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Adaptive H_∞ Consensus Control for Distributed Parameter Systems of Parabolic Type

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Abstract: Design methodologies of adaptive H_∞ consensus control of multi-agent systems composed of a class of infinite-dimensional systems are provided in this paper. The proposed control schemes are derived as solutions of certain H_∞ control problems, where the effects of neglected infinite-dimensional modes and the imperfect knowledge of the leader are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable consensus tracking is achieved approximately via adaptation schemes.

1. INTRODUCTION

Among plenty of cooperative control problems of multi-agent systems, distributed consensus tracking of multi-agent systems with limited communication networks, has been a basic and important topic, and various research results have been reported for various processes and under various conditions such as Kingston et al. [2005], Olfati-Saber et al. [2007], Ren et al. [2007], Cao and Ren [2011]. In those research works, adaptive control or sliding mode control methodologies were also applied in order to deal with uncertainties of agents, and stability of control systems was assured via Lyapunov function analysis. Furthermore, robustness properties of the control schemes were also discussed. However, those results are restricted to simple and low-order systems, or finite-dimensional mechanical systems, and those approaches do not have been applied to the control of infinite-dimensional (or high-order) systems via finite-dimensional (or low-order) compensators.

On the contrary, there have been several researches in the fields of adaptive control for infinite-dimensional systems (Miyasato and Kitamori [1985], Kobayashi [1988], Miyasato [1990], Orlov [1997], Böhm et al. [1998], Krstić [1999], Hong and Bentsman [1994], Ilchmann et al. [2002], Krstić et al. [2006]). In our previous work (Miyasato [2006]), we developed design methods of adaptive control for distributed parameter systems of parabolic type via finite-dimensional controllers. In the proposed methodologies, stabilizing control signals are added to regulate the effects of infinite-dimensional modes, and those are derived as a solution of certain H_∞ control problem where the effect of infinite-dimensional modes are regarded as external disturbances to the processes.

The purpose of the present paper is to apply our previous result (Miyasato [2006]) to the consensus tracking problems, and present design methods of adaptive consensus control of multi-agent systems composed of distributed parameter systems of parabolic type (a class of infinite-dimensional systems) with finite-dimensional inputs and outputs. The proposed control strategy is composed of finite dimensional compensators, and is derived as a solution of certain H_∞ control problem, where the effects of neglected infinite-dimensional modes and the imperfect knowledge of the leader are regarded as external disturbances to the processes. It is shown that the resulting control systems are robust to uncertain system parameters and neglected infinite-dimensional modes, and that the desirable consensus tracking is achieved approximately via adaptation schemes. This is also an extension of the work (Miyasato [2013]) to distributed parameter systems of parabolic type.

2. MULTI-AGENT SYSTEM AND INFORMATION NETWORK

First, mathematical preliminaries on information network graph of multi-agent systems are summarized (Ren et al. [2007], Cao and Ren [2011]). We consider a weighted undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ as a model of interaction among agents. $\mathcal{V} = \{1, \cdots, N\}$ is a node set, which corresponds to a set of agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set. An edge $(i, j) \in \mathcal{E}$ indicates that the agent i and j can communicate with each other. Associated with $\mathcal{E}$, we introduce a weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{N\times N}$, and the entry a_{ij} of it is defined such as $a_{ij} = a_{ji} > 0$ (when $(i, j) \in \mathcal{E}$) and $a_{ij} = a_{ji} = 0$ (otherwise). A path is a sequence of edges in the form (i_1, i_2), (i_2, i_3), $\cdots$ $(i_j \in \mathcal{V})$, and the undirected graph is called connected, if there is always an undirected path between every pair of distinct nodes. For the adjacency matrix $A = [a_{ij}]$, the Laplacian matrix $L = [l_{ij}] \in \mathbf{R}^{N\times N}$ is defined by $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ $(i \neq j)$. The Laplacian matrix is symmetric and positive-semidefinite, and furthermore, has a simple 0 eigenvalue with the associated eigenvector $\mathbf{1} = [1 \cdots 1]^\mathsf{T}$, and all other eigenvalues are positive, if the corresponding undirected graph is connected.

In this manuscript, we consider a consensus control problem of leader-follower type, and y_0 is a leader which each agent $i \in \mathcal{V}$ (a follower) should follow. For the leader and the followers, a_{i0} is defined such as $a_{i0} > 0$ (when leader's information is available to follower i), and $a_{i0} = 0$ (otherwise), and from a_{i0} and L, the matrix $M \in \mathbf{R}^{N\times N}$ is defined by $M = L + \mathrm{diag}\,(a_{10} \cdots a_{N0})$. M is symmetric and positive definite, if 1. at least one a_{i0} $(1 \le i \le N)$ is positive, and 2. the graph $\mathcal{G}$ is connected (Cao and Ren [2011]). Hereafter, we assume those assumptions 1 and 2.

