Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Stable Active Vibration Control System for Building Structures using PD/PID Control

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Abstract: Proportional-Derivative (PD) and Proportional-Integral-Derivative (PID) controllers are the most popular algorithms in industrial applications. Although most of real building structure controllers are in the form of PD/PID, there are few published theory results of PD/PID on the structural vibration control. In this paper, by analyzing the stability of the controllers we give explicit conditions to choose the gains of PD/PID controller. Experimental studies on a two-story building prototype with the controllers are addressed. The experimental results give validation of our theory analysis.

Keywords: active control, building structures, PID control, vibration control

1. INTRODUCTION

Protection of large civil structures and human occupants from natural hazards like earthquake and wind is very important and challenging. In order to protect the buildings, a vibration control system could be added to the building structure. Structural vibration can be generally controlled by: 1) using smart materials in the buildings, see Housner, *et.al.* (1997); or 2) by adding controlling devices like dampers or actuators to the building, see Datta (2003). Since the force exerted by the earthquake and wind on the structures are very huge and uncertain, these large civil structures require a huge amount of energy to control it. The structural control can be classified as passive control which does not require an external power source, and active control which uses sensors and active actuators to control the unwanted vibrations, see Spencer and Sain (1997). There are many active control devices designed for structural control applications. The active mass damper (AMD) is the most popular actuator, which uses a mass without spring and dashpot, Chang and Soong (1980). In this paper, we use AMD for the active vibration control.

The objective of structural control is to reduce the vibration of the building due to earthquake or large winds through an external control force. In active control system it is essential to design an effective control strategy, which is simple, robust, and fault tolerant. Many attempts have been made to introduce advanced controllers for the active vibration control of building structures. Instead of changing the structure stiffness, a pole-placement H_∞ control corresponding to a target damping ratio is proposed in Park, *et.al.* (2008). In order to avoid the higher order problem in H_∞ control, the balanced truncation is applied in Saragih (2010). In Du and Zhang (2008), the genetic algorithm is used to determine the feedback control. There are several optimal control algorithms applied for the active vibration control of building structures, for example linear quadratic regulator (LQR), Alavinasab and Moharrami (2006). All the above controllers are model-based, which are complex and demand the exact model of the building structure. Some model-free controllers, such as sliding mode control, Yang, *et.al.* (1997), neural network control, Kim, *et.al.* (2000), and fuzzy logic control, Shook, *et.al.* (2008), are still complex.

PID control is widely used in industrial applications. Without model knowledge, PID control may be the best controller in real-time applications. The great advantages of PID control over the others are that they are simple and have clear physical meanings. Although theory research in PID control algorithms is well established, it is still not well developed in structural vibration control. In Nerves and Krishnan (1995), a simple proportional (P) control is applied to reduce the building displacement due to the wind excitation. In Guclu and Yazici (2008), PD and PID controllers were used in the numerical simulations. However, the control results are not satisfactory, because it is difficult to tune the PID gains to guarantee good performances such as the rise-time, overshoot, settling time, and steady-state error. Moreover, these works do not discuss the stability analysis of these active control systems.

In this paper, we use standard industrial PD and PID controllers for the active vibration control. The main contribution of the present work is that we give theory analysis of these PD/PID controllers. Both the linear and nonlinear cases for structural stiffness were considered in the analysis. Bouc-Wen model is used here to model the nonlinear hysteresis phenomenon. The sufficient conditions for asymptotic stability are derived using Lyapunov stability theorem, which are simple and explicit. Thus, the designer can choose the controller gains directly from these conditions. An active vibration control system for a two-story building structure equipped with an AMD is constructed for the experimental study. The experimental results obtained using the PD and PID controllers were compared and the effectiveness of our theory results has been demonstrated.

2. MODELING AND CONTROL OF BUILDING STRUCTURES

In order to control a structure effectively, it is important to have the knowledge about its dynamics. A mathematical model of the structure determines whether a controller is able to produce the desired dynamics in the building structure within a stable region.

A simple building structure can be modeled by Chopra (2001),

$$m\ddot{x} + c\dot{x} + kx = f_e \tag{1}$$

where m is the mass component, c is the damping component, k is the stiffness component, f_e is an external force applied to the structure, and x, $\dot{x}$, and $\ddot{x}$ are the displacement, velocity and acceleration, respectively.

