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Decentralized Networked Control for Vehicle-String Velocity and Spacing Distance Bias System

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Abstract: Decentralized overlapping feedback control laws are designed for a vehicle-string bias system which controlled over networks in this paper. The dynamic model of vehicle-string bias system is treated as an interconnected system with overlapping states. Using the mathematic framework of the Inclusion Principle, the interconnected system is expanded into a higher dimensional space in which the subsystems appear to be disjoint. Then, for the locally extracted subsystems with respect to network-induced time-delay and packet-dropout, by modeling the networked control system (NCS) as an asynchronous dynamic system (ADS) with rate constraints on events, the state feedback controllers are designed and the sufficient exponential stability criterion is derived. The design procedure is based on linear matrix inequalities (LMI). As a final step, the decentralized controllers are contracted back to the original space for implementation. The simulation result is given to show the effectiveness of the method.

1. INTRODUCTION

Intelligent vehicle/highway systems (IVHS) have attracted a considerable attention among researchers (Jonathan A. Rogge and Dirk Aeyels, 2008; Gianluca Antonelli and Stefano Chiaverini , 2006; Jonathan A. Rogge and Dirk Aeyels, 2008; Varaiya P ,1993). The IVHS architecture is usually based on the notion of platoons, vehicle-string following the leading vehicle with small intra-platoon separation and the focus is on developing control methods to allow platoons of vehicles to automatically move at a desired velocity with a specified spacing distance between vehicles. Control of platoons of vehicles has been studied from different viewpoints (D. N. Godbole and J. Lygeros., 1994; S. Sheikholeslam and C. A. Desoer ,1992; D. Swaroop and J. K. Hedrick, 1996).The so-called *spacing* and *headway* control strategies are discussed (Srdjan S. Stankovic, Milorad J. Stanojevic, and Dragoslav D. Siljak , 2000; D. Swaroop, J. K. Hedrick, C. C. Chien, and P. Ioannou, 1994) . Generally, decentralized control schemes are chosen since they are superior in terms of reliability with respect to structural reconfigurations to centralized control schemes.

From a viewpoint, model of platoon can be treated as an interconnected system of overlapping subsystems (the subsystems share common components). This allows one to consider control structures based on overlapping. A general mathematical framework for overlapping decompositions and decentralized control is the Inclusion Principle (M. Ikeda, D. D. Siljak and D. E. White, 1984; Chu, D., and Siljak, D. D., 2005). A dynamic system with overlapping information structure constraints is expanded into a larger state space where the subsystems appear as disjoint. Then, the estimation and control laws are designed in the expanded space using standard methods for disjoint subsystems. Under the inclusion conditions, the laws can be contracted to the smaller space for implementation in the original system (X.-B. Chen and S.S. Stankovic, 2005; Zecevic, A. I., and Siljak, D. D. , 2005). The motivation to use decentralized overlapping control comes from the fact that it has already been successfully applied to control a model of a platoon of vehicles (Srdjan S. Stankovic, Milorad J. Stanojevic, and Dragoslav D. Siljak , 2000).

Recently, new methods and algorithms have been proposed to include communication issues into the decentralized control design framework (Smith, R. S. and Hadaegh, F. Y. ,2007; Roberts, D. G. and Stilwell, D. J.,2005; Stubbs, A., Vladimerou, V., Fulford, A., Strick, J., and Dullerud, G. E. ,2006). Though a variety of structures and models in this framework have been analyzed, there remains a gap between decentralized control and control over networks (Lubomir Bakule, 2008). To the authors' knowledge, strategies taking systematically into account networked control systems (NCSs) which controlled over realistic decentralized communication channels have not yet been reported.

Network-induced delay and packet-dropout are two main problems in NCSs. In this paper, a novel strategy is presented for decentralized overlapping state feedback design with respect to network-induced time-delay and packet-dropout. Firstly, The dynamic model of vehicle-string bias system is expanded into a higher dimensional space in which the subsystems appear to be disjoint using the Inclusion Principle ; Then, for the locally extracted subsystems, by modeling the networked control systems as an asynchronous dynamical system （ADS） with rate constraints on events, the sufficient time-delay and data packet dropout criterion for the exponential stability of the networked control systems are presented and proved by using the Lyapunov stability theory. Last, the decentralized controllers are contracted back to the original space for implementation based on the Inclusion Principle.

