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A Magnetic Levitation System for Advanced Control Education

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Abstract: This paper describes a magnetic levitation system for use in graduate controls education. We explain how to use this system to show the nonlinear system modeling, and how to used advanced control techniques for this interesting and visually impressive equipment. Several open problems in areas of electrical and control engineering are offered. Also, the paper presents some initial outcomes in creating a laboratory environment for remote monitoring of the magnetic levitation equipment.

Keywords: magnetic levitation system, control education, modelling, PID control, intelligent control

1. INTRODUCTION

Magnetic levitation techniques apply a magnetic field to levitate or suspend a magnetic object based on the interaction between the magnetic object and the applied magnetic field. A magnetic object can be levitated and stabilized in a magnetic system with an electronic feedback control to dynamically adjust one or more electromagnets in the system to stabilize the magnetically levitated object at a desired location. A servo control is provided to control the magnetic field that levitates the magnetic platform to stabilize the levitated magnetic platform, Wong (1986).

The portable magnetic levitation system (MagLev) is a classroom demonstration device. It is helpful in teaching engineering courses particularly for automatic control. MagLev system is so simple and small that is very convenient to be carried from class to class, Hurley (1997).

The mathematical model is very important for control education. The advanced controller usually needs a mathematical model of the real system. Once a good model is obtained and verified, a suitable control law can be implemented to compensate the plant instability and improve performance. The objects of the control education usually includes, Barie and Chiasson (1996)

- System modeling
- Controller design

Due to its nonlinear and unstable nature, the Maglev system is a very challenging plant. Normal feedback control, such as PID control, can be used to control magnetic field that levitates the platform to stabilize the platform. This controller is very simple and does not require any knowledge of mechanical systems. However it gives very large actuation and cannot guarantee zero tracking error with the existence of disturbances, Kelly (1998). Various advanced control schemes have been developed in the literature for magnetic levitation systems, aee Qu and Dawson (1996), and Ortega and Spong (1989). While model-based nonlinear control can remove this error, it is usually restricted to the case that the model is exactly known. Adaptive control can compensate unknown dynamics if the structure is known, Slotine and Lin (1988). The robust version of adaptive control may achieve a good performance with the system uncertainties and external disturbances, Singh (1985). Non-adaptive control may also get high quality performance with the parameters or structure uncertainties. Robust feedback control, Dawson, et.al. (1993), and optimal control, Li and Brandt (1996), may guarantee closed-loop stable if the disturbances are bounded. All of these works assume exact or partial knowledge of the nonlinear dynamics. Obviously, this is a requirement that generally cannot be met in practice.

In this paper we modify the prototype of magnetic levitation system, InTeCo (2005), to test different modeling and control methods.

2. SYSTEM DESCRIPTION

The MagLev system consists of the electromagnet, the suspended hollow steel sphere, the sphere position sensors, computer interface board and drivers, a signal conditioning unit, real time control toolbox, see Fig. 1 and Fig.2.

This is a nonlinear, open-loop unstable and time varying dynamic system. The basic principle of MLS operation is to apply the voltage to an electromagnet to keep a ferromagnetic object levitated. The object position is determined through a sensor. Additionally the coil current is measured to explore identification and multi loop or nonlinear control strategies. To levitate the sphere a real-time controller is required. The equilibrium stage of two forces (the gravitational and electromagnetic) has to be maintained by this controller to keep the sphere in a desired distance from the magnet. When two electromagnets are used the lower one can be used for external excitation

Fig. 1. A magnetic levitation system -Real System

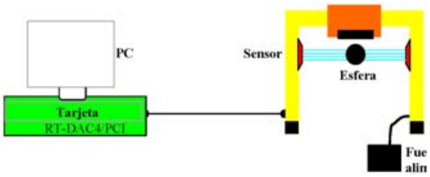

Fig. 2. A magnetic levitation system-Scheme.

or as contraction unit. The position of the sphere may be adjusted using the set-point control and the stability may be varied using different controllers.

Furthermore, changes of the parameters of the plant, such as change of mass and suspension of the variations of resistance and inductance due magnetic due to heat, must also be taken into account. The magnetic levitation system includes:

1) Measurement System: measuring the position of the object to levitate, and with this information feedback to the control stage. This system is built with light-receiving sensors

2) Control System: this system is responsible for regulating the position of the object based on information received by the measurement system and a previously set reference.