3. PROBLEM STATEMENT

We consider a multi-agent system composed of distributed parameter systems (DPS) of parabolic type. Let Ω_i be a bounded open domain in a finite dimensional Euclidian space, and $L^2(\Omega_i)$ is defined as the Hilbert space of all square integrable functions with the inner product

$$(u_i, v_i) = \int_{\Omega_i} u_i(x_i)v_i(x_i)dx_i. \tag{1}$$

We consider a single-input, single-output distributed parameter system of parabolic type in $L^2(\Omega_i)$ (Kobayashi [1987], Kobayashi [1988]) described by

$$\frac{d}{dt}u_i(t) = A_iu_i(t) + g_if_i(t), \tag{2}$$
$$y_i(t) = (c_i,\, u_i(t)) \equiv C_iu_i(t), \tag{3}$$
$$(i = 1, \cdots, N),$$

where $u_i(t)$ $(\in L^2(\Omega_i))$ is a state, $f_i(t)$ (an input) and $y_i(t)$ (an output) are scalar functions on $t \in [0, \infty)$, g_i $(\in L^2(\Omega_i))$ is an input influence function, and c_i $(\in L^2(\Omega_i))$ is a sensor influence function. The operator A_i is a self-adjoint operator bounded from above with compact resolvent whose eigenvalues λ_{ij}

$$\infty > \gamma \ge -\lambda_{i1} > -\lambda_{i2} > \cdots, \qquad (\lim_{j\to\infty} \lambda_{ij} = \infty), \tag{4}$$

are assumed to be simple. The normalized eigenfunctions of A_i are denoted by ϕ_{ij} such that

$$A_i\phi_{ij} = -\lambda_{ij}\phi_{ij}, \quad (j = 1, 2, \cdots). \tag{5}$$

The set ϕ_{ij} $(j = 1, 2, \cdots)$ forms a complete orthonormal system in $L^2(\Omega_i)$. Then, A_i generates a strongly continuous semigroup written as

$$U_i(t)\cdot = \exp(A_it)\cdot = \sum_{j=1}^{\infty} \mathrm{e}^{-\lambda_{ij}t}(\,\cdot,\, \phi_{ij})\phi_{ij}, \tag{6}$$

and a unique solution for the system (2) is described as follows:

$$u_i(t) = U_i(t)u_{i0} + \int_0^t U_i(t-\tau)g_if_i(\tau)d\tau, \tag{7}$$

where $u_{i0} = u_i(0)$.

For the controlled process (2), (3), only the input $f_i(t)$ and the output $y_i(t)$ are assumed to be available for measurement, but the state $u_i(t)$ and system parameters included in A_i, g_i, c_i are unknown.

The control objective is to design an adaptive consensus control system for a swarm of infinite-dimensional systems (2), (3) in which consensus tracking is achieved via adaptation schemes such that $y_i \to y_j, y_i \to y_0, (i,\, j = 1, \cdots, N)$.

Throughout this paper, the index i corresponds to each agent, and $i = 1, \cdots, N$.

4. SYSTEM REPRESENTATION

In the present section, we derive an input-output representation of the process (2), (3). First, let $\tilde{\lambda}_{iN}$ (> 0) be a given damping constant. We take an integer N_i such that

$$0 < \tilde{\lambda}_{iN} < \lambda_{N_i+1}, \tag{8}$$

and define orthogonal projection operators P_{iN}, Q_{iN} by

$$P_{iN}\cdot = \sum_{j=1}^{N_i}(\,\cdot,\, \phi_{ij})\phi_{ij}, \tag{9}$$

$$Q_{iN}\cdot = (I - P_{iN})\cdot = \sum_{j=N_i+1}^{\infty}(\,\cdot,\, \phi_{ij})\phi_{ij}. \tag{10}$$

Then, $u_i(t)$ and $y_i(t)$ in (2), (3) are expressed as follows:

$$u_i(t) = P_{iN}u_i(t) + Q_{iN}u_i(t) \equiv u_{iN}(t) + \tilde{u}_{iN}(t), \tag{11}$$
$$y_i(t) = C_i\{u_{iN}(t) + \tilde{u}_{iN}(t)\} \equiv y_{iN}(t) + \tilde{y}_{iN}(t), \tag{12}$$
$$C_i\cdot \equiv (c_i,\, \cdot). \tag{13}$$

The controlled process (2), (3) is decomposed into two subsystems $[S_{i1}]$ and $[S_{i2}]$ described in the following equations:

$[S_{i1}]$

$$\frac{d}{dt}\bar{u}_{iN}(t) = \bar{A}_{iN}\bar{u}_{iN}(t) + \bar{g}_{iN}f_i(t), \tag{14}$$
$$y_{iN}(t) = \bar{C}_{iN}\bar{u}_{iN}(t), \tag{15}$$
$$\bar{u}_{iN}(0) = [(u_{i0}, \phi_{i1}),\, (u_{i0}, \phi_{i2}),\, \cdots,\, (u_{i0}, \phi_{iN_i})]^{\mathsf{T}}, \tag{16}$$
$$u_{iN}(t) = \bar{u}_{iN}(t)^{\mathsf{T}}[\phi_{i1},\, \phi_{i2},\, \cdots,\, \phi_{iN_i}]^{\mathsf{T}}, \tag{17}$$
$$\bar{A}_{iN} = \mathrm{diag}\,(-\lambda_{i1},\, -\lambda_{i2},\, \cdots,\, -\lambda_{iN_i})\,(\in \mathbf{R}^{N_i\times N_i}), \tag{18}$$
$$\bar{g}_{iN} = [g_{i1},\, g_{i2},\, \cdots,\, g_{iN_i}]^{\mathsf{T}}\,(\in \mathbf{R}^{N_i}), \quad g_{ij} = (g_i,\, \phi_{ij}), \tag{19}$$
$$\bar{C}_{iN} = [c_{i1},\, c_{i2},\, \cdots,\, c_{iN_i}]\,(\in \mathbf{R}^{1\times N_i}), \quad c_{ij} = (c_i,\, \phi_{ij}), \tag{20}$$
$$\bar{u}_{iN}(t) \in \mathbf{R}^{N_i}. \tag{21}$$

