Model predictive control (MPC) has evolved to become an industrial standard in advanced process control with a steady increasing number of applications [15]. One main reason for this success is the explicit use of a plant model to predict future system behavior and determine the control action by solving an optimization problem respecting the actuator constraints. Obviously the closed loop performance critically depends on the quality of the available plant model. For continuous operation of process systems, a linear model representation is often sufficient while systems dominated by a nonlinear transient operation may require the use of nonlinear prediction models. While MPC is becoming a mature control technology there are still several domains where research is needed. One of the main challenges for many applications is the development of suitable models of the process system. Tuning of offset-free controllers, performance diagnostics for retuning or remodeling, economical performance objective and very large or nonlinear applications are other challenges that keeps researcher and vendors of control system software busy.

A wide range of process systems are often subjected to unknown sustained disturbances. In such cases it is required that the MPC incorporates integrating modes to render offset-free tracking. Methods for introducing such integrating modes are given in Muske and Rawlings [9] and Pannocchia and Rawlings [13]. One drawback from introducing integrating modes in the controller is that it leads to a deliberate model plant mismatch which complicates the tuning procedure. Through a number of recent publications, we have advocated for a methodology for offset free ARX based linear MPC that is simple to implement [2, 4, 3, 12, 5]. This methodology is based on estimation

of linear MISO models by standard convex optimization tools, a simple noise model to ensure offset-free tracking and a state space system formulation in innovation form. The state space representation is convenient for closed loop system analysis and tuning. So far studies have only been performed based on linear process systems.

The purpose of the paper is therefore twofold:

- To illustrate the proposed offset-free MPC methodology for a realistic nonlinear process example.
- To show by simple means how advanced control can be realized for the process industry.

This paper starts in Section 2 with an introduction to MPC based on ARX models and a formulation of the control problem. Then the case study is briefly motivated and introduced in Section 3. In Section 4 linear models for the MPC are identified and finally closed loop simulations are presented in Section 5. Section 6 provides the conclusions.

2. ARX-BASED MPC FOR MIMO SYSTEMS

In this section, we derive a state space representation for an unconstrained MPC based on MISO ARX-models modified with a filtered integrated white noise stochastic model. First, we represent the MISO ARX model as a state space model in innovation form. Subsequently, we use this state space model in innovation form to derive the filter equation and the MPC control problem.

2.1 State Space Model in Innovation Form

The discrete time MISO ARX model with time index k

$$A_i(q^{-1})y_{i,k} = B_i(q^{-1})u_k + \varepsilon_{i,k} \quad i = 1, \dots, n_y \quad (1)$$

with $y_{i,k} \in \mathbb{R}$, $\varepsilon_{i,k} \in \mathbb{R}$ for $i = 1, \dots, n_y$ and $u_k \in \mathbb{R}^{n_u}$, has been used in a number of MPC applications.

The advantage of this model parameterization is that the parameters may be identified using standard system identification techniques based on convex optimization. To have offset-free control from the MPC based on this model, the stochastic part of the model is modified to be a filtered white noise process

$$\varepsilon_{i,k} = \frac{1 - \alpha_i q^{-1}}{1 - q^{-1}} e_{i,k} \quad i = 1, \dots, n_y \quad (2)$$

where the coefficients α_i are design parameters of the MPC and $e_{i,k} \sim \mathcal{N}_{iid}(0, R_{ee})$ with the covariance matrix R_{ee} .