3) Magnetics System: It consists in providing the necessary magnetic force induced in the object, in order to counteract the force of gravity and maintain an equilibrium of forces. It is noteworthy that the force which is generated by this system varies depending on the position in which the sphere is suspended and motionless.

4) Signal System: This system is responsible for adapting and interpreting the signals provided by different systems that make this prototype, so they can be processed properly.

5) Power System: This is responsible for transforming the control signals of low voltage and low current signals useful for the actuator, in this particular case, are transformed into electric currents flowing through electromagnet .

6) Computer Control System: It is responsible for processing all the input-output information, which determines the state of the state in which the system is located. It is worth mentioning that this levitation system has a fast dynamics, then the response speed of this stage should be very high and a large bandwidth

3. MODELING

Mathematical mode of this magnetic levitation system includes three parts: mechanical model, electrical model and sensor model. The mechanical model uses Newton law

$$m\ddot{x} = mg - F_{em} \tag{1}$$

where F_{em} is the electromagnetic force, m and x are the mass and position of the ball.

The electrical model uses Kirchhoff law

$$v(t) = Ri(t) + L(x)\frac{di(t)}{dt} - c \tag{2}$$

where R is the resistance, $L(x)$ is the induction, $i(t)$ and $v(t)$ are current and voltage, c is a constant current. Here the induction $L(x)$ is a nonlinear function of the ball position x, Wong (1986),

$$L(x) = L_1 + \frac{L_0 x_0}{x} \tag{3}$$

where x_0 is the reference position, L_1 and $L_0\left(x\right)$ are system parameters.

The magnetic energy of the system is a function of the coil current i and the separation of the ball ferromagnetic x

$$W(i,x) = \frac{1}{2}L(x)i^2 \tag{4}$$

The strength of electromagnetic on the ferromagnetic ball is given by

$$f_{em} = -\frac{\partial W}{\partial x} = -\frac{i^2}{2}\left[\frac{dL(x)}{dx}\right] \tag{5}$$

Considering (3)

$$f_{em} = -\frac{1}{2}L_0 x_0 \left(\frac{i}{x}\right)^2 \tag{6}$$

The electromagnetic force F_{em} can be approximated by Hurley (1997)

$$f_{em} \approx -\frac{L_r}{2a} i^2 e^{-\frac{x}{a}} \tag{7}$$

where L_r and a are system parameters. Or by Moon (1994)

$$f_{em} \approx \frac{i}{a\left(x+b\right)^N} \tag{8}$$

where a, b and N are system parameters. Or by Hajjaji and Guladsine (2001)

$$f_{em} = \frac{x^2}{1.1994x^4 - 0.9165x^3 + 0.7159x + 0.0304} \tag{9}$$

So (2) becomes

$$v(t) = Ri + L\frac{di}{dt} - L_0 x_0 \frac{i}{x^2}\frac{dx}{dt} - c \tag{10}$$

If we define

$$C\left(x\right) = \frac{L_0\left(x\right)x_0}{2} \tag{11}$$

x_1 as position of the sphere, x_2 as velocity of the sphere, x_3 as current, $u\left(t\right)$ as control voltage. The final model is obtained from (1), (2) and (10)

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= g - \frac{C\left(x\right)}{m}\left(\frac{x_3}{x_1}\right)^2 \\ \dot{x_3} &= -\frac{R}{L(x)}x_3 + \frac{2C}{L(x)}\frac{x_2 x_3}{x_1^2} + \frac{u\left(t\right)}{L(x)} \end{aligned} \tag{12}$$

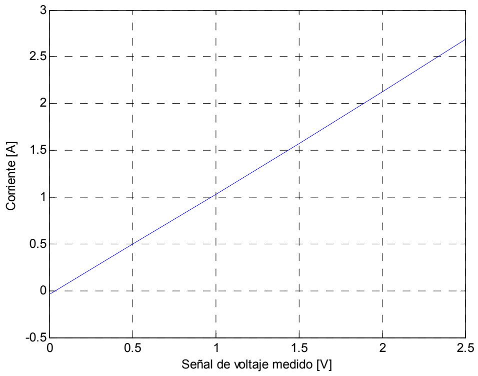

Fig. 3. PWM control u via current i

To determine the value of parameter C in (11), an approximation experiment is needed, see Cho, et.al. (1993). The ball is located on a magnetic shelf. It is not directly below the electromagnet. The exact position is recorded by the position sensor, then a smooth ramp voltage is applied to the magnetic circuit and the resulting position and the current consumed by the electromagnet is measured. Considering magnetic force equals the gravity force, $C(x)$ is calculated as