The organization of the paper is as follows. In section 2, the linear bias model of a platoon that can be treated as an interconnected system with state coupled is described. In section 3, the Inclusion Principle and corresponding expansion and contraction procedures are presented, and in section 4, a decentralized networked controller design procedure and sufficient conditions for exponentially stable are derived in terms of LMI algorithms. In section 5, simulation results for a platoon consisting of 4 vehicles are studied.

2. MODEL DESCRIPTION

Let i_{th} automotive vehicle in a platoon be represented by the following model (Levine, W. S. and M. Athans, 1966):

$$\begin{aligned} \Delta\dot{v}_i &= -\Delta v_i + \Delta u_i, i = 1,2,\cdots,N \\ \Delta\dot{d}_{i-1,i} &= \Delta v_{i-1} - \Delta v_i, i = 2,3,\cdots,N \end{aligned} \tag{1}$$

Where Δv_i , $\Delta d_{i-1,i}$ and Δu_i are derivation of the velocity, spacing distance and control input respectively, for sake of simplicity, the above formula can be denoted

$$\begin{aligned} \dot{v}_i &= -v_i + u_i, i = 1,2,\cdots,N \\ \dot{d}_{i-1,i} &= v_{i-1} - v_i, i = 2,3,\cdots,N \end{aligned} \tag{2}$$

It will be initially assumed that the measurements of $d_{i-1,i}$, v_i , v_{i-1} are available in i_{th} vehicles. To illustrate this, consider an interconnected system with three subsystems as shown in Fig. 1.

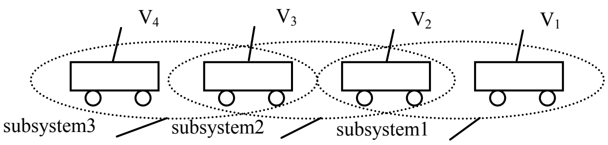

Fig.1. Plot of a platoon with information structure constraint

Based on (2), the following state-space model S of the entire platoon can be formulated (assuming that all the vehicles have identical model) as follow:

$$S: \dot{x} = Ax + Bu, y = Cx \tag{3}$$

Where:

$$x = \left(v_1,\ d_{1,2},\ v_2,\ d_{2,3},\ \cdots,\ d_{N-1,N},\ v_N\right)^T$$

$$u = \left(u_1,\ u_2,\ \cdots,\ u_N\right)^T$$

$$y = \left(y_{v_1},\ y_{d_{1,2}},\ y_{v_2},\ y_{d_{2,3}},\ \cdots,\ y_{d_{N-1,N}},\ y_{v_N}\right)^T$$ and

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

The overlapping interconnections between subsystems are depicted with dashed lines.

3. OVERLAPPING STRUCTURE DECOMPOSITION

3.1 Inclusion Principle

Consider a pair $\left(S,\tilde{S}\right)$ of linear time-invariant continuous-time dynamic systems represented by

$$\begin{aligned} S &: \dot{x} = Ax + Bu, y = Cx \\ \tilde{S} &: \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}, \tilde{y} = \tilde{C}\tilde{x} \end{aligned} \tag{4}$$

Where $x(t_0) = x_0$ and $\tilde{x}(t_0) = \tilde{x}_0$, $x \in R^n$ and $\tilde{x} \in R^{\tilde{n}}$ are the states, $u \in R^p$ and $\tilde{u} \in R^{\tilde{p}}$ the inputs, $y \in R^q$ and $\tilde{y} \in R^{\tilde{q}}$ the outputs of S and $\tilde{S}$,respectively.

It is assumed that $n \le \tilde{n}, p \le \tilde{p}, q \le \tilde{q}$.

Definition 1: The system $\tilde{S}$ includes the system S if there exists a quadruplet of full rank matrices $\{U,V,R,S\}$ satisfying $UV = I_n$, such that for any x_0 and u in S the conditions $\tilde{x}_0 = Vx_0$ and $\tilde{u} = Ru$ imply $x = U\tilde{x}$ and $y = S\tilde{y}$.

There are different combinations of state, input and output contractions/expansions (X.-B. Chen and S.S. Stankovic, 2005).We shall focus our attention to one particular case of restriction.