The representation of the MIMO system from these MISO systems is not unique. One straightforward representation leading to a compact notation is

$$A(q^{-1})y_k = B(q^{-1})u_k + (I - Iq^{-1})^{-1} (I - \mathcal{A}q^{-1})e_k \quad (3)$$

with $A(q^{-1}) = \text{diag}([A_1(q^{-1}); \dots; A_{n_y}(q^{-1})])$, $B(q^{-1}) = [B_1(q^{-1}); \dots; B_{n_y}(q^{-1})]$, and $\mathcal{A} = \text{diag}([\alpha_1; \dots; \alpha_{n_y}])$. This model can be represented as an ARMAX model

$$\bar{A}(q^{-1})y_k = \bar{B}(q^{-1})u_k + \bar{C}(q^{-1})e_k \quad (4)$$

with

$$\bar{A}(q^{-1}) = (I - Iq^{-1})A(q^{-1}) \quad (5a)$$

$$\bar{B}(q^{-1}) = (I - Iq^{-1})B(q^{-1}) \quad (5b)$$

$$\bar{C}(q^{-1}) = I - \mathcal{A}q^{-1} \quad (5c)$$

Denote the coefficients of $\bar{A}(q^{-1})$ and $\bar{B}(q^{-1})$ as

$$\bar{A}(q^{-1}) = I + \bar{A}_1 q^{-1} + \bar{A}_2 q^{-2} + \dots + \bar{A}_n q^{-n} \quad (6a)$$

$$\bar{B}(q^{-1}) = \bar{B}_1 q^{-1} + \bar{B}_2 q^{-2} + \dots + \bar{B}_n q^{-n} \quad (6b)$$

Then the system (1)-(2) may be represented as a state space model in innovation form [6]. I.e. perfect correlation between measurement and process noise.

$$x_{k+1} = \hat{A}x_k + \hat{B}u_k + \hat{K}e_k \quad (7a)$$

$$y_k = \hat{C}x_k + e_k \quad (7b)$$

with the state space matrices $(\hat{A}, \hat{B}, \hat{K}, \hat{C})$ realized in observer canonical form

$$\hat{A} = \begin{bmatrix} -\bar{A}_1 & I & 0 & \dots & 0 \\ -\bar{A}_2 & 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\bar{A}_{n-1} & 0 & 0 & \dots & I \\ -\bar{A}_n & 0 & 0 & \dots & 0 \end{bmatrix} \hat{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \vdots \\ \bar{B}_{n-1} \\ \bar{B}_n \end{bmatrix} \hat{K} = \begin{bmatrix} \mathcal{A} - \bar{A}_1 \\ -\bar{A}_2 \\ \vdots \\ -\bar{A}_{n-1} \\ -\bar{A}_n \end{bmatrix}$$

$$\hat{C} = [I \ 0 \ 0 \ \dots \ 0]$$

2.2 The MPC Control Problem

The filtered state estimation and the one-step prediction may for state space models in innovation form (7) be combined to give the following expressions for computation of the innovation, e_k :

$$\hat{x}_{k|k-1} = \hat{A}\hat{x}_{k-1|k-2} + \hat{B}u_{k-1} + \hat{K}e_{k-1} \quad (8a)$$

$$\hat{y}_{k|k-1} = \hat{C}\hat{x}_{k|k-1} \quad (8b)$$

$$e_k = y_k - \hat{y}_{k|k-1} \quad (8c)$$

Initially, $\hat{x}_{0|-1}$ is known and the one-step prediction (8a) is not needed. Knowing the innovation, e_k , the predictions in the state space model in innovation form may be represented as

$$\hat{x}_{k+1|k} = \hat{A}\hat{x}_{k|k-1} + \hat{B}\hat{u}_{k|k} + \hat{K}e_k \quad (9a)$$

$$\hat{x}_{k+1+j|k} = \hat{A}\hat{x}_{k+j|k} + \hat{B}\hat{u}_{k+j|k}, \quad j = 1, \dots, N-1 \quad (9b)$$

$$\hat{y}_{k+j|k} = \hat{C}\hat{x}_{k+j|k}, \quad j = 1, \dots, N \quad (9c)$$

It is important to notice the term $\hat{K}e_k$ in (9a) due to the innovation form. This term is important for derivation of the optimal control law and is often ignored [4]. Let the objective of the MPC be

$$\phi = \frac{1}{2} \sum_{j=0}^{N-1} \|\hat{y}_{k+j+1|k} - r_{k+j+1|k}\|_Q^2 + \|\Delta\hat{u}_{k+j|k}\|_S^2 \quad (10)$$