$$C(x) = \frac{mg}{i^2} x^2 \tag{13}$$

There are several simplified models for $\dot{x}_3$, for example in Hajjaji and Guladsine (2001)

$$\dot{x}_3 = \frac{u(t)}{L} - \frac{R}{L} x_3 \tag{14}$$

In arry and Casey (1999),

$$\dot{x}_3 = \frac{u(t)}{L} - \frac{R}{L} x_3 - M_z \frac{di_r}{dt} \tag{15}$$

In Taghirad, et.al. (2010)

$$\dot{x}_3 = \frac{1}{k} x_1 u(t) - \frac{R}{k} x_1 x_3 + \frac{x_2 x_3}{x_1} \tag{16}$$

he parameters of the above models are $m = 0.0571Kg$, $L_0 = 0.038514H$, $L_r = 0.017521H$, $R = 0.0243$, $a = 0.005831$, $k_i = 2.5165$, $g = 9.81$

3.1 Actuator and sensor

The pulse width modulation (PWM) of a signal or power source is a technique in which modifies the duty cycle of a periodic signal (a sinusoidal or a square, for example), either to transmit information or to control the amount of energy that is sent to a load. The pulse width modulation is a technique used to control devices by providing a DC voltage. The PWM response is nonlinear with respect to control voltage and the current. However, in working area the control current i can be approximated by the following polynomial

$$i = 0.0168u(t)^2 + 1.045u(t) - 0.0317 \tag{17}$$

It is almost linear in $[0, 2.5]$, see Fig.3.

After the sensor calibration, the relation between the sensor output (voltage) and the position of the levitated object is still nonlinear, see Fig.4. It can also be approximated by

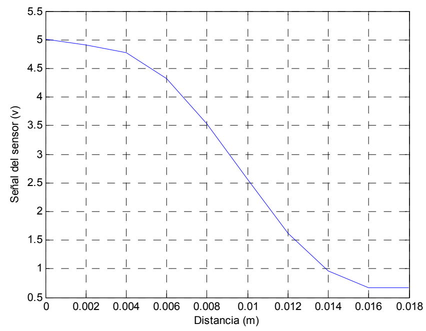

Fig. 4. Sensor output (voltage) and the position.

$$\begin{aligned} V_{position} &= -25697073504.59x^5 + 1245050011.25x^4 \\ &-18773635.92x^3 + 79330.24x^2 - 150.21x + 5.015 \end{aligned} \tag{18}$$

4. ADVANCED CONTROL

Besides the classical PID control, in this session we use several advanced control techniques for the magnetic levitation system. These controller design methods will be used for graduated students education. The PID controller is

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_D \frac{de(t)}{dt} \tag{19}$$

where

$$e(t) = x_1 - x_1^*$$

x_1^* is the desired position of the ball, x_1 is real position. K_p, K_i and K_d are proportional, integral and derivative gains of the PID controller, respectively.

4.1 Linear Control

The nonlinear model (12) can be approximated in the equilibrium point $(x_{10}, x_{20}, x_{30}) = (x_0, 0, i_0)$

$$\begin{aligned} \delta\dot{x} &= A\delta x + B\delta u \\ y &= C\delta x \end{aligned} \tag{20}$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2Cx_{30}^2}{mx_{10}^3} & 0 & -\frac{2Cx_{30}}{mx_{10}^2} \\ 0 & \frac{2Cx_{30}}{L(x)x_{10}^2} & -\frac{R}{L} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ L(x)^{-1} \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