$[S_{i2}]$

$$\frac{d}{dt}\tilde{u}_{iN}(t) = \tilde{A}_{iN}\tilde{u}_{iN}(t) + \tilde{g}_{iN}f_i(t), \tag{22}$$
$$\tilde{y}_{iN}(t) = \tilde{C}_{iN}\tilde{u}_{iN}(t), \tag{23}$$
$$\tilde{u}_{iN}(0) = Q_{iN}u_{i0}, \tag{24}$$
$$\tilde{A}_{iN} = A_iQ_{iN}, \tag{25}$$
$$\tilde{g}_{iN} = Q_{iN}g_i, \tag{26}$$
$$\tilde{C}_{iN} = C_iQ_{iN}, \tag{27}$$
$$\tilde{u}_{iN}(t) \in L^2(\Omega_i). \tag{28}$$

$[S_{i1}]$ is a finite dimensional system (N_i dimension), and its solution is written as follows:

$$\bar{u}_{iN}(t) = \bar{U}_{iN}(t)\bar{u}_{iN}(0) + \int_0^t \bar{U}_{iN}(t-\tau)\bar{g}_{iN}f_i(\tau)d\tau, \tag{29}$$

$$\bar{U}_{iN}(t) = \exp(\bar{A}_{iN}t) = \mathrm{diag}(\mathrm{e}^{-\lambda_{i1}t},\, \mathrm{e}^{-\lambda_{i2}t},\, \cdots,\, \mathrm{e}^{-\lambda_{iN_i}t}). \tag{30}$$

On the other hand, $[S_{i2}]$ is an infinite dimensional system and its solution is given by

$$\tilde{u}_{iN}(t) = \tilde{U}_N(t)\tilde{u}_{iN}(0) + \int_0^t \tilde{U}_{iN}(t-\tau)\tilde{g}_{iN} f_i(\tau)d\tau, \quad (31)$$

$$\tilde{U}_{iN}(t)\cdot = \exp(\tilde{A}_{iN}t)\cdot = \sum_{j=N_i+1}^{\infty} \mathrm{e}^{-\lambda_{ij}t}(\cdot,\, \phi_{ij})\phi_{ij}. \quad (32)$$

$\bar{U}_{iN}(t)$ and $\tilde{U}_{iN}(t)$ are evaluated in the following:

$$\|\bar{U}_{iN}(t)\|_{\mathbf{R}^{N_i}} \le \mathrm{e}^{-\lambda_{i1}t}, \quad (33)$$

$$\|\tilde{U}_{iN}(t)\|_{L^2(\Omega_i)} \le \mathrm{e}^{-\tilde{\lambda}_{iN}t}. \quad (34)$$

The next assumption is introduced for $[S_{i1}]$.

Assumption 1. The subsystem $[S_{i1}]$ $(\bar{C}_{iN}, \bar{A}_{iN}, \bar{g}_{iN})$ (N_i dimension) is completely controllable and observable, that is

$$c_{ij} \ne 0, \quad g_{ij} \ne 0, \quad (1 \le j \le N_i). \quad (35)$$

Then, on Assumption 1, we can construct a finite dimensional observer for $[S_{i1}]$, and it is denoted by $[S_{i1}^{'}]$.

$[S_{i1}^{'}]$

$$\frac{d}{dt}\hat{\bar{u}}_{iN}(t) = \bar{F}_{iN}\hat{\bar{u}}_{iN}(t) + \bar{g}_{iN}f_i(t) + \bar{K}_{iN}y_{iN}(t), \quad (36)$$

$$y_{iN}(t) = \bar{C}_{iN}\bar{u}_{iN}(t), \quad (37)$$

$$\hat{y}_{iN}(t) = \bar{C}_{iN}\hat{\bar{u}}_{iN}(t), \quad (38)$$

$$\hat{\bar{u}}_{iN}(0) = \hat{\bar{u}}_{iN0}\,(\in \mathbf{R}^{N_i}), \quad (39)$$

where $\bar{F}_{iN}$ $(\in \mathbf{R}^{N_i \times N_i})$ is a stable matrix defined by

$$\bar{F}_{iN} = \bar{A}_{iN} - \bar{K}_{iN}\bar{C}_{iN}, \quad (40)$$

and $\bar{K}_{iN}$ $(\in \mathbf{R}^{N_i \times 1})$ is an observer gain matrix selected properly. Since $\bar{F}_{iN}$ is stable, the following relation holds.