in which the second term, $\|\Delta\hat{u}_{k+j|k}\|_S^2$, is a regularization term. We assume the reference parameterization, $\{r_{k+j|k}\}_{j=1}^N = \{r_k, \dots, r_k\}$. The tuning parameters in this objective function are the matrices Q and S which may be diagonal matrices. Box constraints may be formulated for the sequence of $\hat{u}_{k+j|k}$ and $\Delta\hat{u}_{k+j|k}$. The unconstrained MPC can be represented as the convex quadratic optimization problem with the solution given in Jørgensen et al. [4] and a tuning algorithm based on this formulation is presented in Olesen et al. [11, 12].

3. THE NEWELL AND LEE EVAPORATOR

The case study used in this manuscript is the Newell and Lee forced circulation evaporator model [10]. This nonlinear, multi loop process is a very convenient example for illustration and benchmarking of process control technologies and has been used in numerous textbooks and articles [7, 14, 17, 1]. The process and the process variables are depicted in Fig. 1 and the model equations will briefly be presented in the following subsection. The evaporator model by Newell and Lee is representative for a number of industrial evaporation processes. Evaporation is found in sugar and paper mills as well as in a range of food and pharmaceutical industries [8]. In these processes, some solvent (typically water) is removed to concentrate a stream before drying or crystallization.

The principle in the evaporation process is separation by evaporation from a liquid mixture where at least one component is not volatile. The evaporation chamber is designed as a heat exchanger where latent heat from condensation of steam is used to heat up and evaporate a fraction of the circulating process stream. The gas and liquid phase from the evaporator is separated and the gas is condensed before leaving the system. A fraction of the liquid is taken out as product before the remaining fraction is mixed with fresh feed and fed to the evaporator again. Due to the material loop created by the forced circulation of the liquid stream, the dynamic coupling of the system is known to be strong and disturbances anywhere in the system can propagate throughout the process.

3.1 The model equations

As illustrated in Fig. 1, the model for the forced circulation evaporator can be divided into four parts: The separator, the evaporator, the steam jacked and the condenser. The equations are in the following given as in [10] and the reader is referred to this text for more details on the modeling. The following nomenclature will be used: F_i ,

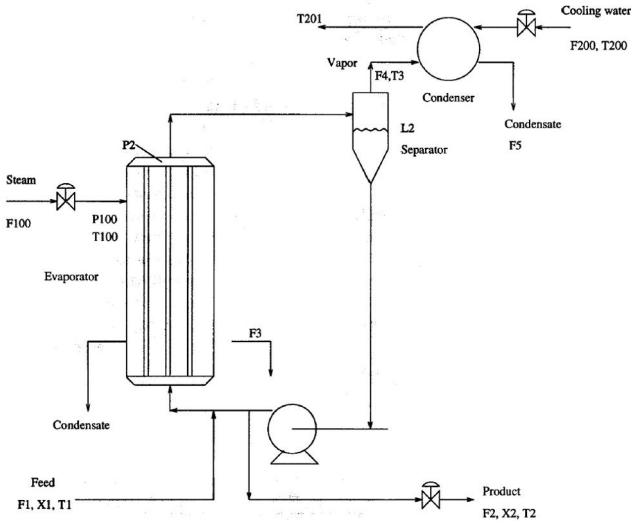


Fig. 1. The evaporator system from [10]

X_i, T_i refers to flow rates, composition and temperature of stream i . L_i, P_i and Q_i are levels, pressures and duties in unit i .

The separator A total mass balance around the separator gives

$$\rho A \frac{dL2}{dt} = F1 - F4 - F2 \quad (11)$$

where ρ is the liquid density and A is the cross sectional area of the separator and ρA is assumed to be 20 kg/m .