Since In the equilibrium point $mx_1 = C\frac{x_{30}^2}{x_{10}^2}$, $u_0 = Ri_0$, $\delta x_1 = x - x_0$, $\delta x_2 = v$, $\delta x_3 = x_3 - i_0$, $\delta u = v - Ri_0$. The PID controller (19) becomes Barie and Chiasson (1996),

$$\begin{aligned} u &= K_0 \int_0^t (x_{1ref} - x_1)dt + K_1(x_{1ref} - x_1) \\ &+K_2(x_{2ref} - x_2) + K_3(x_{3ref} - x_3) + u_{ref} \end{aligned} \tag{21}$$

where

$$u_{ref} = Rx_{3ref} + \left(\frac{Lx_{30}}{x_{10}} - \frac{2mg}{x_{30}}\right) x_{2ref} - \left(\frac{Lx_{30}}{2g}\right) x_{1ref}$$

4.2 Feedback Linearization

Consider the following transform coordinator

$$\begin{aligned} z_1 &= x_1, \quad z_2 = x_2 \\ z_3 &= g - \left(\frac{C}{m}\right)\left(\frac{x_3}{x_1}\right)^2 \end{aligned} \tag{22}$$

The system states are restricted to region where the state space is $x_1 > 0$ and $x_3 > 0$ to ensure that the transformation (22) is invertible. After the transformation (22), (12) becomes

$$\begin{aligned} \dot{z_1} &= z_2 \\ \dot{z_2} &= z_3 \\ \dot{z_3} &= \alpha(x) + \beta(x)u \end{aligned} \tag{23}$$

where $\alpha(x) = \frac{2C}{m}\left(\left(1 - 2\frac{C}{L(x)x_1}\right)\frac{x_3^2 x_2}{x_1^3} + \frac{R}{L(x)}\frac{x_3^2}{x_1^2}\right)$, $\beta(x) = -\frac{2Cx_3}{L(x)mx_1^2}$.

If the feedback linearization control is

$$u = \left(-\frac{\alpha(x) + w}{\beta(x)}\right) \tag{24}$$

The closed-loop system is

$$\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \\ \dot{z_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}$$

The reference w is chosen as

$$\begin{aligned} w = K_0 &\int_0^t (z_{1ref} - z_1)dt + K_1(z_{1ref} - z_1) \\ &+ K_2(z_{2ref} - z_2) + K_3(z_{3ref} - z_3) + j_{ref} \end{aligned} \tag{25}$$

where $z_{1ref}(t) = x_{ref}(t)$, $z_{2ref}(t) = \frac{dz_{1ref}(t)}{dt}$, $z_{3ref}(t) = \frac{dz_{2ref(t)}}{dt}$, $j_{ref}(t) = \frac{dz_{3ref}(t)}{dt}$, $j_{ref}(t)$ is input reference. In this way, the closed-loop systems is stable.

4.3 Lyapunov Design

The Lyapunov design is to calculate analytically a global feedback asymptotically stabilizes. The final control u will be calculated in n steps, where n is the number of state variables for the proposed case.

1) The first step, calculate tracking error for x_1. Define the tracking error as

$$\epsilon_1 = x_1 - x_1^* \tag{26}$$

where x_1^* is the desired trajectory of x_1. The Lyapunov function is selected as

$$V_1(x) = \frac{1}{2}\epsilon_1^2 \tag{27}$$

The control object is force the Lyapunov function satisfy

$$\dot{V_1}(x_1) = -k_1\epsilon_1^2 < 0 \tag{28}$$

After calculation, we find if we define a virtual control ϕ_{x2} as

$$\phi_{x2} = \dot{x_{1ref}} - k_1(x_1 - x_{1ref}) \tag{29}$$

and replace x_2 with ϕ_{x2}, (28) is established.

2) In the second step, we define the tracking error of x_2 as

$$\epsilon_2 = x_2 - \phi_{x2} \tag{30}$$

The Lyapunov function is selected as

$$V_2(x_1, x_2) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2 \tag{31}$$

we hope

$$\dot{V_2}(x_1, x_2) = -k_1\epsilon_1^2 - k_2\epsilon_2^2 < 0 \tag{32}$$

After calculation, we find if we define a virtual control ϕ_{x3} as

$$\begin{aligned} \phi_{x3} = -&\frac{m_3(c - x_1)^2}{C}[(k_1 + k_2)(x_2 - \dot{x_{1ref}}) \\ &+ (1 + k_1k_2)(x_1 - x_{1ref}) - g - \ddot{w_{1ref}}] \end{aligned} \tag{33}$$

and replace x_3 with ϕ_{x3}, (32) is established.