Consider a linear multi-story structure with n-degree-of-freedom (n-DOF), where it is assumed that the mass of the structure is concentrated at each floor. Neglecting gravity force and assuming that a horizontal force is acting on the structure base, the equation of motion of the n-floor structure can be expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + F_s = -F_e \tag{2}$$

where

$$M = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{bmatrix} \in \Re^{n\times n},$$

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & \cdots & 0 & 0 \\ -c_2 & c_2 + c_3 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \cdots & c_{n-1} + c_n & -c_n \\ 0 & 0 & \cdots & -c_n & c_n \end{bmatrix} \in \Re^{n\times n},$$

$F_s \in \Re^n$ is the structure stiffness force, and $F_e \in \Re^n$ is the external force vector acting on the structure, such as earthquake and wind. If the relationship between the lateral force F_s and the resulting deformation x is linear, then F_s is

$$F_s = Kx(t) \tag{3}$$

where $K = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \cdots & k_{n-1} + k_n & -k_n \\ 0 & 0 & \cdots & -k_n & k_n \end{bmatrix}.$

If the relationship between the lateral force F_s and the resulting deformation x is nonlinear, then the stiffness component is said to be inelastic. This happens when the structure is excited by a very strong force, that deforms the structure beyond its limit of linear elastic behavior.

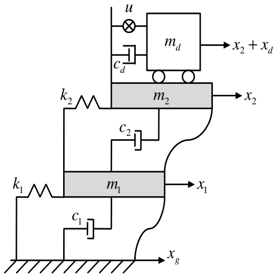

Fig. 1. Building structure equipped with AMD.

Then the stiffness force F_s in (2) can be described using Bouc-Wen model, see Wen (1976)

$$F_s(x, \dot{x}) = \tilde{\alpha}kx + (1 - \tilde{\alpha})k\eta f_r \tag{4}$$

where $\tilde{\alpha}, k$ and η are positive numbers and f_r is the nonlinear restoring force which satisfies

$$\dot{f}_r = \eta^{-1}\left[\delta\dot{x} - \beta|\dot{x}||f_r|^{p-1}f_r + \gamma\dot{x}|f_r|^p\right] \tag{5}$$

where δ, β, and γ are positive numbers and p is an odd number.

In order to attenuate the vibrations caused by the external force, an AMD is installed on the structure, see Fig. 1. The closed-loop system with the control force u is defined as

$$M\ddot{x}(t) + C\dot{x}(t) + F_s + F_e = \Gamma(u - d) \tag{6}$$

where $u \in \Re^n$ is the control signal applied to the dampers, $d \in \Re^n$ is the damping and friction force of the dampers, and $\Gamma \in \Re^{n\times n}$ is the location matrix of the dampers, defined as follows.

$$\Gamma_{i,j} = \begin{cases} 1 \text{ if } i = j = s \\ 0 \quad \text{otherwise} \end{cases}, \forall i, j \in \{1, ..., n\}, s \subseteq \{1, ..., n\}$$

where s are the floors on which the dampers are installed. In the case of a two-story building, if the damper is placed on second floor, $s = \{2\}$, $\Gamma_{2,2} = 1$. If the damper is placed on both first and second floor, then $s = \{1, 2\}$, $\Gamma_{2\times 2} = I_2$.

The force exerted by the q-th damper on the structure is

$$F_{dq} = m_{dq}(\ddot{x}_s + \ddot{x}_{dq}) = u_q - d_q \tag{7}$$

where m_{dq} is the mass of the q-th damper, $\ddot{x}_s$ is the acceleration of s-th floor on which the damper is installed, $\ddot{x}_{dq}$ is the acceleration of q-th damper, u_q is the control signal to the q-th damper, and

$$d_q = c_{dq}\dot{x}_{dq} + \epsilon_q m_{dq} g \tanh\left[\beta_h \dot{x}_{dq}\right] \tag{8}$$

where c_{dq} and $\dot{x}_{dq}$ are the damping coefficient and velocity of the q-th damper respectively and the second term is the Columb friction represented using a hyperbolic tangent dependent on a large positive constant β_h where ϵ_q is the friction coefficient between the q-th damper and the floor on which it is attached and g is the gravity constant.