$$\|\bar{u}_{iN}(t) - \hat{\bar{u}}_{iN}(t)\|_{\mathbf{R}^{N_i}},\, |y_{iN}(t) - \hat{y}_{iN}(t)| \sim \|\exp(\bar{F}_{iN}t)\|_{\mathbf{R}^{N_i}} \to 0. \quad (41)$$

Here we introduce the following signal $f_{if}(t)$

$$\frac{d}{dt}f_{if}(t) = -\lambda_{i0}f_{if}(t) + f_i(t), \quad (42)$$

where λ_{i0} is a positive constant. Then, we derive the input-output representation of the process (2), (3) from the subsystems $[S_{i1}^{'}]$ and $[S_{i2}]$ in the following:

$$\begin{aligned}
\frac{d}{dt}y_i(t) &= \frac{d}{dt}y_{iN}(t) + \frac{d}{dt}\tilde{y}_{iN}(t)\\
&= \frac{d}{dt}\hat{y}_{iN}(t) + \frac{d}{dt}\tilde{y}_{iN}(t) + \epsilon_i(t)\\
&= \bar{C}_{iN}\bar{F}_{iN}\hat{\bar{u}}_{iN}(t) + \bar{C}_{iN}\bar{g}_{iN}f_i(t) + \bar{C}_{iN}\bar{K}_{iN}y_{iN}(t)\\
&\quad +\tilde{C}_{iN}\tilde{A}_{iN}\tilde{u}_{iN}(t) + \tilde{C}_{iN}\tilde{g}_{iN}f_i(t) + \epsilon(t)\\
&= \bar{C}_{iN}\bar{F}_{iN}(\bar{F}_{iN} + \lambda_{i0}I)\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{g}_{iN}f_{if}(\tau)d\tau\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{K}_{iN}y_{iN}(\tau)d\tau\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\bar{g}_{iN}f_{if}(t) + \bar{C}_{iN}\bar{K}_{iN}y_{iN}(t)\\
&\quad +\tilde{C}_{iN}\tilde{A}_{iN}\tilde{u}_{iN}(t) + C_i g_i f_i(t) + \epsilon_i(t),
\end{aligned} \quad (43)$$

where the integration by parts is taken by utilizing (42). The term $\epsilon_i(t)$ is a linear combination of decaying exponentials and is determined by the initial conditions of the process (2), (3), the subsystem $[S_{i1}^{'}]$ and $f_{if}(t)$; it is evaluated as follows:

$$|\epsilon_i(t)| \sim \|\exp(\bar{F}_{iN}t)\|_{\mathbf{R}^{N_i}},\, \mathrm{e}^{-\lambda_{i0}t} \to 0. \quad (44)$$

Hereafter, all exponentially decaying terms are denoted by ϵ_i in the manuscript. The substitution of the next relation

$$y_{iN} = y_i(t) - \tilde{y}_{iN}(t), \quad (45)$$

into (43) yields

$$\begin{aligned}
&\frac{d}{dt}y_i(t)\\
&= \bar{C}_{iN}\bar{F}_{iN}(\bar{F}_{iN} + \lambda_{i0}I)\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{g}_{iN}f_{if}(\tau)d\tau\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{K}_{iN}y_i(\tau)d\tau\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\bar{g}_{iN}f_{if}(t) + \bar{C}_{iN}\bar{K}_{iN}y_i(t)\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{K}_{iN}\{-\tilde{y}_{iN}(\tau)\}d\tau\\
&\quad +\bar{C}_{iN}\{-K_{iN}\tilde{y}_{iN}(t)\} + \tilde{C}_{iN}\tilde{A}_{iN}\tilde{u}_{iN}(t)\\
&\quad +C_i g_i f_i(t) + \epsilon_i(t).
\end{aligned} \quad (46)$$

Here we introduce the following finite-dimensional (N_i dimension) state variable filters.

$$\begin{cases}
\dfrac{d}{dt}\bar{v}_{i1}(t) = \bar{F}_{iN0}\bar{v}_{i1}(t) + \bar{g}_{i0}f_{if}(t),\\
\dfrac{d}{dt}\bar{v}_{i2}(t) = \bar{F}_{iN0}\bar{v}_{i2}(t) + \bar{g}_{i0}y_i(t),
\end{cases} \quad (47)$$

where $(\bar{F}_{iN0},\, \bar{g}_{i0})$ $(\bar{F}_{iN0} \in \mathbf{R}^{N_i \times N_i},\, \bar{g}_{i0} \in \mathbf{R}^{N_i})$ is a controllable pair, and $\bar{F}_{iN0}$ is chosen such that the following equality holds.

$$\det(sI - \bar{F}_{iN}) = \det(sI - \bar{F}_{iN0}). \quad (48)$$

For any stable $\bar{F}_{iN0}$ $(\in \mathbf{R}^{N_i \times N_i})$, there exists $\bar{K}_{iN}$ satisfying (48), since $(\bar{C}_{iN},\, \bar{A}_{iN})$ is observable. Then, owing to controllability of $(\bar{F}_{iN0},\, \bar{g}_{i0})$, the following relation holds for properly selected θ_{i1}, θ_{i2} $(\in \mathbf{R}^{N_i})$ (Ioannou and Sun [1996]).

