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Novel Anti-windup PID Controller Design under Holonomic Endpoint Constraints for Euler-Lagrange Systems with Actuator Saturation

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Abstract: A novel anti-windup PID controller design method under holonomic constraints is proposed for nonlinear Euler-Lagrange systems with actuator saturation. The controller design is based on passivity, quasi-natural potential and saturated-position feedback. According to four saturation cases, switching of four integrating functions in the control law is utilized and four Lyapunov functions, such as hybrid control, are derived. Global asymptotic stability is ensured by energy dissipation between the four Lyapunov functions. The control performance is verified by numerical simulations using a two-link robot arm.

1. INTRODUCTION

Every plant system is subject to input limitations. When the controller requires an input that exceeds these limitations, the control performance is degraded and becomes unstable. Actuator saturation caused by exceeded input limitations gives rise to windup phenomena, which degrade the control performance. Over the past decade, several anti-windup controller design methods have been proposed for linear systems to suppress windup degradation (*e.g.*, Kanamori *et al*., 2007). However, studies of specifically nonlinear robot systems are few. Several studies have investigated anti-windup controller design for robot systems, including those by Kanamori (2011, 2013a,b), Khan *et al*. (2010), Lopez-Araujo *et al*. (2012), Loria *et al*. (1997), Morabito *et al*. (2004), Suntibanez *et al*. (1996), Teo *et al.* (2009), and Zavara-Rio *et al.* (2009). Kanamori (2011) proposed an anti-windup PID position controller for nonlinear Euler-Lagrange systems with input saturation. In this work, global asymptotic stability was guaranteed by the Lyapunov theorem based on the passivity described by Arimoto (1996). This anti-windup method was extended to adaptive tracking control considering input saturation (Kanamori, 2013a), and the control performance was demonstrated by experimentation (Kanamori, 2013b). A series of such works based on passivity imply that passivity is extremely suitable to stabilization that considers input saturation for nonlinear Euler-Lagrange systems. Since, in general, holonomic constraints have given rise to almost all robot systems (Khan *et al*., 2010), an anti-windup control method based on passivity is expected.

In the present paper, the anti-windup adaptive law by Kanamori (2013a) is extended to anti-windup PID position control under holonomic constraints for nonlinear Euler-Lagrange systems with input saturation. According to four saturation cases, switching of four integrating functions in the control law is utilized and four Lyapunov functions, such as hybrid control, are derived. Global asymptotic stability is ensured by energy dissipation between the four Lyapunov functions. The control performance of the proposed controller is verified by numerical simulations using a two-link robot arm under holonomic constraints.

2. PRELIMINARY

Let us consider the case where the endpoint of the manipulator is constrained on a surface (Arimoto, 1996). The surface is described by a scalar function as

$$\phi(x_1, x_2, x_3) = 0\,, \tag{1}$$

where

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \tag{2}$$

denotes the Cartesian coordinates fixed at the internal reference frame. The contact force arises in the direction of the normal vector to the surface at point $\boldsymbol{x}$ and the contact friction arises in the direction of $-\dot{\boldsymbol{x}}$ with magnitude $\xi(\|\dot{\boldsymbol{x}}\|)\|\dot{\boldsymbol{x}}\|$, where $\xi(\cdot)$ is a positive scalar function. Then, the robot dynamics are described by

$$\begin{aligned} &\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \{\boldsymbol{B}_0 + \frac{1}{2}\dot{\boldsymbol{H}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \\ &\qquad = \boldsymbol{J}_\phi(\boldsymbol{q})^T f - \xi(\|\dot{\boldsymbol{x}}\|)\boldsymbol{J}_x(\boldsymbol{q})^T \dot{\boldsymbol{x}} + \boldsymbol{\sigma}(\boldsymbol{u})\,, \end{aligned} \tag{3}$$

where

$$\boldsymbol{J}_x(\boldsymbol{q}) = \left(\frac{\partial x_i(\boldsymbol{q})}{\partial q_j} \right), i = 1,2,3.,\ j = 1,\ldots,n, \tag{4}$$

$$\boldsymbol{J}_\phi(\boldsymbol{q}) = \frac{\left(\dfrac{\partial \phi}{\partial \boldsymbol{x}} \right)^T \boldsymbol{J}_x(\boldsymbol{q})}{\left\| \dfrac{\partial \phi}{\partial \boldsymbol{x}} \right\|^2}\,, \tag{5}$$