Obviously, the building structures are stable when there is no external force, $F_e = 0$. In this case, the active control is not needed, hence $u = 0$. The ideal active control is $\Gamma u = F_e$. However, it is impossible because F_e is not always measurable and F_e is much bigger than any AMD force. Hence, the objective of the active control is to maintain the vibration as small as possible by minimizing the relative movement between the structural floors. In the next section, we will discuss the simple PD and PID controllers and their stability analysis.

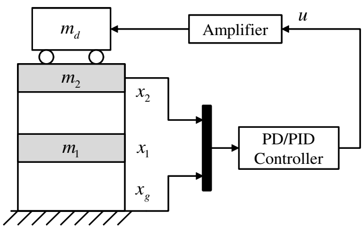

Fig. 2. PD/PID control for a two-story building.

2.1 PD control

PD control may be the simplest controller for the structural system, see Fig. 2. It provides high robustness with respect to uncertainties. PD control has the following form

$$u = -K_p(x - x^d) - K_d(\dot{x} - \dot{x}^d) \quad (9)$$

where K_p and K_d are positive-definite constant matrices, which correspond to the proportional and derivative gains, respectively and x^d is the desired position. In active vibration control of building structures, the references are $x^d = \dot{x}^d = 0$, hence (9) becomes

$$u = -K_p x - K_d \dot{x} \quad (10)$$

The aim of the controller design is to choose suitable gains K_p and K_d in (10), such that the closed-loop system is stable. Without loss of generality, we use a two-story building structure as in Fig. 2 to demonstrate the designing of a stable PD controller.

When the building structure (6) is completely known, i.e. there are no uncertainties and F_s is linear as in (3) then the building structure is a linear determined system. Many papers have used this model for the structural vibration control design, however, they did not discuss the stability problem.

Assuming $d = 0$, the closed-loop system with the PD control (10) is

$$M\ddot{x} + C\dot{x} + Kx + F_e = -\Gamma(K_p x + K_d \dot{x}) \quad (11)$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} > 0$, $C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} > 0$, $K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} > 0$, $F_e = \begin{bmatrix} m_1 \ddot{x}_g \\ m_2 \ddot{x}_g \end{bmatrix}$, $K_p = \begin{bmatrix} k_{p1} & 0 \\ 0 & k_{p2} \end{bmatrix} > 0$, and $K_d = \begin{bmatrix} k_{d1} & 0 \\ 0 & k_{d2} \end{bmatrix} > 0$. The damper is installed on the second floor, then $\Gamma = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Now (11) can be written in the state-space form

$$\dot{z} = A_{cl} z + F_{cl} \quad (12)$$

where $A_{cl} = \begin{bmatrix} 0_{2\times 2} & I_2 \\ -M^{-1}(K + \Gamma K_p) & -M^{-1}(C + \Gamma K_d) \end{bmatrix} \in \Re^{4\times 4}$, $z = \left[x^T, \dot{x}^T\right]^T \in \Re^4$, and $F_{cl} = \begin{bmatrix} 0_{1\times 2} & -F_e^T \end{bmatrix}^T \in \Re^4$.

The stability of the closed-loop system only depends on A_{cl}. Its characteristic polynomial is

$$\det(sI - A_{cl}) = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 \quad (13)$$

where

$$\begin{aligned}
a_1 &= \frac{1}{m_1}(c_1 + c_2) + \frac{1}{m_2}(c_2 + k_{d2}) \\
a_2 &= \frac{1}{m_1 m_2}\begin{pmatrix} c_1 k_{d2} + c_2 k_{d2} + m_1 k_{p2} \\ +c_1 c_2 + k_1 m_2 + k_2 m_1 + k_2 m_2 \end{pmatrix} \\
a_3 &= \frac{1}{m_1 m_2}(k_1 k_{d2} + k_2 k_{d2} + c_1 k_{p2} + c_2 k_{p2} + c_1 k_2 + c_2 k_1) \\
a_4 &= \frac{1}{m_1 m_2}(k_1 k_{p2} + k_2 k_{p2} + k_1 k_2)
\end{aligned}$$

Using Lienard-Chipart criterion, Poznyak (2009), the closed-loop system A_{cl} is stable if and only if

$$a_i > 0, i = 1, 2, 3, 4 \text{ and } a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 > 0 \quad (14)$$