Theorem 1 the system S is a restriction of $\tilde{S}$ if there exist full rank matrices $\{V,R,T\}$ such that

$$\tilde{A}V = VA; \tilde{B}R = VB; \tilde{C}V = TC$$

If the pairs of matrices (U, V), (Q, R), and (S, T) are specified, the matrices $\tilde{A},\tilde{B},\tilde{C}$ can be expressed as

$$\tilde{A} = VAU + M_A, \tilde{B} = VBQ + M_B,$$
$$\tilde{C} = TCU + M_C$$

Where M_A, M_B, and M_C are complementary matrices of appropriate dimensions. For $\tilde{S}$ to be an expansion of S, a proper choice of M_A, M_B, and M_C is required and satisfying

$$M_A V = 0; M_B R = 0; M_C V = 0$$

If static feedback control laws for both systems are assumed to be in the following form:

$$u = Kx, K \in R^{m\times n} \ ; \tilde{u} = \tilde{K}\tilde{x}, \tilde{K} \in R^{\tilde{m}\times\tilde{n}}$$

Then the condition for close-loop system $\bar{S} : \dot{x} = \left(A + BK\right)x$ is included in $\bar{\tilde{S}} : \dot{\tilde{x}} = \left(\tilde{A} + \tilde{B}\tilde{K}\right)\tilde{x}$ are given as

Theorem 2: $\bar{S}$ is a restriction of $\bar{\tilde{S}}$ if one of the following is true:

(a) $\tilde{A}V = VA, \tilde{B}R = VB, \tilde{K}V = RK$

(b) $\tilde{A}V = VA, B = VBQ, K = Q\tilde{K}V$

3.2 Overlapping Structure Decomposition

Let us consider a linear system (3). We regard the system as composed of *N*-1 overlapping subsystems. By choose expansion matrices

$$V = T = diag\left(I_2,\ O_1,\ 1,\ O_1,\ 1,\ \cdots,\ O_1,\ I_2\right)$$

$$U = S = diag\left(I_2,\ O_{\beta 1},\ 1,\ O_{\beta 1},\ 1,\ \cdots,\ O_{\beta 1},\ I_2\right)$$

$$R = diag\left(1,\ O_1,\ O_1,\ \cdots,\ O_1,\ 1\right)$$

$$Q = diag\left(1,\ O_{\beta 1},\ O_{\beta 1},\ \cdots,\ O_{\beta 1},\ 1\right)$$

Satisfying

$$UV = I_{2N-1}; QR = I_N; ST = I_{2N-1}$$

Where

$$O_1 = \begin{bmatrix}1 & 1\end{bmatrix}^T, O_{\beta 1} = \begin{bmatrix}\beta & 1-\beta\end{bmatrix}$$

β is the dynamic balance factor of overlapping structure decomposition, $0 < \beta < 1$.

Complement matrix can be chosen

$$M_A = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_{d2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_{a2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_{d1} & 0 & m_{d2} & \cdots & 0 & 0 \\ 0 & 0 & 0 & m_{a3} & \cdots & 0 & 0 \\ 0 & 0 & 0 & m_{d1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & m_{d2} & 0 \\ 0 & 0 & 0 & 0 & \cdots & m_{aN-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & m_{d1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$M_B = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_{b2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & m_{b3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & m_{bN-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$M_C = diag\left[0,\ m_{c2},\ 0,\ m_{c3},\ \cdots,\ m_{cN-1},\ 0\right]$$

Where

$$m_{d1} = \begin{bmatrix}-\beta & \beta\end{bmatrix},\ m_{d2} = \begin{bmatrix}-(1-\beta) & (1-\beta)\end{bmatrix}$$

$$m_{ai} = \begin{bmatrix}-(1-\beta) & (1-\beta) \\ \beta & -\beta\end{bmatrix}$$

$$m_{bi} = \begin{bmatrix}(1-\beta) & -(1-\beta) \\ -\beta & \beta\end{bmatrix}$$

$$m_{ci} = \begin{bmatrix}(1-\beta) & -(1-\beta) \\ -\beta & \beta\end{bmatrix},\ i = 2,3,\cdots,N-1$$

Thus, we obtain overlapping structural decomposition pair-wise subsystem. That is

$$\tilde{S}_{i-1} : \begin{bmatrix}\dot{v}_{i-1} \\ \dot{d}_{i-1,i} \\ \dot{v}_i\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}v_{i-1} \\ d_{i-1,i} \\ v_i\end{bmatrix} + \begin{bmatrix}1 & 0 \\ 0 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}u_{i-1} \\ u_i\end{bmatrix}$$

$$\begin{bmatrix}y_{v_i-1} \\ y_{d_i-1} \\ y_{v_i}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_{i-1} \\ d_{i-1,i} \\ v_i\end{bmatrix}, i = 2,3,\cdots,N$$