$$\begin{aligned}
&\bar{C}_{iN}\bar{F}_{iN}(\bar{F}_{iN} + \lambda_0 I)\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{g}_{iN}f_{if}(\tau)d\tau\\
&\quad +\bar{C}_{iN}\bar{F}_{iN}\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{K}_{iN}y_i(\tau)d\tau\\
&\quad = \theta_{i1}^{\mathsf{T}}\bar{v}_{i1}(t) + \theta_{i2}^{\mathsf{T}}\bar{v}_{i2}(t) + \epsilon_i(t).
\end{aligned} \quad (49)$$

By considering (49), (46) is rewritten into

$$\begin{aligned}
\frac{d}{dt}y_i(t) &= \theta_{i1}^{\mathsf{T}}\bar{v}_{i1}(t) + \theta_{i2}^{\mathsf{T}}\bar{v}_{i2}(t) + \theta_{i3}f_{if}(t) + \theta_{i4}y_i(t)\\
&\quad +\theta_{i0}f_i(t) + \delta_i(t) + \epsilon_i(t),
\end{aligned} \quad (50)$$

$$\theta_{i3} = \bar{C}_{iN}\bar{F}_{iN}\bar{g}_{iN}, \tag{51}$$

$$\theta_{i4} = \bar{C}_{iN}\bar{K}_{iN}, \tag{52}$$

$$\theta_{i0} = C_i g_i, \tag{53}$$

$$\delta_i(t) = \bar{C}_{iN}\bar{F}_{iN}\int_0^t \{\exp \bar{F}_{iN}(t-\tau)\}\bar{K}_{iN}\{-\tilde{y}_{iN}(\tau)\}d\tau$$
$$+\bar{C}_{iN}\{-K_{iN}\tilde{y}_{iN}(t)\} + \tilde{C}_{iN}\tilde{A}_{iN}\tilde{u}_{iN}(t). \tag{54}$$

The input-output representation of the controlled process is given by (50), and it is composed of two parts, that is; $\theta_{i1}^{\mathsf{T}}\bar{v}_{i1}(t) + \theta_{i2}^{\mathsf{T}}\bar{v}_{i2}(t) + \theta_{i3}f_{if}(t) + \theta_{i4}y_i(t) + \theta_{i0}f_i(t)$, and $\delta_i(t)$. The former half is constructed by finite dimensional systems (47), and is considered as a primal part of the process. On the contrary, the latter half $\delta_i(t)$ is derived from the infinite dimensional system $[S_{i2}]$, and is dealt with as a residual part.

In the rest of this section, the residual term $\delta_i(t)$ is to be evaluated. First, we define the next state variable filters whose dimensions are all one.

$$\begin{cases} \dfrac{d}{dt}w_{i1}(t) = -\tilde{\lambda}_{iN}w_{i1}(t) + |f_{if}(t)|, \\ \dfrac{d}{dt}w_{i2}(t) = -\lambda_{if}w_{i2}(t) + w_{i1}(t), \\ \dfrac{d}{dt}w_{i3}(t) = -\lambda_{if}w_{i3}(t) + |f_{if}(t)|, \end{cases} \tag{55}$$

where λ_{if} is chosen such that the following relation holds.

$$\begin{aligned} &\|\exp(\bar{F}_{iN}t)\| \le M_{iF}\,\mathrm{e}^{-\lambda_{if}t}, \\ &\|\exp(\bar{F}_{iN0}t)\| \le M_{iF0}\,\mathrm{e}^{-\lambda_{if}t}, \\ &\qquad (0 < M_{iF},\, M_{iF0} < \infty). \end{aligned} \tag{56}$$

In order to evaluate $\delta_i(t)$ in (50), it is assumed that the sensor influence function c_i and the input influence function g_i are smooth in the following meaning.

Assumption 2. The following relations hold.

$$\sum_{j=1}^{\infty}|\lambda_{ij}^k c_{ij}g_{ij}| < \infty \ \ (k=1,\,2), \qquad \sum_{j=1}^{\infty}\lambda_{ij}^2 c_{ij}^2 < \infty. \tag{57}$$

Then, $\delta_i(t)$ in (50) is evaluated as follows:

Lemma 3. On Assumption 2, $\delta_i(t)$ is evaluated from above as follows:

$$|\delta_i(t)| \le g_{i\delta}(t)^{\mathsf{T}}d_{i\delta} + |\epsilon_i(t)|, \tag{58}$$

$$g_{i\delta} = [\,|f_{if}(t)|,\, w_{i1}(t),\, w_{i2}(t),\, w_{i3}(t)\,]^{\mathsf{T}}, \tag{59}$$

$$d_{i\delta} = [\,M_{i1},\, M_{i2},\, M_{i3},\, M_{i4}\,]^{\mathsf{T}}, \tag{60}$$

$$0 < M_{i1} \sim M_{i4} < \infty,$$
$$\epsilon_i(t) \sim \mathrm{e}^{-\tilde{\lambda}_{iN}t},\ \mathrm{e}^{-\lambda_{if}t},\ \mathrm{e}^{-\lambda_{i0}t} \to 0.$$

(Proof) See Miyasato [2006].