3) In the third step, we define the tracking error of x_3 as

$$\epsilon_3 = x_3 - \phi_{x3} \tag{34}$$

The Lyapunov function is selected as

$$V_3(x_1, x_2, x_3) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2 + \frac{1}{2}\epsilon_3^2 \tag{35}$$

If the control is selected as

$$\begin{aligned} u &= R_2x_3 - 2C\frac{x_2x_3}{x_1^2} - \frac{L(x_1}{2}(k_1 + k_2 + k_3)x_3 \\ &+ \frac{L(x_1}{2x_3}\left[\frac{C}{x_2x_1^2}\left[x_2 - \dot{x_{1ref}} + k_1(x_1 - x_{1ref})\right]\right] \\ &+ \frac{m_2x_1}{C}\left[\begin{array}{c} (1 + k_1k_2)(x_1 - x1ref) + g \\ +(k_1 + k_2)(x_2 - \dot{x_{1ref}}) - \ddot{x}_{1ref} \end{array}\right](k_3x_1 + 2x_2) \\ &+ \frac{m_2x_1^2}{C}[(1 + k_1k_2)(x_2 - \dot{x}_{1ref}) \\ &+ (k_1 + k_2)(g - \ddot{x}_{1ref}) - x_{1ref}^{(3)}] \end{aligned}$$

then

$$\dot{V_3}(x_1, x_2, x_3) = -k_1\epsilon_1^2 - k_2\epsilon_2^2 - k_3|\epsilon_3| < 0 \tag{36}$$

4.4 Sliding mode controller

The tracking error is defined as

$$e_1 = x_1 - x_1^*$$

Let us define the sliding surface as $S = \left(\frac{d}{dt} + \lambda\right)^n e_1$,when $n = 2$

$$S = \left(\frac{d}{dt} + \lambda\right)^2 e_1 = \ddot{e_1} + \lambda_1\dot{e_1} + \lambda_2 e_1 \tag{37}$$

We select Lyapunov function as

$$V = \frac{1}{2}S^2 \tag{38}$$

We use (12) and (37)

$$S = g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1x_2 + \lambda_2(x_1 - x_{1d}) \tag{39}$$

We hop the derivative of 38 satisfies

$$\dot{V} = S\dot{S} < 0 \tag{40}$$

We should use the control as

$$\begin{aligned} u = \frac{1}{g_1(x)}&\left(-f_1(x) - \lambda_1\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right)\right) \\ &-\lambda_2x_2 - SM \end{aligned} \tag{41}$$

where SM=$W sign(S)$, $f_1(x) = \frac{R}{L(x)}x_3 + \frac{2C}{L(x)}\frac{x_2x_3}{x_1^2}$, $g_1(x) = L^{-1}(x)$. In order to reduce chattering, saturation function is used as $SM = W sat(S)$. When the sliding mode controller (41) is applied, the asymptotic stability is gauntness

$$\lim_{t\to\infty} e_1 = 0$$

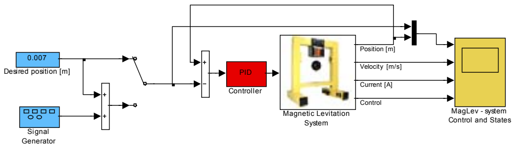

Fig. 5. Real-time simulink program for PID control.

4.5 Neural control

A neural network can approximate the controller (41). Here we use radial base function neural network

$$u_{nn} = \sum_{i=1}^{n} w_i exp\left(-\frac{\parallel S - c_1 \parallel^2}{z\rho_1}\right) = W^T \phi \qquad (42)$$

where n is the number of hidden layers. An adaptive law is used to adjust the weights for the search of optimal values of the weights and obtain a stable convergence property

$$W_{new} = W_{old} - \eta \frac{\partial S\dot{S}}{\partial W} \mid W = W_{old}$$

That is $W_{new} = W_{old} - \eta S \frac{\partial S}{\partial W} \mid W = W_{old}$. Because $\frac{\partial \dot{S}}{\partial W} = \frac{\partial \dot{S}}{\partial u} \times \frac{\partial u}{\partial W}$, and $\dot{S} = \dot{e} + \lambda_1 \ddot{e} + \lambda_2 \dot{e}$, we find the update law as follows

$$W_{new} = W_{old} - \eta S \frac{-2C}{Lmx_1^2} \times \frac{\partial u}{\partial W} \simeq W = W_{old} \qquad (43)$$

This gradient update law can assure u_{nn} go to u in (41).