and f is the contact force. In addition, $\boldsymbol{q}=[q_1 \ \ .... \ \ q_n]^T$ represents the angular vector of each joint, $\boldsymbol{H}(\boldsymbol{q})\in \boldsymbol{R}^{n\times n}$ is the positive definite inertia matrix, $\boldsymbol{B_0}\in \boldsymbol{R}^{n\times n}$ is the positive diagonal constant matrix derived from the Raleigh dissipation function, $\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\in \boldsymbol{R}^{n\times n}$ is the skew-symmetric matrix, and $\boldsymbol{g}(\boldsymbol{q})$ is the gravity term derived from the relation $\frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}}=\boldsymbol{g}(\boldsymbol{q})$, where $U(\boldsymbol{q})$ is the potential energy. Furthermore, $\boldsymbol{u}=[u_1 \ \ .... \ \ u_n]^T$ is the control input torque vector, and $\boldsymbol{\sigma}(\boldsymbol{u})=[\sigma(u_1) \ \ ..... \ \ \sigma(u_n)]^T$ is the saturated input torque vector, defined as follows:

$$\begin{aligned}
&\text{if } u_{i\min}\le u_i\le u_{i\max},\ \sigma(u_i)=u_i,\\
&\text{if } 0<u_{i\max}<u_i,\quad \sigma(u_i)=u_{i\max},\\
&\text{if } u_i<u_{i\min}<0,\quad \sigma(u_i)=u_{i\min},
\end{aligned} \tag{6}$$

where $|u_{i\max}|=|u_{i\min}|$ for $i=1,...,n$. The estimation value of the input saturation $\boldsymbol{\psi}(\boldsymbol{u})=[\psi(u_1) \ \ ... \ \ \psi(u_n)]^T$ is introduced as

$$\psi(\boldsymbol{u})=\boldsymbol{u}-\sigma(\boldsymbol{u}), \tag{7}$$

where

$$\begin{aligned}
&\text{if } u_{i\min}\le u_i\le u_{i\max},\ \psi(u_i)=0,\\
&\text{if } 0<u_{i\max}<u_i,\quad \psi(u_i)=u_i-u_{i\max}>0,\\
&\text{if } u_i<u_{i\min}<0,\quad \psi(u_i)=u_i-u_{i\min}<0.
\end{aligned} \tag{8}$$

The closed-loop system is described as

$$\boldsymbol{L}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})+\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}),\boldsymbol{\sigma}(\boldsymbol{u}),f)=0, \tag{9}$$

where

$$\boldsymbol{L}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})=\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\{\boldsymbol{B}_0+\frac{1}{2}\dot{\boldsymbol{H}}(\boldsymbol{q})+\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\}\dot{\boldsymbol{q}} +\xi(\|\dot{\boldsymbol{x}}\|)\boldsymbol{J}_x(\boldsymbol{q})^T\boldsymbol{J}_x(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{10}$$

$$\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}),\boldsymbol{\sigma}(\boldsymbol{u}),f)=\boldsymbol{g}(\boldsymbol{q})-\boldsymbol{\sigma}(\boldsymbol{u})-\boldsymbol{J}_\phi(\boldsymbol{q})^T f. \tag{11}$$

To achieve the state $(\boldsymbol{q},\dot{\boldsymbol{q}},f)\to(\boldsymbol{q}_d,0,f_d)$, where $\boldsymbol{q}_d$ and f_d are the target vectors of each joint angle and target contact force, respectively, the following assumption is introduced.

Assumption 1: *Assume that the following conditions are satisfied for* $i=1,...,n$ *for any target posture:*

$$u_{i\min}<g_i(\boldsymbol{q})<u_{i\max}, \tag{12}$$

$$u_{i\min}<[\boldsymbol{g}(\boldsymbol{q}_d)+\boldsymbol{J}_\phi(\boldsymbol{q}_d)^T f_d]_i<u_{i\max}. \tag{13}$$

Take the appropriate output $\boldsymbol{y}$ and assume that the inner product of $\boldsymbol{y}$ and the closed-loop system (9) has the following form:

$$\boldsymbol{y}^T\boldsymbol{L}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})=\frac{d}{dt}V_0+W_0, \tag{14}$$

$$\boldsymbol{y}^T\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{u}),\boldsymbol{\sigma}(\boldsymbol{u}),f)\ge\frac{d}{dt}V_{ci}+W_{ci}, \tag{15}$$

where V_{ci} and W_{ci} are switched for $i=1,...,l$. Adding (14) and (15), the following relation holds:

$$0\ge\frac{dV_i}{dt}+W_i, \tag{16}$$

where

$$V_i=V_0+V_{ci} \text{ and } W_i=W_0+W_{ci}. \tag{17}$$

Assume that V_i and W_i for $i=1,...,l$ are positive definite and V_j is the minimum function, $V_j=\min(V_1,.....,V_l)$. Let us denote the difference between V_i and V_j for $i\ne j$ by $U_i=V_i-V_j$. For asymptotic stability, the following lemma may be introduced (Kanamori, 2013a).