Remark 4. In this manuscript, the N_i dimensional state variable filters (47) are utilized as finite dimensional controllers. On the contrary, $\delta_i(t)$ corresponds to a spill-over term which is not considered fully in the conventional controller designs of DPS.

Hereafter, the input-output representation of each agent is written in the following form.

$$\frac{d}{dt}y_i(t) = \Theta_i^{\mathsf{T}}\omega_i(t) + \theta_{i0}f_i(t) + \delta_i(t) + \epsilon_i(t), \tag{61}$$

$$\Theta_i = \left[\theta_{i1}^{\mathsf{T}},\, \theta_{i2}^{\mathsf{T}},\, \theta_{i3},\, \theta_{i4}\right]^{\mathsf{T}}, \tag{62}$$

$$\omega_i(t) = \left[v_{i1}(t)^{\mathsf{T}},\, v_{i2}(t)^{\mathsf{T}},\, f_{if}(t),\, y_i(t)\right]^{\mathsf{T}}. \tag{63}$$

5. ADAPTIVE H_∞ CONSENSUS CONTROL

5.1 Assumptions

By utilizing the system representations in the previous section, the proposed adaptive H_∞ consensus control schemes are constructed via finite dimensional controllers. The next assumptions are introduced.

Assumption 5. $\theta_{i0} \neq 0$, and $\mathrm{sgn}\,\theta_{i0}$ is known. In the following, it is assumed that $\theta_{i0} > 0$ without loss of generality.

Assumption 6. There exist M_{if0} and M_{if1} such that

$$|f_{if}(t)| \le M_{if0} + M_{if1}\sup_{0\le\tau\le t}\{|y_i(\tau)|\|\},$$
$$(0 \le M_{if0} < \infty,\ \ 0 < M_{if1} < \infty). \tag{64}$$

Remark 7. Assumption 5 states that the relative degree of the process is one, and Assumption 6 asserts that the process has a stable inverse.

5.2 Control Law and Error Equation

The communication structure among agents and a leader is prescribed by the information network graph $\mathcal{G}$ with the adjacency matrix A, the Laplacian matrix L, and the matrix M. Associated with the information network graph $\mathcal{G}$, we employ the following control law.

$$f_i(t) = \hat{p}_i(t)\left[-\hat{\Theta}_i(t)^{\mathsf{T}}\omega_i(t) - \alpha\sum_{\substack{j=0\\ j\neq i}}^{N} a_{ij}\{y_i(t) - y_j(t)\}\right.$$
$$\left.+n_{i0}\dot{y}_0(t)\right] + v_i(t)$$
$$\equiv \hat{p}_i(t)f_{i0}(t) + v_i(t), \tag{65}$$

where a_{ij} $(1 \le i \le N,\ 0 \le j \le N)$ is defined as the entry of the adjacency matrix A and a_{i0}. $\hat{(\cdot)}$ is denoted as a current estimate of $(\cdot)$, and p_i is defined by

$$p_i = 1/\theta_{i0}. \tag{66}$$

Concerned with a_{i0}, n_{i0} is defined as follows:

$$n_{i0} = \begin{cases} 1 : a_{i0} > 0, \\ 0 : \text{otherwise}. \end{cases} \tag{67}$$

Furthermore, v_i is a stabilizing signal to be determined later based on H_∞ control criterion. A tracking error between the leader y_0 and the follower y_i is defined by

$$\tilde{y}_i(t) \equiv y_i(t) - y_0(t), \tag{68}$$

and the substitution of (65) and (68) into (61) yields the total representation of the multi-agent system

$$\dot{\tilde{y}}(t) = \Omega(t)\{\Theta - \hat{\Theta}(t)\} + F_0(t)\Theta_0\{\hat{p}(t) - p\} - \alpha\, M\, \tilde{y}(t) \\ +\Theta_0 v(t) + \delta(t) + (N_0 - \mathbf{1})\dot{y}_0(t) + \epsilon(t), \quad (69)$$

$$\tilde{y} = [\tilde{y}_1, \cdots, \tilde{y}_N]^\mathsf{T}, \quad (70)$$

$$\Omega = \text{block diag}\,(\omega_1^\mathsf{T}, \cdots, \omega_N^\mathsf{T}), \quad (71)$$

$$\Theta = [\Theta_1^\mathsf{T}, \cdots, \Theta_N^\mathsf{T}]^\mathsf{T}, \quad (72)$$