5. OUR EXPERIMENTS

Here interfacing is based on a Xilinx FPGA microprocessor, comprising a multifunction analog and digital I/O board dedicated to real-time data acquisition and control in the Windows XP environment. The control program operated in Windows XP with Matlab 6.5/Simulink. All of the controllers employed a sampling frequency of $1kHz$. All experiments can be repeated as experiments in real time. In this way one can verify the accuracy of the model.

5.1 PID Control

Fig.5is a block diagram in Simulink. which is used for testing in real time of PID control. The hardware platform is shown in Fig.6

The PID gains are chosen as $K_p = 200$, $K_I = 10$, $K_D = 10$. The regulation results are shown in Fig.7 and Fig.8.

5.2 Nonlinear control

We use feedback linearization control

$$u = \frac{1}{b}\begin{bmatrix} -a_1 - a_2 - a_3 - a_4 + k_1(x_1 - referencia) + k_2 x_2 \\ +k_3(-26.3474 x_3^2 \exp(\frac{-x_1}{0.0058231}) + 9.81) \end{bmatrix} \qquad (44)$$

where $k_1 = -125$, $k_2 = -75$, $k_3 = -15$. The results are shown in Fig.9.

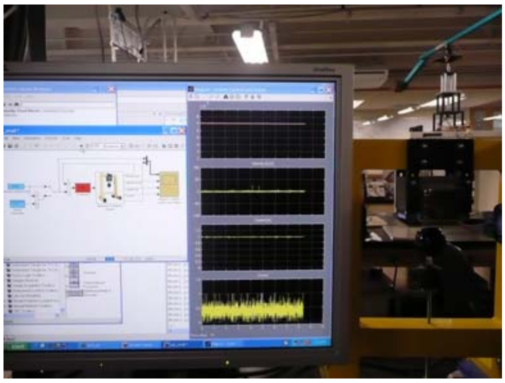

Fig. 6. Hardware platform.

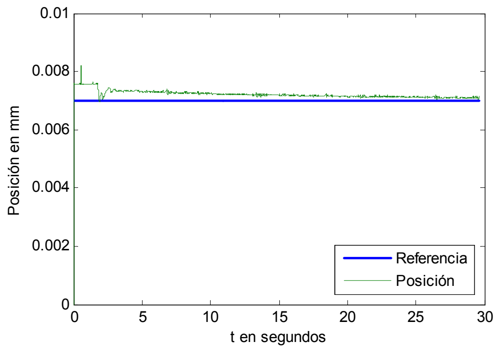

Fig. 7. PID regulation

Fig. 8. PID regulation

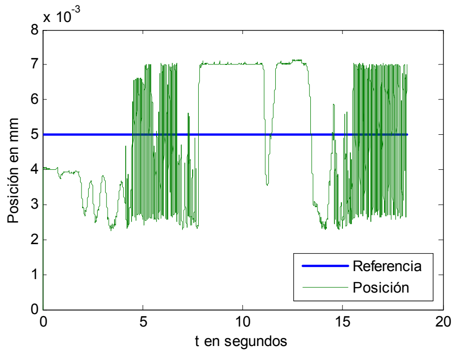

Fig. 9. Feedback linearization

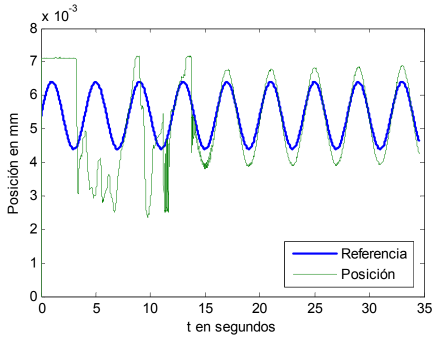

Fig. 10. Neural control-tracking

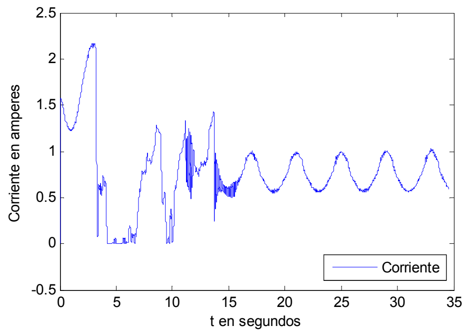

Fig. 11. Neural control- control input

5.3 Neural Control

The PID control and nonlinear feedback linearization cannot realize tracking. However, neural control can finish this job. The neural controller is