Lemma 1*: If we have the following two states for* $i=1,...,l$ *:*

$$V_i>V_j>0 \text{ and } \dot{V}_i<0 \text{ for } i\ne j \text{ : state 1} \tag{18}$$

$$V_j>0 \quad \text{and } \dot{V}_j<0 \qquad \text{: state 2} \tag{19}$$

the minimum function V_j *is a Lyapunov function of the whole trajectory of the closed-loop system and the equilibrium state is asymptotically stable.*

Outline Proof of Lemma 1: *For the proof, it is sufficient to show that* V_j *is dissipative on state 1 shown in (18). Taking the time integral of (16), the following relation holds:*

$$V_j(t)-V_j(0)\le U_i(0)-U_i(t)-\int_0^t W_i dt. \tag{20}$$

If U_i *increases, then* V_j *decreases. If* U_i *decreases, then* V_j *might increase; however, the increase value is bounded by* $U_i(0)$*. Since* W_i *is positive definite, the time integral of* W_i*, which is the dissipated energy, always increases with the integral of time so that the increase in* V_j *turns into a decrease soon. This implies that* V_j *is a Lyapunov function and the equilibrium state of the closed-loop system is asymptotically stable.*

Let us choose the following output $\boldsymbol{y}$ for the evaluation in Arimoto (1996):

$$\boldsymbol{y}=\dot{\boldsymbol{q}}+\alpha\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q}), \tag{21}$$

where α is a positive definite scalar, $\boldsymbol{Q}_\phi(\boldsymbol{q})$ is defined as

$$\boldsymbol{Q}_\phi(\boldsymbol{q})=\boldsymbol{I}-\frac{\boldsymbol{J}_\phi(\boldsymbol{q})^T\boldsymbol{J}_\phi(\boldsymbol{q})}{\|\boldsymbol{J}_\phi(\boldsymbol{q})\|^2}, \tag{22}$$

which is regarded as a projection that projects vectors in joint space onto the plane tangent to the surface $\phi(\boldsymbol{q})=0$ at the contact point $\boldsymbol{q}$, and $\boldsymbol{I}$ denotes the identity matrix. The function $\boldsymbol{s}(\Delta\boldsymbol{q})$ is the output saturation function described in Arimoto (1996) as follows:

$$\boldsymbol{s}(\Delta\boldsymbol{q}) = [s_1(\Delta q_1) \quad s_2(\Delta q_2) \quad \ldots\ldots \quad s_n(\Delta q_n)]^T , \qquad (23)$$

where $\Delta\boldsymbol{q}$ is the control error defined by

$$\Delta\boldsymbol{q} = \boldsymbol{q} - \boldsymbol{q}_d . \qquad (24)$$

Property 1: *Concering the projection* $\boldsymbol{Q}_\phi(\boldsymbol{q})$*, the following relations hold.*

$$\boldsymbol{J}_\phi(\boldsymbol{q})\dot{\boldsymbol{q}} = 0 , \qquad (25)$$

$$\boldsymbol{Q}_\phi(\boldsymbol{q})\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}} , \qquad (26)$$

$$\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{J}_\phi(\boldsymbol{q})^T = 0 , \qquad (27)$$

$$\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{Q}_\phi(\boldsymbol{q}) = \boldsymbol{Q}_\phi(\boldsymbol{q}) . \qquad (28)$$

Property 2: *For the output saturation function in (23), the following properties are ensured.*

(a) There exists a positive convex function $S_i(x_i)$ *such that the following relation is satisfied:*

$$\frac{dS_i(x_i)}{dx_i} = s_i(x_i) . \qquad (29)$$

Then, the following relations hold:

$$\dot{S}_i(\Delta q_i) = s_i(\Delta q_i)\Delta\dot{q}_i \text{ and } \sum_i \dot{S}_i(\Delta q_i) = \boldsymbol{s}(\Delta\boldsymbol{q})^T \dot{\boldsymbol{q}} . \qquad (30)$$