4. DECENTRALIZED NETWORKED CONTROL

4.1 Modelling of networked control

For the locally extracted subsystems, the considered NCS with both time-delay and packet-dropout is shown in Fig. 2

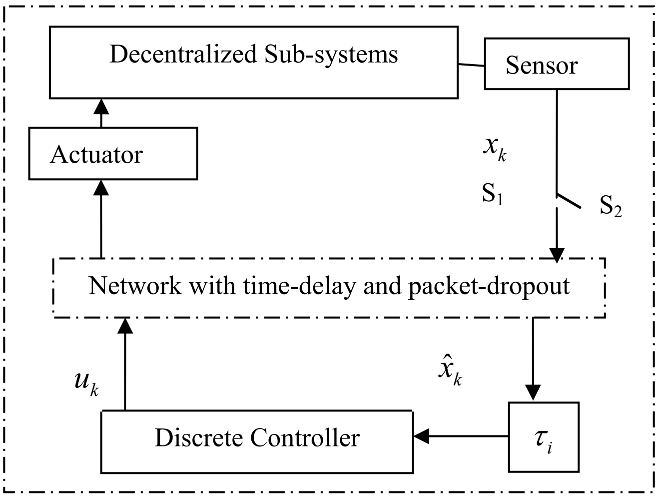

Fig.2. Laconic Model of NCS with time-delay and data-packet dropout

Throughout the paper, the following assumptions are needed for the considered NCS:

A1: The sensor is time-driven, and its sampling interval is T, both the controller and the actuator are event driven;

A2: The constant network-induced delay satisfying $0 \le \tau_i < T$, where $\tau_i = \tau_{sc} + \tau_{ca}$, and τ_{sc} is the sensor-to-controller delay while τ_{ca} is the controller-to-actuator delay.

We use S_1 to denote the event that a packet is successfully transmitted; use S_2 to denote the event that a packet is dropout.

We denote $A_d = e^{\tilde{A}T}$, $B_{d0}(\tau_i) = \int_0^{T-\tau_i} e^{\tilde{A}s}\tilde{B}ds$

$B_{d1}(\tau_i) = \int_{T-\tau_i}^{T} e^{\tilde{A}s}\tilde{B}ds$, $B_d(\tau_i) = \int_0^{T} e^{\tilde{A}s}\tilde{B}ds$

(1) When event S_1 occurs, we choose

$$\hat{x}_{ik} = \begin{cases} \hat{x}_{i(k-1)}, & kT < t \le kT + \tau_i \\ \hat{x}_{ik}, & kT + \tau_i < t \le (k+1)T \end{cases}$$

(2) When event S_2 occurs, namely there is packet-dropout, we have $\hat{x}_{ik} = \hat{x}_{i(k-1)}$

Thus, the model of the NCS under consideration is then described as follows:

$$S_1:\ \begin{aligned} x_{k+1} &= A_d x_k + B_{d0}u_k + B_{d1}u_{k-1} \\ y_k &= C\hat{x}_k \\ \hat{x}_k &= x_k \end{aligned} \tag{5}$$

$$S_2:\ \begin{aligned} x_{k+1} &= A_d x_k + B_d u_{k-1} \\ y_k &= C\hat{x}_k \\ \hat{x}_k &= x_k \end{aligned} \tag{6}$$

Applying the controller $u_k = \tilde{K}\hat{x}_k$ to system (5), (6) and results in the following close-loop NCS

$$S_1: \begin{bmatrix} x_{k+1} \\ \hat{x}_k \end{bmatrix} = \begin{bmatrix} A_d + B_{d0}\tilde{K} & B_{d1}\tilde{K} \\ I & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_{k-1} \end{bmatrix} \tag{7}$$

$$S_2: \begin{bmatrix} x_{k+1} \\ \hat{x}_k \end{bmatrix} = \begin{bmatrix} A_d & B_{d1}\tilde{K} \\ I & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_{k-1} \end{bmatrix} \tag{8}$$