The evaporator The evaporator itself is modelled by the following 5 equations:

$$M \frac{dX2}{dt} = F1X1 - F2X2 \quad (12)$$

$$C \frac{dP2}{dt} = F4 - F5 \quad (13)$$

$$T2 = 0.5616P2 + 0.3126X2 + 48.43 \quad (14)$$

$$T3 = 0.507P2 + 55.0 \quad (15)$$

$$F4 = \frac{Q100 - F1Cp(T2 - T1)}{\lambda} \quad (16)$$

where M is a constant liquid hold up in the evaporator of 20 kg . Cp and λ are the heat capacity and the latent heat of evaporation of the process liquid which is assumed constant at $0.07 \text{ kW/K(kg/min)}$ and 38.5 kW/(kg/min) respectively. The constant $C = 4 \text{ kg/kPa}$ is used to convert a mass of vapor into a pressure in the vessel.

The steam jacketed The steam side of the evaporator is model with three algebraic equations as the dynamics are assumed to be very fast

$$T100 = 0.1538P100 + 90.0 \quad (17)$$

$$Q100 = 0.16(F1 + F3)(T100 - T2) \quad (18)$$

$$F100 = \frac{Q100}{\lambda_s} \quad (19)$$

where $\lambda_s = 36.6 \text{ kW/(kg/min)}$ is the latent heat for steam. The term $0.16(F1 + F3)$ correlates the flow to the evaporator to the overall heat transfer coefficient times the area, $UA1$, at the given process conditions.

The condenser The condenser is also modelled as a set of algebraic equations

$$Q200 = \frac{UA2(T3 - T200)}{1 + UA2/(2CpF200)} \quad (20)$$

$$T201 = T200 + \frac{Q200}{F200Cp} \quad (21)$$

$$F5 = \frac{Q200}{\lambda} \quad (22)$$

where $UA2 = 6.84 \text{ kW/K}$ is the overall heat transfer coefficient times the area and the heat of evaporation λ is 38.5 kW/(kg/min) .

Operational conditions The evaporator model has 8 degrees of freedom which can be classified as manipulated variables, u , and disturbance variables, d . The states of the system, x , is seen from the differential equations in the model and these are all measured. These three process variables are the desired control variable of the system. This gives the following general system description of the model.

$$\frac{dx}{dt} = f(x, u, d), \quad x(t=0) = x_{ss} \quad (23a)$$

where

$$x = [L2 \ X2 \ P2]^T \quad (23b)$$

$$u = [F2 \ P100 \ F200]^T \quad (23c)$$

$$d = [F1 \ T1 \ X1 \ F3 \ T200]^T \quad (23d)$$

and x_{ss} is the steady state solution for the nominal operation. Table 1 lists the system variables for the nominal system.

It will be assumed in this manuscript that the true system, i.e. the nonlinear simulation model, is monitored by a control system which logs data for all three states every minute. Each discrete time measurement, y_k , of the states is the current state value for the process variable, x_k , corrupted by Gaussian distributed noise, i.e.

$$y_k = x_k + v_k, \quad (24a)$$

Table 1. Nominal steady state process conditions for the evaporator system.

Variable	Description	Value	Unit
$F1$	Feed flowrate	10.0	kg/min
$F2$	Product flowrate	2.0	kg/min
$F3$	Circulating flowrate	50.0	kg/min
$F4$	Vapor flowrate	8.0	kg/min
$F5$	Condensate flowrate	8.0	kg/min
$X1$	Feed composition	5.0	%
$X2$	Product composition	25.0	%
$T1$	Feed temperature	40.0	°C
$T2$	Product temperature	84.6	°C
$T3$	Circulating temperature	80.6	°C
$L2$	Separator level	1.0	m
$P2$	Operating pressure	50.5	kPa
$F100$	Steam flowrate	9.3	kg/min
$T100$	Steam temperature	119.9	°C
$P100$	Steam pressure	194.7	kPa
$Q100$	Heater duty	339.0	kW
$F200$	Cooling water flowrate	208.0	kg/min
$T200$	Cooling water inlet temp.	25.0	°C
$T201$	Cooling water outlet temp.	46.1	°C
$Q200$	Condenser duty	307.9	kW

where

$$v_k \in \mathcal{N}_{iid} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.1^2 & 0 & 0 \\ 0 & 0.15^2 & 0 \\ 0 & 0 & 0.25^2 \end{bmatrix} \right) \quad (24b)$$