(b) There exists a positive real constant c_i *such that the following inequality is satisfied:*

$$S_i(\Delta q_i) \ge c_i s_i(\Delta q_i)^2 . \qquad (31)$$

(c) There exists a positive real constant $\bar{c}_0$ *such that the following inequality is satisfied:*

$$-\dot{\boldsymbol{s}}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{s}(\Delta\boldsymbol{q})^T \{-\frac{1}{2}\boldsymbol{Q}_\phi(\boldsymbol{q})\dot{\boldsymbol{H}}(\boldsymbol{q}) - \dot{\boldsymbol{Q}}_\phi(\boldsymbol{q})\boldsymbol{H}(\boldsymbol{q}) + \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\}\dot{\boldsymbol{q}} + \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\}\dot{\boldsymbol{q}} \ge -\bar{c}_0 \|\dot{\boldsymbol{q}}\|^2 . \qquad (32)$$

Property 3: *Based on the characteristics of the output saturation function* $\boldsymbol{s}(\cdot)$ *and condition (12), there exists a positive real constant* c_{0i} *such that the following inequality is satisfied.*

$$\min\{|u_{i\max} - g_i(\boldsymbol{q})|, |u_{i\min} - g_i(\boldsymbol{q})|\} \ge c_{0i} |\boldsymbol{s}(\Delta\boldsymbol{q})|_{i\max} \ge c_{0i} |\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q})|_i . \qquad (33)$$

For the inner product, as shown on the left-hand side of (15), notice that the term $\boldsymbol{y}^T \boldsymbol{J}_\phi(\boldsymbol{q})^T f$ is zero with the relations (25) and (27) in Property 1 in Arimoto (1996). According to the polarity of the inner product $\boldsymbol{y}^T \boldsymbol{\psi}(\boldsymbol{u})$, the input saturation function $\boldsymbol{\sigma}(\cdot)$ is eliminated, as shown in Property 1 in Kanamori (2013a). Replacing $\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q})$ for $\boldsymbol{y}$ in Property 1 in Kanamori (2013a) and using (33), the following property holds so that the margins between input limits and the gravity term are utilized for the stability.

Property 4: *The following inequality holds:*

If $\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{\psi}(\boldsymbol{u}) < 0$*, then*

$$\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) , \ge c_{0\min} \boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q}) , \qquad (34)$$

where $c_{0\min}$ *is the positive minimum value of* c_{0i} *for* $i = 1,\ldots,n$ *as follows.*

$$c_{0\min} = \min[c_{01} \quad \ldots\ldots \quad c_{0n}] . \qquad (35)$$

Property 5: *The following inequalities are satisfied for saturation cases (I) through (IV) so that the input saturation function is eliminated by the inequalities (Kanamori, 2013a):*

(I) If $\dot{\boldsymbol{q}}^T \boldsymbol{\psi}(\boldsymbol{u}) \ge 0$ *and* $\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{\psi}(\boldsymbol{u}) \ge 0$*, then*

$$\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) \ge \boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f) , \qquad (36)$$

(II) If $\dot{\boldsymbol{q}}^T \boldsymbol{\psi}(\boldsymbol{u}) \ge 0$ *and* $\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{\psi}(\boldsymbol{u}) < 0$*, then*

$$\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) \ge \dot{\boldsymbol{q}}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f) + \alpha c_{0\min} \boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q}) , \qquad (37)$$

(III) If $\dot{\boldsymbol{q}}^T \boldsymbol{\psi}(\boldsymbol{u}) < 0$ *and* $\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{\psi}(\boldsymbol{u}) \ge 0$*, then*

$$\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) \ge a\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f) , \qquad (38)$$

(IV) If $\dot{\boldsymbol{q}}^T \boldsymbol{\psi}(\boldsymbol{u}) < 0$ *and* $\boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{\psi}(\boldsymbol{u}) < 0$*, then*

$$\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) \ge \alpha c_{0\min} \boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q}) . \qquad (39)$$

The inner product, as shown in (14), is determined by the given robot system. V_0 and W_0 are independent of the controller design and are obtained as follows (Arimoto, 1996).

Property 6: V_0 *and* W_0 *as shown in (14) become*

$$V_0 \ge \frac{1}{4}\dot{\boldsymbol{q}}^T \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha(b_{0\min}c_{\min} - \alpha\gamma_M)\|\boldsymbol{s}(\Delta\boldsymbol{q})\|^2 , \qquad (40)$$

$$W_0 \ge \{b_{0\min} - \alpha\bar{c}_0 + \frac{1}{2}\xi(\|\dot{x}\|)\boldsymbol{J}_x(\boldsymbol{q})^T \boldsymbol{J}_x(\boldsymbol{q})\}\|\dot{\boldsymbol{q}}\|^2 - \frac{1}{2}\alpha^2 \boldsymbol{s}(\Delta\boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})[\frac{1}{2}\Delta\boldsymbol{B}_0 + \xi(\|\dot{x}\|)\boldsymbol{J}_x(\boldsymbol{q})^T \boldsymbol{J}_x(\boldsymbol{q})]\boldsymbol{Q}_\phi(\boldsymbol{q})\boldsymbol{s}(\Delta\boldsymbol{q}) , \qquad (41)$$