4.2 Design of controller

Definition 2 the close-loop NCS is said to be exponentially stable with decay rate $\alpha > 0$ if

$$\lim_{t \to +\infty} e^{\alpha t} \left\| x(t) \right\| = 0 \text{ is true}$$

Lemma 1 (Arash Hassibi, Stephen P.Boyd and Jonathan P.How., 1999) given a difference equation

$$x_{k+1} = f_s(x_k), s = 1, 2, \cdots, N ,$$

a sufficient condition for exponential stability is the existence of $V: R^n \to R_+$, that V is continuously differentiable $\beta_1 \|x\|^2 \le V(x) \le \beta_2 \|x\|^2$, where $\beta_1 > 0, \beta_2 > 0$, and $\alpha_1, \alpha_2, \cdots, \alpha_N > 0$ satisfying

$$V(x_{k+1}) - V(x_k) \le (\alpha_s^{-2} - 1)V(x_k) \text{ and}$$

$$\alpha_1^{r_1} \alpha_2^{r_2} \cdots \alpha_N^{r_N} > \alpha > 1$$

Lemma 2 [Schur complement]: given symmetric matrices Σ_1, Σ_2 and matrix Σ_3 , where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0 .$$

Lemma 3 (Xie, L. 1996) given matrices M_1, M_2, M_3, Δ_k , where arbitrary matrix Δ_k satisfying $\Delta_k^T \Delta_k \le I$, M_1 is symmetry matrix, then $M_1 + M_3 \Delta_k M_2 + M_2^T \Delta_k^T M_3^T < 0$, if and only if there exist constant $\varepsilon > 0$, such that $M_1 + \varepsilon^{-1} M_3 M_3^T + \varepsilon M_2^T M_2 < 0$

The exponential stability result for NCS (7), (8) is presented in the following theorem.

Theorem 3: For given positive scalar *r* denoting event rate, applying the controller $u_k = \tilde{K}\hat{x}_k$, if there exist matrices T, X, $\tilde{K}$ and scalar a_1, a_2 , such that the following inequalities

$$a_1^{\ r} \times a_2^{\ 1-r} > \alpha > 0$$

$$\begin{bmatrix} T - a_1^{-2} X & 0 & (A_d X + B_{d0}\tilde{K}X)^T \\ 0 & -a_1^{-2} T & (B_{d1} Y)^T \\ A_d X + B_{d0}\tilde{K}X & B_{d1}\tilde{K}X & -X \end{bmatrix} < 0$$

$$\begin{bmatrix} -a_2^{-2} X & 0 & (A_d X)^T \\ 0 & T - a_2^{-2} T & (B_d \tilde{K}X)^T \\ A_d X & B_d \tilde{K}X & -X \end{bmatrix} < 0$$

hold, then the closed-loop NCS (7), (8) is exponentially stable.

Proof: Choose the Lyapunov function

$$V(k) = x_k^T P x_k + x_{k-1}^T Q x_{k-1}$$

When event S_1 occurs, by (7) we have

$$V(k+1) - a_1^{-2} V(k) = \begin{bmatrix} x_k^{\ T} & \hat{x}_{k-1}^{\ T} \end{bmatrix} \Omega_1 \begin{bmatrix} x_k \\ \hat{x}_{k-1} \end{bmatrix}$$

Where

$$\Omega_1 = \begin{bmatrix} (A_d + B_{d0}\tilde{K})^T P (A_d + B_{d0}\tilde{K}) + Q - a_1^{-2} P & (A_d + B_{d0}\tilde{K})^T P B_{d1}\tilde{K} \\ (B_{d1}\tilde{K})^T P (A_d + B_{d0}\tilde{K}) & (B_{d1}\tilde{K})^T P B_{d1}\tilde{K} - a_1^{-2} Q \end{bmatrix}$$

By Lemma 2 and Lemma 3, the following inequality holds

$$V(k+1) - a_1^{-2} V(k) < 0$$

This implies

$$\begin{bmatrix} Q - a_1^{-2} P & 0 & (A_d + B_{d0}\tilde{K})^T \\ 0 & -a_1^{-2} Q & (B_{d1}\tilde{K})^T \\ (A_d + B_{d0}\tilde{K}) & B_{d1}\tilde{K} & -P^{-1} \end{bmatrix} < 0$$

Left and right multiply by $diag\left\{P^{-1},\ P^{-1},\ I\right\}$ respectively, and let

$$X = P^{-1},\ T = P^{-1} Q P^{-1}$$

We derive

$$\begin{bmatrix} T-a_1^{-2}X & 0 & (A_dX+B_{d0}\tilde{K}X)^T \\ 0 & -a_1^{-2}T & (B_{d1}Y)^T \\ A_dX+B_{d0}\tilde{K}X & B_{d1}\tilde{K}X & -X \end{bmatrix} < 0$$