Since a_i are functions of K_p and K_d, these gains can be searched with the five inequality in (14). Or we can first select the PD gains and substitute them into (14) to check if they are satisfied. For example, we consider a linear two-story building with the following set of parameters: the matrix M is $m_1 = 3.3\,\text{kg}$ and $m_2 = 6.1\,\text{kg}$, C is given by $c_1 = 2.5\,\text{N s/m}$ and $c_2 = 1.4\,\text{N s/m}$, and K is given by $k_1 = 4080\,\text{N/m}$ and $k_2 = 4260\,\text{N/m}$. An AMD of mass with $0.45\,\text{kg}$ is installed on the second floor as shown in Fig. 2. If the PD control (10) gains are, $k_{p2} = 350$ and $k_{d2} = 45$, then they satisfy the conditions given in (14), hence a stable closed-loop system.

In practice, the parameters of the building structure are partly known and the structure model might have nonlinearities such as the hysteresis phenomenon. It is convenient to express (6) as

$$M\ddot{x} + C\dot{x} + F = \Gamma u \quad (15)$$

where

$$F = F_s(x, \dot{x}) + F_e + \Gamma d \quad (16)$$

The building structure with the PD control (10) can be written as

$$M\ddot{x} + C\dot{x} + F = -\Gamma(K_p x + K_d \dot{x}) \quad (17)$$

Since (17) is a nonlinear system and M, C, and F are unknown, Routh-Hurwitz stability criterion (13) cannot be applied here. The following theorem, based on Lyapunov stability theorem, gives the stability analysis of the PD control (10). In order to simplify the proof, we first consider $\Gamma_{n\times n} = I_n$, *i.e.*, each floor has an actuator.

Theorem 1. Consider the structural system (15) controlled by the PD controller (10), the closed-loop system (17) is stable, provided that the control gains satisfy

$$K_p > 0, \quad K_d > 0 \quad (18)$$

The derivative of the regulation error x converges to the residual set

$$D_{\dot{x}} = \left\{\dot{x} \mid \|\dot{x}\|_Q^2 \le \bar{\mu}_F\right\} \quad (19)$$

where $\bar{\mu}_F \ge F^T \Lambda_F^{-1} F$ and $C > \Lambda_F > 0$.

It is well known that the regulation error becomes smaller while increasing the gain K_d. The cost of large K_d is that the transient performance becomes slow. Only when $K_d \to \infty$, the regulation error converges to zero. However, it would seem better to use a smaller K_d, if the system contains high-frequency noise signals.

2.2 PID control

From the analysis conducted above, we know that the gain K_d needs to be increased for reducing the regulation error, but results in a slow system response. In the control viewpoint, the regulation error can be removed by introducing an integral component to the PD control, i.e., change the PD control into PID control. The PID control law can be expressed as

$$u = -K_p(x - x^d) - K_i \int_0^t (x - x^d)d\tau - K_d(\dot{x} - \dot{x}^d) \quad (20)$$

where $K_i > 0$ corresponds to the integration gain. Since the vibration control of building structures is a regulation process, $x^d = \dot{x}^d = 0$, then (20) becomes

$$u = -K_p x - K_i \int_0^t x d\tau - K_d \dot{x} \quad (21)$$

In order to analyze the stability of the PID controller, (21) is expressed by

$$\begin{aligned} u &= -K_p x - K_d \dot{x} - \xi \\ \dot{\xi} &= K_i x, \quad \xi(0) = 0 \end{aligned} \quad (22)$$

Now substituting (22) in (15), the closed-loop system can be written as

$$M\ddot{x} + C\dot{x} + F = -K_p x - K_d \dot{x} - \xi \quad (23)$$

In matrix form, the closed-loop system is

$$\frac{d}{dt}\begin{bmatrix} \xi \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} K_i x \\ \dot{x} \\ -M^{-1}\left(C\dot{x} + F + K_p x + K_d \dot{x} + \xi\right) \end{bmatrix} \quad (24)$$

The equilibrium of (24) is $[\xi, x, \dot{x}] = [\xi^*, 0, 0]$. Since at equilibrium point x, $\dot{x} = 0$, the equilibrium is $[F(0,0), 0, 0]$. In order to move the equilibrium to origin, we define

$$\tilde{\xi} = \xi - F(0,0) \quad (25)$$

The final closed-loop equation becomes

$$\begin{aligned} M\ddot{x} + C\dot{x} + F &= -K_p x - K_d \dot{x} - \tilde{\xi} - F(0,0) \\ \dot{\tilde{\xi}} &= K_i x \end{aligned} \quad (26)$$

In order to analyze the stability of (26), we first give the following two properties.