where $c_{\min}$ *is the minimum value of* c_i *as follows:*

$$c_{\min} = \min\{c_1 \quad \ldots\ldots \quad c_n\} , \qquad (42)$$

$b_{0\min}$ *is the minimum value of the diagonal component of the positive diagonal matrix* $\boldsymbol{B}_0$ *in (3),* γ_M *is the maximum eigenvalue of* $\boldsymbol{H}(\boldsymbol{q})$*, and* $\Delta\boldsymbol{B}_0$ *is the difference between* $\boldsymbol{B}_0$ *and* $b_{0\min}\boldsymbol{I}$ *as follows:*

$$\Delta\boldsymbol{B}_0 = \boldsymbol{B}_0 - b_{0\min}\boldsymbol{I} \ge 0 . \qquad (43)$$

Assumption 2: *There exists positive real numbers* α *and* $c_{0\min}$ *such that the following inequalities are satisfied:*

$$b_{0\min} c_{\min} - \alpha \gamma_M > 0, \quad (44)$$

$$b_{0\min} - \alpha \bar{c}_0 > 0, \quad (45)$$

$$2c_{0\min} \boldsymbol{I} - \alpha \{ \frac{1}{2} \Delta \boldsymbol{B}_0 + \xi(\|\dot{x}\|) \boldsymbol{J}_x(\boldsymbol{q})^T \boldsymbol{J}_x(\boldsymbol{q}) \} > 0. \quad (46)$$

Property 7: *There exists a positive real number* a *such that the following inequalities are satisfied:*

$$U(\boldsymbol{q}) - U(\boldsymbol{q}_d) - \Delta \boldsymbol{q}^T \boldsymbol{g}(\boldsymbol{q}_d) + \frac{a}{2} \|\Delta \boldsymbol{q}\|^2 > 0, \quad (47)$$

$$\boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q}) \{ \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_d) \} + \frac{a}{2} \|\boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q})\|^2 > 0. \quad (48)$$

Property 8: The contact force f is given by the following form:

$$f = \left[\boldsymbol{J}_\phi(\boldsymbol{q}) \boldsymbol{H}(\boldsymbol{q})^{-1} \boldsymbol{J}_\phi(\boldsymbol{q})^T \right]^{-1} \left[-\boldsymbol{J}_\phi(\boldsymbol{q}) \boldsymbol{H}(\boldsymbol{q})^{-1} \boldsymbol{X} - \dot{\boldsymbol{J}}_\phi(\boldsymbol{q}) \dot{\boldsymbol{q}} \right], \quad (49)$$

where

$$\boldsymbol{X} = -\{ \boldsymbol{B}_0 + \frac{1}{2} \dot{\boldsymbol{H}}(\boldsymbol{q}) + \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \} \dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\sigma}(\boldsymbol{u}) - \xi(\|\dot{x}\|) \boldsymbol{J}_x(\boldsymbol{q})^T \dot{\boldsymbol{x}}. \quad (50)$$

3. CONTROLLER DESIGN

Let us consider the following proportional, integral and derivative (PID) control law:

$$\boldsymbol{u} = -a \Delta \boldsymbol{q} - b \dot{\boldsymbol{q}} - c \boldsymbol{Q}_\phi(\boldsymbol{q}) \int_0^t \bar{\boldsymbol{y}}(\tau) d\tau - \boldsymbol{J}_\phi(\boldsymbol{q})^T f_d + \beta \boldsymbol{J}_\phi(\boldsymbol{q})^T \int_0^t \Delta f(\tau) d\tau, \quad (51)$$

where a, b, c, and β are positive definite real numbers that are regarded as feedback gain, and Δf is the error between contact force f and its target force f_d represented as $\Delta f = f - f_d$. Using the notation

$$\Delta F = \int_0^t \Delta f(\tau) d\tau, \quad (52)$$

$$\boldsymbol{z} = \int_0^t \bar{\boldsymbol{y}}(\tau) d\tau - \int_0^\infty \bar{\boldsymbol{y}}(\tau) d\tau \text{ and } \int_0^\infty \bar{\boldsymbol{y}}(\tau) d\tau = -c^{-1} \boldsymbol{g}(\boldsymbol{q}_d), \quad (53)$$