4. IDENTIFICATION OF A LINEAR MODEL

In the book introducing the evaporator model, Newell and Lee [10] also gives a linear state space model of the system. This linear model is obtained from Taylor series expansion of the nonlinear model around the nominal steady state. In this manuscript it is assumed that the true system model is unknown for the design of the controller. Hence a linear system description of the general model (23) need to be provided by other means than linearization of a mechanistic model. Therefore sets of MISO models will be found using open loop system identification and data from simulation of the true system model.

4.1 Open loop simulations

Unfortunately, the process cannot be operated in open loop as the level in separator is an integrating process. Hence, this variable need to be controlled and a discrete time implementation of a PI control loop is added to the simulation using $F2$ to control $L2$. This level is assumed constrained in the interval 0-2 m by the equipment. The transfer function for this open loop system was found simulating a step in $F2$ for a short time and realizing that the process gain was -0.15 m/(kg/min) from the system response. The PI controller is tuned using the SIMC tuning rules with a desired closed loop time constant of $\tau_c = 5 \text{ min}$ leading to a control gain $K_c = -1.33 \text{ kg/(min.m)}$ and a integral time $\tau_I = 20 \text{ min}$ [16].

An experiment is made to generate data for the system identification of the ARX models for the transfer functions between the inputs $\{P100, F200\}$ and the outputs $\{X2, P2\}$. The two inputs are perturbed simultaneously for 300 min around the nominal values using PRBS signals. The signal is allowed to vary 20% for $P100$ and 25% for $F200$ and both signals are designed with a clock period of 10 samples. The inputs and outputs from the experiment are shown in Fig. 2. The PI controller to stabilize the level $L2$ is in automatic during the experiment. The input-output data from the experiment is divided in 2 series. $2/3$ is used for estimation and $1/3$ for validation. MISO ARX models are estimated from the data series, one at a time for each output. The parameters are presented in Table 2 together with the 95% confidence limits of the estimates. The model order was fixed to 2 and the numerator polynomials have only one coefficient and a delay of two samples. This model structure was found testing a range of model orders in terms for their ability to fit the validation data and considering correlation analysis of the residuals. From the 95% confidence limits, it is in general seen that all estimates are significant. The only exception is b_2 for the dynamics between $F200$ to $X2$ which could in principle be set equal to zero if it was not the only coefficient in the numerator. One step ahead and pure simulation prediction of the response for the validation data showed a 91.88% and 45.69% fit for the output $X2$ respectively and 77.55% and 17.11% for $P2$. The quality of the fits and the

Table 2. ARX parameter estimates and their 95% confidence limits

I/O	Par	Estimate	low lim.	upper lim.
$X2$	a_1	-1.706	-1.779	-1.632
	a_2	0.7285	0.6574	0.7997
	b_2	0.003475	0.002529	0.004422
	σ	0.3182		
$P2$	a_1	-0.6863	-0.8276	-0.5450
	a_2	-0.2811	-0.4183	-0.1439
	b_2	0.01044	0.00842	0.01246
	σ	0.4113		

confidence limits reported here are reasonable since simple linear models are estimated for a nonlinear process system. This model will be accepted for the MPC controller.

5. CLOSED LOOP SIMULATION

Given the estimated parameters for the dynamics between the inputs $\{P100, F200\}$ and the outputs $\{X2, P2\}$ a state space model in innovation form is constructed as described in section 2. The model is expanded with a noise model (2) to ensure offset free control and the tuning parameters α_i are all chosen to 0.7 as recommended in Huusom et al. [3]. An MPC controller is constructed using the state space model. It is made with a prediction and control horizon of 60 samples and the tuning matrices in the objective function are

$$Q = \begin{bmatrix} 1/X2_{set} & 0 \\ 0 & 1/P2_{set} \end{bmatrix}, \quad S = \begin{bmatrix} 5/P100_{nom} & 0 \\ 0 & 5/F200_{nom} \end{bmatrix} \quad (25)$$

I.e. all inputs and outputs are considered with approximately equal importance. Constraints on the manipulated variables are not needed in the MPC of this paper, hence it is equivalent to a finite horizon LQG.