P1. The positive definite matrix M satisfies the following condition

$$0 < \lambda_m(M) \leq \|M\| \leq \lambda_M(M) \leq \bar{m}, \ \bar{m} > 0 \quad (27)$$

where $\lambda_m(M)$ and $\lambda_M(M)$ are the minimum and maximum Eigen values of the matrix M, respectively.

P2. The F is Lipschitz over $\tilde{x}$ and $\tilde{y}$

$$\|F(\tilde{x}) - F(\tilde{y})\| \leq k_F \|\tilde{x} - \tilde{y}\| \quad (28)$$

Most of uncertainties are first-order continuous functions. Since $F_s(x, \dot{x})$, F_e, and d are first-order continuous and satisfy Lipschitz condition, **P2** can be established using (16). Now we calculate the lower bound of $\int F\,dx$.

$$\int_0^t F\,dx = \int_0^t F_s(x,\dot{x})dx + \int_0^t F_e\,dx + \int_0^t d\,dx \quad (29)$$

We define the lower bound of $\int_0^t F_s(x,\dot{x})dx$ is $-\bar{F}_s$ and for $\int_0^t d\,dx$ is $-\bar{d}$. Compared with $F_s(x,\dot{x})$ and d, F_e is much bigger in the case of earthquake. We define the lower bound of $\int_0^t F_e\,dx$ is $-\bar{F}_e$. Finally the lower bound of $\int_0^t F\,dx$ is

$$k_F = -\bar{F}_s - \bar{F}_e - \bar{d} \quad (30)$$

The following theorem, based on Lyapunov stability theorem, gives the stability analysis of the PID controller (22).

Theorem 2. Consider the structural system (15) controlled by the PID controller (22), the closed-loop system (26) is asymptotically stable at the equilibrium $\left[x^T, \dot{x}^T, \tilde{\xi}^T\right]^T = 0$, provided that the control gains satisfy

$$\begin{aligned} \lambda_m(K_p) &\geq \frac{3}{2}\left[k_F + k_c\right] \\ \lambda_M(K_i) &\leq \phi \frac{\lambda_m(K_p)}{\lambda_M(M)} \\ \lambda_m(K_d) &\geq \phi\left[1 + \frac{k_c}{\lambda_M(M)}\right] - \lambda_m(C) \end{aligned} \quad (31)$$

where $\lambda_m(A)$ and $\lambda_M(A)$ are the minimal and maximal eigenvalues of A, respectively, $k_c \geq \|C\|$, and $\phi = \sqrt{\frac{\lambda_m(M)\lambda_m(K_p)}{3}}$.

It is well known that without the uncertainty and external force $F = 0$, the PD control (10) with any positive gains can guarantee an asymptotically stable closed closed-loop system. The main objective of the integral action can be regarded to cancel F. In order to decrease integral gain, an estimated F is applied to the PID control (22). The PID control with an approximate force compensation $\hat{F}$ is

$$u = -K_p x - K_d \dot{x} - \xi + \hat{F}, \quad \dot{\xi} = K_i x \quad (32)$$

The above theorem is also correct for the PID control with an approximate F compensation (32). The condition for PID gains (31) becomes $\lambda_m(K_p) \geq \frac{3}{2}\left[\tilde{k}_F + k_c\right]$ and $\lambda_M(K_i) \leq \frac{3\phi}{2}\frac{\tilde{k}_F + k_c}{\lambda_M(M)}$, where $\tilde{k}_F \ll k_F$.

If the number of dampers installed on the buildings is less than the number of the building floors (n), then the resulting system is termed as underactuated system. In this case, the location matrix Γ should be included along with the gain matrices. In our experiment, there is only one damper installed on the second floor of the structure. Substituting the corresponding Γ in the PID controller results

$$\Gamma u = \begin{bmatrix} 0 \\ -k_{p2}x_2 - k_{i2}\int_0^t x_2 d\tau - k_{d2}\dot{x}_2 \end{bmatrix} \quad (33)$$

where the scalars k_{p2}, k_{i2}, and k_{d2} are the position, integral, and derivative gains, respectively. In this case (31) becomes,

$$\begin{aligned} k_{p2} &\geq \frac{3}{2}\left[k_F + k_c\right] \\ k_{i2} &\leq \tilde{\phi}\frac{\min\{k_{p2}\}}{\lambda_M(M)} \\ k_{d2} &\geq \tilde{\phi}\left[1 + \frac{k_c}{\lambda_M(M)}\right] - \lambda_m(C) \end{aligned} \quad (34)$$

where $\tilde{\phi} = \sqrt{\frac{\lambda_m(M)\min\{k_{p2}\}}{3}}$.