When event S_2 occurs, we have

$$V(k+1)-a_2^{-2}V(k)=\begin{bmatrix} x_k{}^T & \hat{x}_{k-1}{}^T \end{bmatrix}\Omega_2\begin{bmatrix} x_k \\ \hat{x}_{k-1} \end{bmatrix}$$

Where

$$\Omega_2=\begin{bmatrix} A_d{}^TPA_d-a_2^{-2}P & A_d{}^TPB_{d1}\tilde{K} \\ (B_{d1}\tilde{K})^TPA_d & (B_{d1}\tilde{K})^TPB_{d1}\tilde{K}+Q-a_2^{-2}Q \end{bmatrix}$$

Similar to event S_1, we derive

$$\begin{bmatrix} -a_2^{-2}X & 0 & (A_dX)^T \\ 0 & T-a_2^{-2}T & (B_d\tilde{K}X)^T \\ A_dX & B_d\tilde{K}X & -X \end{bmatrix} < 0$$

So, NCS is exponentially stable.

5. SIMULATION RESULT

Let us consider a vehicle-string bias system which is composed of 4 vehicles. The system model can be described as (3), where

$$A=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

According to inclusion principle, by choosing the transformation matrices $\{U,V,R,S\}$ satisfying *theorem* 1, system S can be expanded to $\tilde{S}$, where

$$\tilde{A}=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\tilde{B}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \tilde{C}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, for the locally extracted subsystems, we choose sample interval $T=0.1s$, time-delay $\tau_1=0.06s$, $\tau_2=0.03s$, $\tau_3=0.07s$ respectively, and assume that initial velocity $v_{10}=3m/s$; $v_{20}=1m/s$; $v_{30}=5m/s$; $v_{40}=2m/s$; de -sired velocity $v_d=3.5m/s$, initial spacing distance of four vehicles are (2,1,2) meters respectively, specified spacing distance $l_d=5m$.

Suppose packet-dropout rate $\gamma=0.9$, according to *Theorem*3, we have $a_1=1.07, a_2=0.6$ and state feedback gain matrix

$$\tilde{K}=\begin{bmatrix} -0.374 & -0.2137 & 0.159 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1592 & 0.2045 & -0.3439 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.374 & -0.2137 & 0.159 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1592 & 0.2045 & -0.3439 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.374 & -0.2137 & 0.159 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1592 & 0.2045 & -0.3439 \end{bmatrix}$$

for implementation, choose $\beta=0.5$, by *Theorem*2, we obtain control laws

$$K=\begin{bmatrix} -0.374 & -0.2137 & 0.159 & 0 & 0 & 0 & 0 \\ 0.0796 & 0.1023 & -0.359 & -0.107 & 0.08 & 0 & 0 \\ 0 & 0 & 0.0796 & 0.1023 & -0.359 & -0.107 & 0.08 \\ 0 & 0 & 0 & 0 & 0.1592 & 0.2045 & -0.3439 \end{bmatrix}$$

Simulation results are presented in Figs. 3 and 4.

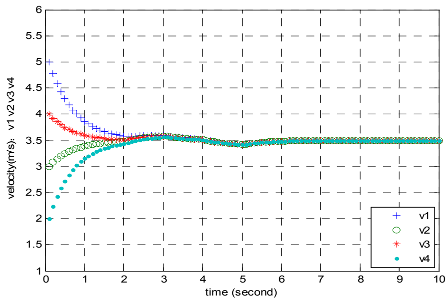

Fig.3. velocity of 4 vehicles

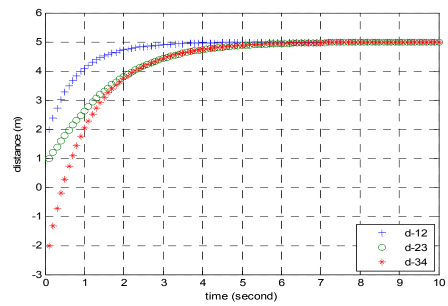

Fig.4. spacing distance of 4 vehicles

6. CONCLUSIONS

In this paper, an efficient method to design decentralized overlapping control laws for a platoon of vehicles with overlapping information structure in the networked control framework has been proposed. The platoon is modeled as an interconnected system. Static state feedback control laws were designed in the expanded space using the Inclusion Principle with respect to Network-induced delay and packet-dropout, and then contracted back to the original space for implementation. Since the algorithm is formulated in the expanded space where subsystems are disjoint, this method offers significant reduction in computational time due to the possibility of parallel processing. As an example, the procedure was applied to a platoon of four vehicles, and the obtained results are promising.

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