Positive step changes are simulated in three of the potential five disturbance variables, $\{F1, X1, T200\}$, and the responses are shown in Fig. 3. It is clearly seen that the control system consisting of both the MPC and the PI loops is able to achieve offset free tracking of all the controlled variables. An acceptable closed loop performance is achieved despite the sustained unmeasured disturbances and the reduced order linear models used for the MPC. The PI controller further maintains the level $L2$ within the physical constraints of the gas/liquid separator unit.

5.1 Discussion

Using the proposed controller, all controlled variables do not have steady state offsets. The controller is designed using standard system identification tools, a simple noise model, and a combination of PI and MPC technology. The tuning used here has been the standard choices for the noise model, a reasonable long prediction and control horizon and approximately equal penalties to inputs and control moves in the MPC cost function. Additional fine tuning can be used to further improve the performance of the resulting control system. However, the purpose of this paper is to illustrate how an offset free ARX based MPC design methodology can be applied to multivariate

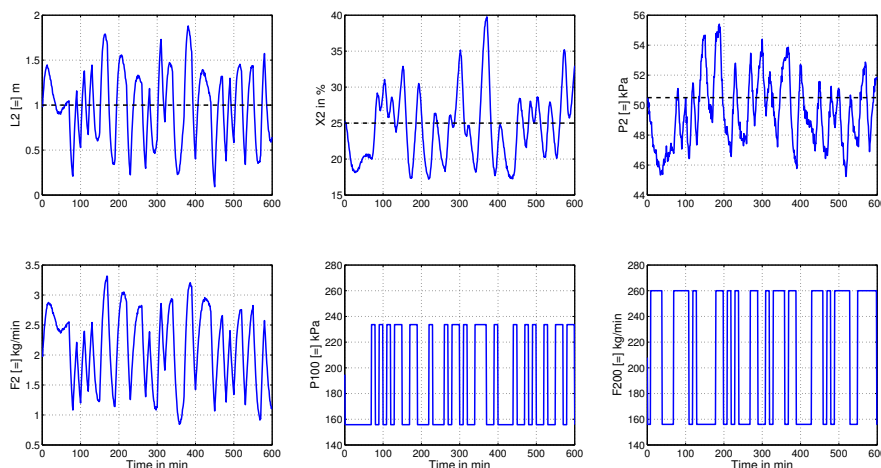


Fig. 2. Open loop data generation for system identification. PRBS perturbations are made in $P100$ and $F200$, while $F2$ is given by a PI controller to stabilize $L2$.

nonlinear processes. And for this purpose, further fine tuning is not needed.

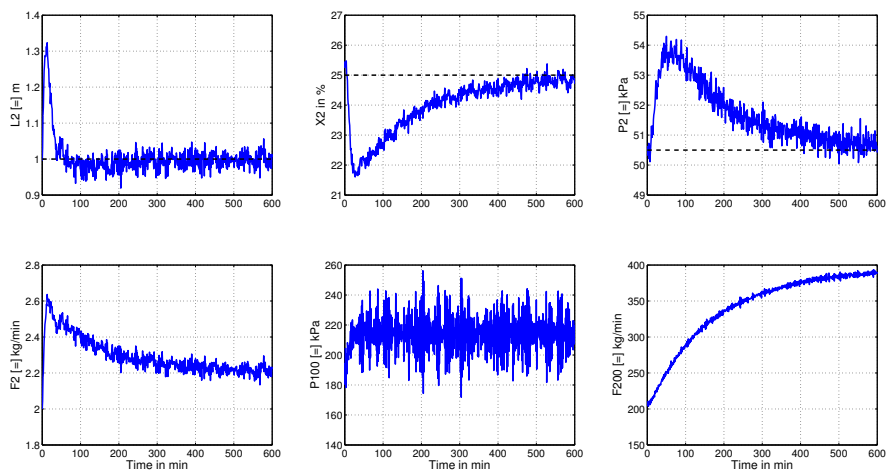
Another control design option would be to allow the MPC to vary the level in the separator by changing the set point to the PI controller between the constraints and thereby increase the degrees of freedom to reject disturbances.