3. EXPERIMENTAL RESULTS

To illustrate the theory analysis results, a two-story building prototype is constructed which is mounted on a shaking table, see Fig. 3. The building structure is constructed of aluminum. The shaking table is actuated using a hydraulic control system (FEEDBACK EHS 160), which is used to generate earthquake signals. A linear magnetic encoder (LM15) position sensor is used to measure the

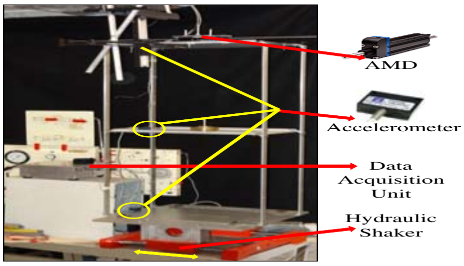

Fig. 3. Two-story building prototype with the shaking table.

floor displacement. The AMD is a linear servo actuator (STB1108, Copley Controls Corp.), which is mounted on the second floor. The moving mass of the damper weights 5% (0.45 kg) of the total building mass. The linear servo mechanism is driven by a digital servo drive (Accelnet Micro Panel, Copley Controls Corp). ServoToGo II I/O board is used for the data acquisition purpose.

The control programs were operated in Windows XP with Matlab 6.5/Simulink. All the control actions were employed at a sampling frequency of 1.0 kHz. The control signal generated by the control algorithm is fed as voltage input to the amplifier. The current control loop is used to control the AMD operation. The amplifier converts its voltage input to a respective current output with a gain of 0.5. The AMD have a force constant of 6.26 N/ A or 3.13 N/ V.

The theorems in this paper give sufficient conditions for the minimal values of the proportional and derivative gains and maximal values of the integral gains. In order to do a fair comparison both the PD and PID controller uses the same proportional and derivative gains. We first design the PID controller based on the identified parameters of the two-story lab prototype. The following set of parameters were used for the control design: $\lambda_M(M) = 6.1$, $\lambda_m(C) = 0.6$, $k_F = 365$, and $k_c = 5.8$. Applying these values in Theorem 2 we get

$$\lambda_m(K_p) \geq 556, \quad \lambda_M(K_i) \leq 3066, \quad \lambda_m(K_d) \geq 65 \quad (35)$$

In order to evaluate the performance, these controllers were implemented to control the vibration on the excited lab prototype. The control performance is evaluated in terms of their ability to reduce the relative displacement of each floor of the building. The proportional, derivative, and integral gains were further adjusted to obtain a higher attenuation. Finally, the PID controller gains are chosen to be

$$k_p = 635, \quad k_i = 3000, \quad k_d = 65$$

Since the above proportional-derivative gains satisfy Theorem 1, we choose same gains. Hence the PD controller gains are

$$k_p = 635, \quad k_d = 65$$

The behaviors of the second floor under PD and PID control are shown in Fig. 4 and Fig. 5. The control action of PID is shown in Fig. 6. It is clear that both PD and PID controllers were able to reduce the displacement due to the external disturbance into a smaller value. The controlled response using PD controller has been reduced significantly by applying a damping using the derivative gain. They also show that the vibration attenuation achieved by adding an integral action to the above PD controller. The results demonstrate that the PID controller performs better than the PD controller.

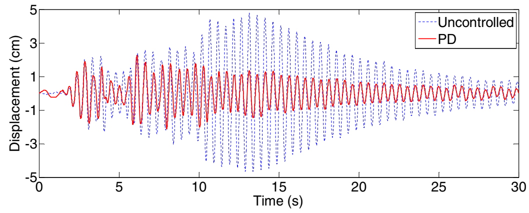

Fig. 4. The displacements of the second floor using PD control.