$\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f)$ defined by (11) becomes

$$\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{g}(\boldsymbol{q}_d) + a \Delta \boldsymbol{q} + b \dot{\boldsymbol{q}} + \boldsymbol{Q}_\phi(\boldsymbol{q}) c \boldsymbol{z} - \boldsymbol{J}_\phi(\boldsymbol{q})^T (\Delta f + \beta \Delta F) + \boldsymbol{\psi}(\boldsymbol{u}). \quad (54)$$

Let us take the following $\bar{\boldsymbol{y}}(t)$ according to the four saturation cases (Kanamori, 2013a):

In CASE (I), $\bar{\boldsymbol{y}}(t) = \dot{\boldsymbol{q}} + \alpha \boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q})$, (55)

In CASE (II), $\bar{\boldsymbol{y}}(t) = \dot{\boldsymbol{q}}$, (56)

In CASE (III), $\bar{\boldsymbol{y}}(t) = \alpha \boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q})$, (57)

In CASE (IV), $\bar{\boldsymbol{y}}(t) = 0$. (58)

The inner products, as shown on the right-hand side of (36) through (39), can be taken easily because $\sigma(\cdot)$ is eliminated and V_{ci} and W_{ci} in (15) are also easily derived. In CASE (I), the inner product $\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f)$ in (36) is represented as follows:

$$\boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{\sigma}(\boldsymbol{u}), f) \geq \boldsymbol{y}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f) = \frac{dV_{c1}}{dt} + W_{c1}, \quad (59)$$

where V_{c1} and W_{c1} are represented as

$$V_{c1} \geq U(\boldsymbol{q}) - U(\boldsymbol{q}_d) - \Delta \boldsymbol{q}^T \boldsymbol{g}(\boldsymbol{q}_d) + \frac{a}{2} \|\Delta \boldsymbol{q}\|^2 + \alpha b c_{\min} \|\boldsymbol{s}(\Delta \boldsymbol{q})\|^2 + \frac{c}{2} \|\boldsymbol{z}\|^2, \quad (60)$$

$$W_{c1} \geq \alpha [\boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q}) \{ \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_d) \} + a \|\boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q})\|^2] + b \|\dot{\boldsymbol{q}}\|^2. \quad (61)$$

The suffix $i = 1$ of V_{c1} and W_{c1} means saturation CASE (I). In the derivation, the following relation, (26) and (28) are utilized:

$$\boldsymbol{y}^T \boldsymbol{Q}_\phi(\boldsymbol{q}) c \boldsymbol{z} = c \boldsymbol{y}^T \boldsymbol{z} = c \bar{\boldsymbol{y}}^T \boldsymbol{z} = c \dot{\boldsymbol{z}}^T \boldsymbol{z} = \frac{c}{2} \frac{d}{dt} \|\boldsymbol{z}\|^2. \quad (62)$$

In the same manner, V_{ci} and W_{ci} for $i = 2, 3$ are obtained by using (56) and (57), respectively. Notice that the second term $\alpha c_{0\min} \boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q})$ on the right-hand side of (37) or (39) must be included in W_{c2} or W_{c4}, respectively. In CASE (IV), there is no inner product on the right-hand side in (39); however, it is regarded that there is the inner product of zero and $\boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f)$ as follows:

$$\boldsymbol{0}^T \boldsymbol{F}(\boldsymbol{g}(\boldsymbol{q}), \boldsymbol{u}, f) = \frac{dV_{c4}}{dt} = 0, \quad (63)$$

where

$$V_{c4} = \frac{c}{2} \|\boldsymbol{z}\|^2 : \text{constant}. \quad (64)$$

Then, W_{c4} becomes

$$W_{c4} \geq \alpha c_{0\min} \boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q}) \boldsymbol{s}(\Delta \boldsymbol{q}). \quad (65)$$

It is confirmed that V_{ci} for $i = 1, \ldots, 4$ is positive definite when the conditions (47) are satisfied, and V_i in the first equation in (17) is also positive definite, since V_0 is positive definite under (44). W_i for $i = 1, \ldots, 4$ in the second equation in (17) is also positive definite under (45), (46), (48) and the following condition:

$$a \boldsymbol{I} - \alpha \{ \frac{1}{2} \Delta \boldsymbol{B}_0 + \xi(\|\dot{x}\|) \boldsymbol{J}_x(\boldsymbol{q})^T \boldsymbol{J}_x(\boldsymbol{q}) \} > 0. \quad (66)$$

The above results imply that we have the following two states according to the saturation cases:

$$V_i \geq V_4 > 0 \, , \; \dot{V}_i < 0 \, , \text{ for } i = 1, 2, 3 : \text{state 1} \qquad (67)$$

$$V_4 > 0 \, , \quad \dot{V}_4 < 0 \, . \qquad : \text{state 2} \qquad (68)$$

where V_i is switched according to saturation cases (I) through (IV). Based on Lemma 1, we have the following main theorem.