$$u_{NNf} = W_f S \tag{45}$$

where $S = \left(\frac{3}{1+\exp(-x_1)}\right)\left(\frac{1}{1+\exp(-x_2)}\right)\left(\frac{2}{1+\exp(-x_3)} - 0.1\right)$. The weight W_f is updated as

$$\dot{W}_f = \left(\frac{3}{1+\exp(-x_1)}\right)\left(\frac{1}{1+\exp(-x_2)}\right) \times \left(\frac{2}{1+\exp(-x_3)} - 0.1\right) e_1$$

The tracking results are shown in Fig.10. The blue line id the reference signal. The green line is the real position of the ball. Fig.11 shows the control input (current).

6. CONCLUSION

In this paper, we use Maglev system to show how to realize the modeling and control. The modeling process includes mechanical model, electrical model and sensor model. The advanced controller design includes: PID control, linear control, nonlinear feedback control, Lyapunov design, sliding mode control, and neural control.

As a teaching aid, the Maglev system enables the implementation of many basic and advanced approaches to both theoretical study and practical investigation of the nonlinear, unstable system control. We hope that it would be of wide interest in the control engineering community. Moreover, in order to support learning of automatic control, a web-based laboratory will be established. The remote user is able to control the magnetic levitation equipment in stand alone graphical user interface.

7. REFERENCES

N.Barry and R.Casey (1999), Elihu Thomson's Jumping Ring in a Levitated Closed-Loop Control Experiment, *IEEE Transactions on Education*, vol. 42, 72-79.

W.Barie and T.Chiasson (1996), Linear and nonlinear state-space controllers for magnetic levitation, *International Journal of System Science* vol.27 No.11, 1153-1163.

D.Cho, et.al. (1993), Sliding mode and classical control of magnetic levitation system, *IEEE Control System*, Vol.13, No.1, 42-48.

D.Dawson, et.al. (1993), Hybrid adaptive control for the tracking of rigid-link flexible-joint robots, *in Modelling and Control of Compliant and rigid Motion Systems*, Atlanta GA, pp. 95-98,

A.E.Hajjaji and M. Guladsine (2001), Modeling and non linear control of magnetic levitation system. *IEEE transaction on industrial electronic,* Vo.48 No.4, 831-838.

W.G. Hurley (1997), Electromagnetic design of a magnetic suspension system, *IEEE transaction on education,* Vo.40 No.2, 124-130.

InTeCo (2005), *Magnetic Levitation System*, Krakow, Poland5.

R.Kelly (1998), Global Positioning on Robot Manipulators via PD control plus a Classs of Nonlinear Integral Actions, *IEEE Trans. Automat. Contr.*, vol.43, No.7, 934-938.

F.Li and R.D.Brandt (1996), An Optimal Control Approach to Robust Control of Robot Manipulators, *IEEE Trans. on Robot. Automat.*, Vol.14, No.1, 69-77.

R.Ortega and M.W.Spong (1989), Adaptive Mation Control of Rigid Robots: a Tutorial, *Automatica*, 877-888.

F. C. Moon (1994), *Superconducting Levitation: Applications to Bearings and Magnetic Transportation*. New York: Wiley.

Z.Qu and D.M. Dawson (1996), *Robust Tracking Control of Robot Manipulators*, IEEE Press, New York.

S.N.Singh (1985), Adaptive model-following control of nonlinear robotic systems, *IEEE Transactions on Automatic Control*, vol. 30, pp. 1099-1100.

J.E.Slotine and W.Lin (1988), Adaptive Manipulator Control: A Case Study, *IEEE Trans. on Automat. Contr.*, Vol. 33, 995-1003.

H.D. Taghirad,et.al. (2010), *Electromagnetic Levitation System : An Experimental Approach*, Department of Electrical Engineering Tehran Iran.

T.H.Wong (1986), Desing and levitation of a magnetic levitation control system- an undergraduate project, *IEEE transaction on education* Vol.29 No.4, 196-200.