6. CONCLUSION

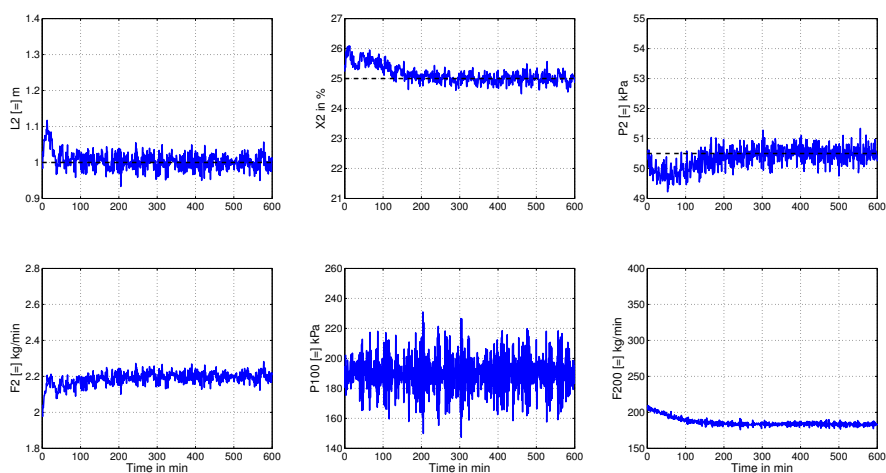
The methodology for offset-free MPC design using identified MISO ARX models has been applied for the nonlinear process example of the Newell and Lee forced circulation evaporator model. The control system consists of a MIMO MPC and a SISO PI loop due to the integration mode of the gas-liquid separator. The overall performance of the controller provides an acceptable closed loop performance and offset-free tracking of all controlled variables based on standard choices for the tuning. The paper illustrates how an advanced control system for a realistic nonlinear multivariate process can be designed.

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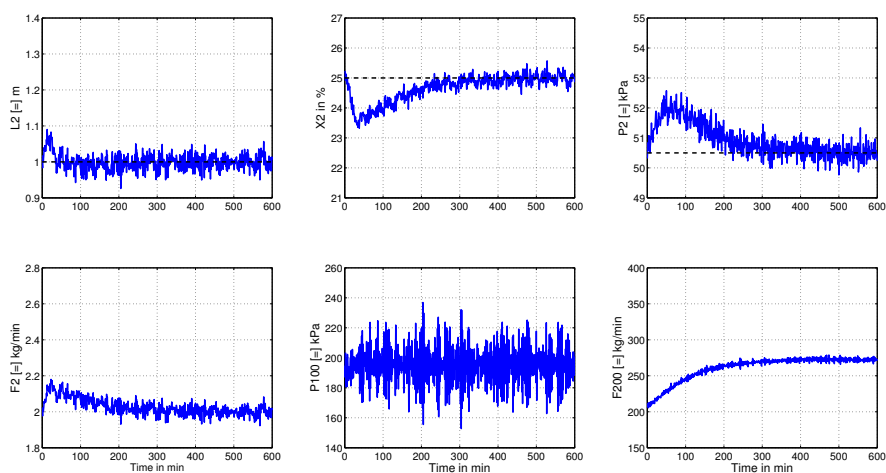
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(a) +10 % step in $F1$.



(b) +10 % step in $X1$.



(c) +10 % step in $T200$.

Fig. 3. Closed loop simulation with step changes in some disturbance parameters. The $L2 \rightarrow F2$ loop is control by the PI controller and the $\{P100, F200\} \rightarrow \{X2, P2\}$ loop is controlled using the MPC.