$$F_0 = \text{diag}\,(f_{10}, \cdots, f_{N0}), \quad (73)$$

$$\Theta_0 = \text{diag}\,(\theta_{10}, \cdots, \theta_{N0}), \quad (74)$$

$$p = [p_1, \cdots, p_N]^\mathsf{T}, \quad (75)$$

$$N_0 = [n_{10}, \cdots, n_{N0}]^\mathsf{T}, \quad (76)$$

$$\mathbf{1} = [1, \cdots, 1]^\mathsf{T}, \quad (77)$$

$$v = [v_1, \cdots, v_N]^\mathsf{T}, \quad (78)$$

$$\delta = [\delta_1, \cdots, \delta_N]^\mathsf{T}, \quad (79)$$

$$\epsilon = [\epsilon_1, \cdots, \epsilon_N]^\mathsf{T}. \quad (80)$$

5.3 Adaptive H_∞ Consensus Control

A positive function W is defined by

$$W(t) = \frac{1}{2}\tilde{y}(t)^\mathsf{T}\, M\, \tilde{y}(t) \\ +\frac{1}{2}\left\{\hat{\Theta}(t) - \Theta\right\}^\mathsf{T}\Gamma_1^{-1}\left\{\hat{\Theta}(t) - \Theta\right\} \\ +\frac{1}{2}\left\{\hat{p}(t) - p\right\}^\mathsf{T}\Theta_0\,\Gamma_2^{-1}\left\{\hat{p}(t) - p\right\} \\ +\frac{1}{2}\left\{\hat{\theta}_0(t) - \theta_0\right\}^\mathsf{T}\Gamma_3^{-1}\left\{\hat{\theta}_0(t) - \theta_0\right\}, \quad (81)$$

$$(\Gamma_1 = \Gamma_1^\mathsf{T} > 0,\ \Gamma_2 = \Gamma_2^\mathsf{T} > 0,\ \Gamma_3 = \Gamma_3^\mathsf{T} > 0),$$

where Γ_2 is especially chosen as a diagonal matrix, and θ_0 is defined as follows:

$$\theta_0 = [\theta_{10}, \cdots, \theta_{N0}]^\mathsf{T}. \quad (82)$$

The tuning laws of $\hat{\Theta}$, $\hat{p}$, $\hat{\theta}_0$ are determined by

$$\begin{cases} \dot{\hat{\Theta}}(t) = \Pr\left\{\Gamma_1\Omega(t)^\mathsf{T}\, M\, \tilde{y}(t)\right\}, \\ \dot{\hat{p}}(t) = \Pr\left\{-\Gamma_2 F_0(t)^\mathsf{T}\, M\, \tilde{y}(t)\right\}, \\ \dot{\hat{\theta}}_0(t) = \Pr\left\{\Gamma_3 V(t)^\mathsf{T}\, M\, \tilde{y}(t)\right\}, \end{cases} \quad (83)$$

$$V = \text{diag}\,(v_1, \cdots, v_N), \quad (84)$$

where $\Pr(\cdot)$ are projection operations in which tuning parameters $\hat{\Theta}$, $\hat{p}$ and $\hat{\theta}_0$ are constrained to bounded regions deduced from upper-bounds of $\|\Theta\|$ and upper-bounds and lower-bounds of each element of p and θ_0, respectively (Ioannou and Sun [1996]). Then, the time derivative of W along its trajectory is given as follows:

$$\dot{W}(t) \le -\alpha\,\tilde{y}(t)^\mathsf{T}\, M^2\,\tilde{y}(t) + \tilde{y}(t)^\mathsf{T}\, M\,\hat{\Theta}_0(t)v(t) \\ +\tilde{y}(t)^\mathsf{T}\, M\left\{(N_0 - \mathbf{1})\dot{y}_0(t) + \epsilon(t)\right\} \\ +\tilde{y}(t)^\mathsf{T}\, M\,\delta(t). \quad (85)$$

From the evaluation of $\dot{W}$ (85), we introduce the next virtual system,

$$\dot{\tilde{y}} = f + g_{11}d_1 + g_{12}d_2 + g_2 v, \quad (86)$$

$$f = -\alpha\, M\tilde{y}, \quad (87)$$

$$g_{11} = G_\delta,\ g_{12} = I,$$

$$G_\delta = \text{block diag}\,(g_{\delta 1}^\mathsf{T}, \cdots, g_{\delta N}^\mathsf{T}),$$

$$g_2 = \hat{\Theta}_0, \quad (88)$$

$$d_1 = \begin{bmatrix} d_{\delta 1} \\ \vdots \\ d_{\delta N} \end{bmatrix},\ d_2 = (N_0 - \mathbf{1})\,\dot{y}_0 + \epsilon, \quad (89)$$

where d_1, d_2 are regarded as external disturbances to the process. Especially, d_1 is related to the neglected infinite-dimensional modes, and d_2 corresponds to the imperfect knowledge of the leader. We are to stabilize the virtual system via a control input v by utilizing H_∞ criterion for those external disturbances d_1, d_2 (Krstić and Deng [1998], Miyasato [2000]). For that purpose, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation and its solution W_0.

$$\mathcal{L}_f W_0 + \frac{1}{4}\left\{\sum_{i=1}^{2}\frac{\|\mathcal{L}_{g_{1i}}W_0\|^2}{\gamma_i^2} - (\mathcal{L}_{g_2}W_0)R^{-1}(\mathcal{L}_{g_2}W_0)^\mathsf{T}\right\} \\ +q = 0, \quad (90)$$

$$W_0 = \frac{1}{2}\tilde{y}^\mathsf{T}\, M\,\tilde{y}, \quad (91)$$

where q and R are a positive function and a positive definite matrix respectively, and those are derived from HJI equation based on inverse optimality (Krstić and Deng [1998], Miyasato [2000]) for the given solution W_0 and the positive constants γ_1, γ_2. By substituting the solution W_0 (91) into HJI equation (90), R and q are obtained such as