Main Theorem (Anti-windup PID control law): *Assume that conditions (44) through (46) are satisfied. Choose feedback gain a as the positive real number such that the conditions (47), (48) and (66) are satisfied. Adopt the switching function as (55) through (58) for the control law (51). Then, the equilibrium state of the closed-loop system is globally asymptotically stable based on Lemma 1.*

Proposition (Practical anti-windup PID control law): *Assume that the conditions in the Main Theorem are satisfied. For the control law (51), choose the function* $\bar{\boldsymbol{y}}(t)$ *as follows:*

$$\bar{\boldsymbol{y}}(t) = \bar{\boldsymbol{y}}_1(t) + \alpha \bar{\boldsymbol{y}}_2(t) \, , \qquad (69)$$

where

$$\text{if } \dot{q}_i \psi(u_i) \geq 0 \, , \text{ then } \bar{y}_{1i} = \dot{q}_i \, , \qquad (70)$$

$$\text{if } \dot{q}_i \psi(u_i) < 0 \, , \text{ then } \bar{y}_{1i} = 0 \, , \qquad (71)$$

$$\text{if } \{\boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\}_i \psi(u_i) \geq 0 \, , \text{ then, } \quad \bar{y}_{2i} = \{\boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\}_i \, , \qquad (72)$$

$$\text{if } \{\boldsymbol{s}(\Delta \boldsymbol{q})^T \boldsymbol{Q}_\phi(\boldsymbol{q})\}_i \psi(u_i) < 0 \, , \text{ then } \bar{y}_{2i} = 0 \, , \qquad (73)$$

for $i = 1, \ldots\ldots, n$ *, where* $\{\cdot\}_i$ *denotes the i-th component of the vector,* $\bar{y}_{ki}$ *denotes the following vector component:*

$$\bar{\boldsymbol{y}}_k = [\bar{y}_{k1} \;\; \ldots \bar{y}_{ki} \ldots \;\; \bar{y}_{kn}]^T \text{ and } k = 1, 2 \, . \qquad (74)$$

Then, the equilibrium state of the closed-loop system is globally asymptotically stable.

4. CONTROL PERFORMANCE

The control performance of the proposed anti-windup PID controller is examined by numerical simulations using a two-link robot arm. A schematic two-link robot arm with contact force is depicted in Fig. 1, where L_1 and L_2 denote the length of the arm. L_0 is the coordinate value of x_1 and the start point of the constraint surface. In this system, the constraint surface is represented as (75) and (1) becomes (76).

$$x_2 = a_s x_1 + b_s \, , \; L_0 \leq x_1 \, , \qquad (75)$$

$$\phi = x_2 - a_s x_1 - b_s = 0 \, , \qquad (76)$$

where a_s and b_s are real constants for the constraint surface. The system equation with actuators and the detail of the two-link robot system have been described in (48) through (51) and TABLE 1 in Kanamori (2013b), respectively. Output saturation function $s_i(x_i)$ in (23) has also been shown in (52) in Kanamori (2013b). Input limits ±2 V are given for the shoulder joint input and ±1 V are given for the elbow joint input. The following values are used for the simulations.

$$f_d = 10\,\text{N}, \; \xi(\|\dot{\boldsymbol{x}}\|) = 0.3\,\text{Nm/s}, \; \gamma_M = 6.38 \, , \qquad (77)$$

$$\alpha = 3.0 \, , \; a = 10.0 \, , \; b = 2.0 \, , \; c = 5.0 \, , \; \beta = 1.0 \, . \qquad (78)$$

Fig. 2 shows the control performance using the normal PID controller. For the strict input limits, the windup phenomenon is observed at elbow joint input u_2, as shown in the upper plots in Fig. 2. As a result, considerable overshoots occur in shoulder joint angle q_1 and elbow joint angle q_2, as shown in the lower plots in Fig. 2. As shown in Fig. 3, the contact force also degrades for the input saturation. In contrast, the windup phenomenon is restrained by using the proposed anti-windup PID controller, as shown in the upper plots in Fig. 4. As a result, considerable overshoots are restrained very well,