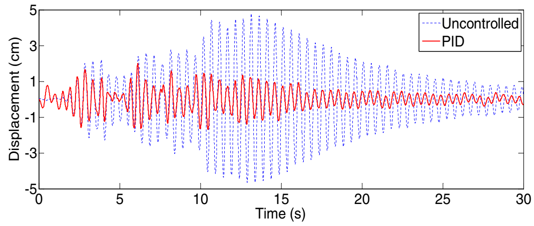

Fig. 5. The displacements of the second floor using PID control.

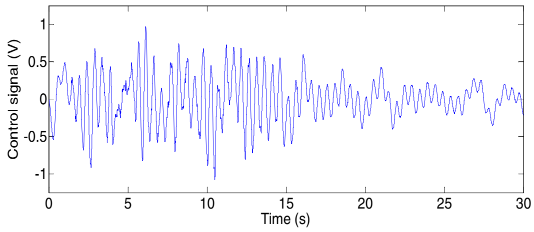

Fig. 6. Control signal of PID control.

Table-1 shows the mean squared error $MSE = \frac{1}{N}\sum_{i=1}^{N} e_i^2$ of the displacement with proposed controllers, here N is the number of data samples and the regulation error, $e = (x^d - x) = -x$, where x is the position achieved using the controllers.

Table-1 Comparison of regulation error

Control Action	PD	PID	No Control
Floor-1 Displacement	0.1699	0.1281	1.0688
Floor-2 Displacement	0.5141	0.3386	3.3051

Remark 1. It is worth to note the frequency characteristics of an integrator. An ideal integrator acts like a low-pass filter. The bode magnitude plot of an ideal integrator is shown in Fig. 7. At 1.6 Hz the integrator attenuates the input power by 20 dB and at 16 Hz it reaches to 40 dB. During earthquake the structure oscillates at its natural frequencies. If the natural frequency is very small then the integrator produces larger output. The structure

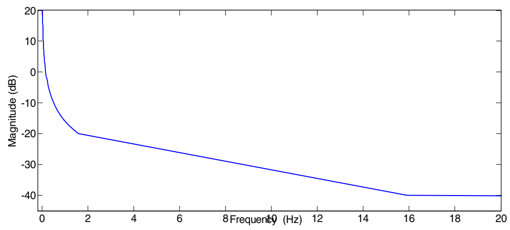

Fig. 7. Bode magnitude plot of an ideal integrator.

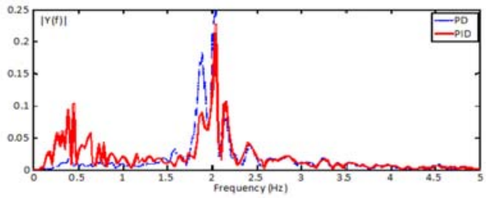

Fig. 8. Fourier spectrums of PD and PID control signals.

prototype we used for the experiments have natural frequencies 2.1 Hz and 8.9 Hz. Since these frequencies has an attenuation more than 20 dB a larger value can be used for K_i. On the other hand if the building have a natural frequency less than 1.6 Hz, then the integral gain should be reduced accordingly. Here the error input to the integrator is the position data. From Fig. 4 to Fig. 6 we can see that the position data for the most part takes successive positive and negative values. Hence, the integrator output for high frequency input signal is small due to the rapid cancellation between these positive and negative values.

Sometimes the integral control result in an actuator saturation. But as discussed in *Remark 1* the output of the integrator is small in our case. From Fig. 8 we can see that the magnitude of PID control signal is less than PD, even though the K_i gain is large.

4. CONCLUSION

In this paper, the model of building structures with an active vibration control has been analyzed. The theoretical contribution of this paper is that the stability of the AMD PD/PID control for building structures is proven. By using Lyapunov theory, sufficient conditions of stability were derived to tune the PD/PID gains. The technical advance of this paper is that a systematic tuning method of PID has been proposed based on the stability analysis. The above new approaches have been successfully applied to a two-story building prototype. The results show that even though the chosen gains are not optimal, the controllers based on Theorems 1 and 2 guarantee a stable control performance.

Acknowledgments

The authors would like to thank Mr. Jesús Meza and Mr. Gerardo Castro for their assistance to complete the experiments. The first author would like to thank Consejo Nacional de Ciencia y Tecnología (CONACyT) of Mexico for the financial support.

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