$$R = \left(\frac{\hat{\Theta}_0^{-1}G_\delta G_\delta^\mathsf{T}\hat{\Theta}_0^{-\mathsf{T}}}{\gamma_1^2} + \frac{\hat{\Theta}_0^{-1}\hat{\Theta}_0^{-\mathsf{T}}}{\gamma_3^2} + K\right)^{-1}, \quad (92)$$

$$q = \alpha\tilde{y}^\mathsf{T}\, M^2\,\tilde{y} + \frac{1}{4}\tilde{y}^\mathsf{T}\, M\,\hat{\Theta}_0 K\hat{\Theta}_0^\mathsf{T}\, M\,\tilde{y}, \quad (93)$$

where K is a diagonal positive definite matrix (a design parameter). From R, v is derived as a solution of the corresponding H_∞ control problem as follows:

$$v = -\frac{1}{2}R^{-1}(\mathcal{L}_{g_2}W_0)^\mathsf{T} = -\frac{1}{2}R^{-1}\hat{\Theta}_0^\mathsf{T}\, M\,\tilde{y}, \quad (94)$$

where the entries of $\hat{\Theta}_0$ are constructed from the elements of $\hat{\theta}_0$ (83). Then, the time derivative of W is evaluated by

$$\dot{W} \le -q - v^\mathsf{T}Rv \\ +\left(v + \frac{1}{2}R^{-1}\hat{\Theta}_0^\mathsf{T}\, M\,\tilde{y}\right)^\mathsf{T} R\left(v + \frac{1}{2}R^{-1}\hat{\Theta}_0^\mathsf{T}\, M\,\tilde{y}\right) \\ +\gamma_1^2\|d_1\|^2 - \gamma_1^2\sum_{i=1}^{N}\left\|d_{\delta i} - \frac{\left|\sum_{j=1}^{N} m_{ij}\tilde{y}_j\right| g_{\delta_i}}{2\gamma_1^2}\right\|^2 \\ +\gamma_2^2\|d_2\|^2 - \gamma_2^2\sum_{i=1}^{N}\left(|d_{2i}| - \frac{\left|\sum_{j=1}^{N} m_{ij}\tilde{y}_j\right|}{2\gamma_2^2}\right)^2, \quad (95)$$

$$M = [m_{ij}] \in \mathbf{R}^{N\times N}, \quad (96)$$

$$d_2 = (N_0 - \mathbf{1})\,\dot{y}_0 + \epsilon = [d_{21}, \cdots, d_{2N}]^\mathsf{T}, \quad (97)$$

and the next theorems are obtained.

Theorem 8. In the adaptive control system (65), (83), (94), the stabilizing signal v is a sub-optimal control input minimizing the upper bound on the cost functional J.

$$J(t) \equiv \sup_{d_1,d_2 \in \mathcal{L}_2} \left[\int_0^t \{q + v^\mathsf{T} R v\} d\tau + W(t) - \sum_{i=1}^{2} \gamma_i^2 \int_0^t \|d_i\|^2 d\tau \right]. \tag{98}$$

Also we have the next inequality.

$$\int_0^t \{q + v^\mathsf{T} R v\} d\tau + W(t) \leq \sum_{i=1}^{2} \gamma_i^2 \int_0^t \|d_i\|^2 d\tau + W(0). \tag{99}$$

Theorem 9. The adaptive control system (65), (83), (94) is uniformly bounded, and if $(N_0 - \mathbf{1})\,\dot{y}_0 = 0$ (that is; $\dot{y}_0(t) = 0$ or the information of the leader $\dot{y}_0$ is available for all followers), then it follows that

$$\lim_{T\to\infty} \sup \frac{1}{T} \int_0^T \|\tilde{y}(t)\|^2 dt \leq \text{const} \cdot \gamma_1^2. \tag{100}$$

Otherwise, if $(N_0 - \mathbf{1})\,\dot{y}_0 \neq 0$ (that is; $\dot{y}_0(t) \neq 0$ and the information of $\dot{y}_0$ is not available for all followers), then the next relation holds.

$$\lim_{T\to\infty} \sup \frac{1}{T} \int_0^T \|\tilde{y}(t)\|^2 dt \leq \text{const} \cdot (\gamma_1^2 + \gamma_2^2). \tag{101}$$

Remark 10. Theorem 9 states that the approximate consensus tracking with the ratio of γ_1 or $\sqrt{\gamma_1^2 + \gamma_2^2}$, is achieved according to the availability of $\dot{y}_0$ or the value of $\dot{y}_0$. Furthermore, the adaptive control scheme is constructed via $M\tilde{y}$ and local informations of each agents, and can be implemented in a distributed fashion.

6. CONCLUDING REMARKS

Design methodologies of adaptive H_∞ consensus control of multi-agent systems composed of a class of infinite-dimensional systems have been provided in the present paper. The proposed control strategy is composed of finite dimensional compensators, and is derived as a solution of certain H_∞ control problem, where the effects of neglected infinite-dimensional modes and the imperfect knowledge of the leader are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and neglected infinite-dimensional modes, and that the desirable consensus tracking is achieved approximately via adaptation schemes. The proposed method would provide a basic and useful strategy to deal with the coordinate control of certain large-scale complicated processes.

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