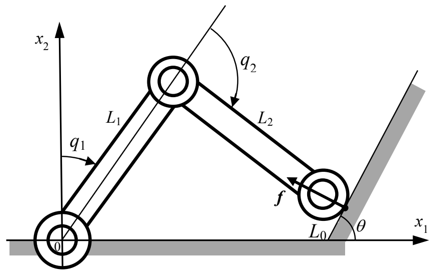

Fig. 1 Two-link robot arm with holonomic constraint

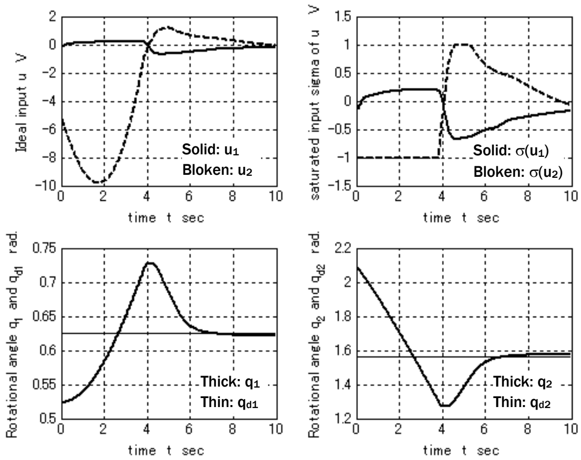

Fig. 2 Control performance using normal PID controller

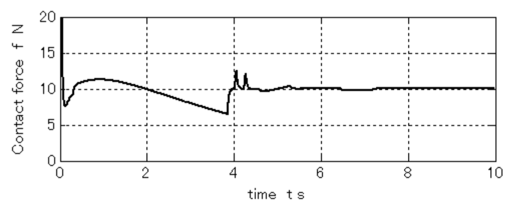

Fig. 3 Contact force using normal PID controller

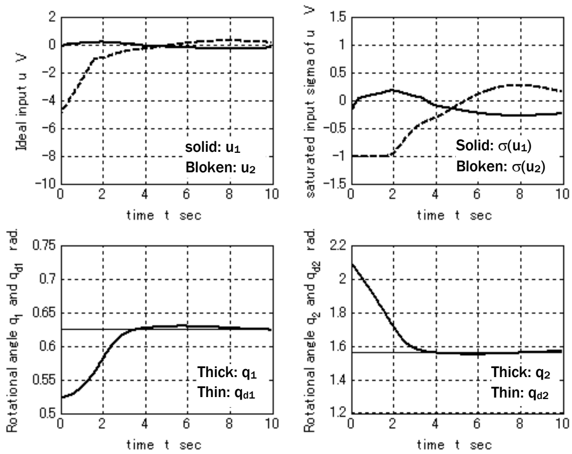

Fig. 4 Control performance using the proposed controller

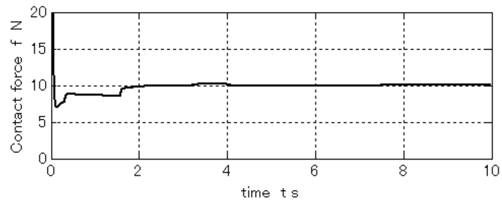

Fig. 5 Contact force using the proposed PID controller

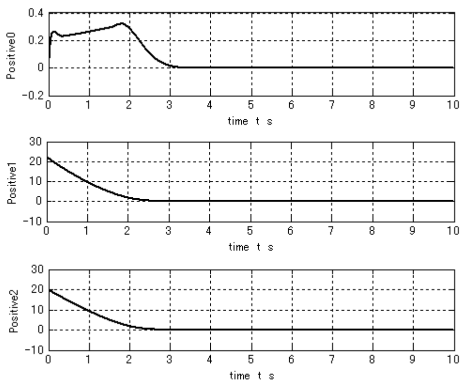

Fig. 6 Confirmation plot of conditions (32), (47) and (48)

as shown in the lower plots in Fig. 4. As shown in Fig. 5, the control performance of the contact force is improved in comparison with that in Fig. 3. Fig. 6 shows the validity of conditions (32), (47), and (48). The top plot shows the left-hand side of (32), where the right-hand side is transposed to the left-hand side. The middle and the lower plots correspond to the left-hand sides of (47) and (48), respectively. Since these values are positive or zero, it is confirmed that conditions (32), (47), and (48) are satisfied. These simulation results validate the analysis and design of the proposed anti-windup PID controller.

5. CONCLUSIONS

An anti-windup PID controller design for Euler-Lagrange systems under holonomic endpoint constraints has been presented. The control performance was examined by numerical simulations using a two-link robot arm with contact force. The simulation results validate the analysis and design of the proposed anti-windup PID controller.

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