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Robust Decentralized PI Control Design ⋆

Petr Hušek, Vladimír Kučera

Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University in Prague, Technicka 2, 166 27, Prague 6, Czech Republic (e-mail: husek, kucera@fel.cvut.cz).

Abstract: A method for decentralized PI controller design for stable MIMO plants is presented. Each loop is designed individually by shaping the Gershgorin bands so that they avoid the circle with a prescribed radius centered in the critical point. The radius is used as a tuning parameter. The proposed procedure guarantees stability and robustness of the closed loop. A simulation example is provided to demonstrate the performance of the proposed method.

Keywords: Decentralized control; Multivariate systems; Nyquist stability analysis

1. INTRODUCTION

Proportional-integral-derivative (PID) controllers have attracted many engineers especially in the industry for a long time because of easy implementation and tuning (Åström and Hägglund (1995, 2005)). Moreover, for SISO systems they achieve acceptable control performance in most applications.

For MIMO systems performance of PID controllers is deteriorated especially when strong interactions occur. In that case utilizing some decoupling control (Wang and Yang (2002)) is inevitable. Nevertheless, in the case of moderate interactions a decentralized PID control is often sufficient and the corresponding design procedures are much simpler.

Most design methods for decentralized PID control are based on a frequency domain approach. Typically, the ultimate points on the frequency responses of diagonal elements are found and the controller parameters are determined using some detuning factor, e.g. biggest log modulus tuning (BLT) intoduced by Luyben (1986). Such procedures are in fact the multivariate counterparts to Ziegler-Nichols tuning rules (Chen and Seborg (2003); Loh et al. (1993)).

The common disadvantage of all the methods mentioned above is that they disregard the off-diagonal terms. An approach taking the interactions into account based on Nyquist stability criterion for MIMO systems (Rosenbrock (1970)) shapes the Nyquist frequency responses of diagonal open-loop transfer functions that are represented by Gershgorin bands that aggregate the interactions (Ho et al. (1995); Husek (2011)). For a non-oscillatory second-order plus dead-time model the decentralized PID parameters guaranteeing properties analogical to gain and phase margin were determined by Ho et al. (2000). Different approach consists in analysis of equivalent transfer functions that include closing of all loops (Xiong et al. (2007)). Procedures that iteratively improve the shape of frequency response have been reported in Lee et al. (2000).

The method presented in this paper shapes the Nyquist plot such that it avoids a circle with the prescribed radius centered in the critical point. The experiments reveal that the radius may serve as an appropriate tuning parameter (Garcia et al. (2005)). The procedure is graphical in nature and plots all the PI controller candidates a curve in the $k_P - k_I$ plane. The presented method is applied on a control of two-input two-output Wood-Berry distillation column model (Wood and Berry (1973)).

2. DIRECT NYQUIST ARRAY DESIGN

Consider a plant with $n \times n$ transfer matrix $G(s) = [g_{ij}(s)]_{n\times n}$ controlled by a decentralized controller with diagonal transfer matrix $C(s) = \mathrm{diag}\{c_1(s), c_2(s), \ldots, c_n(s)\}$, see Fig. 1.

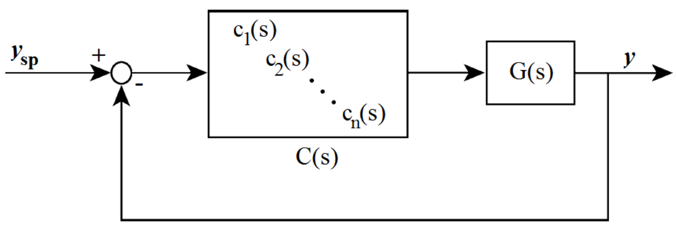

Fig. 1. Decentralized control scheme

The individual control loops are paired according to Relative Gain Analysis (RGA) (Maciejowski (1989)). Denote by

$$L(s) = G(s)C(s), \quad (1)$$
$$H(s) = (I + G(s)C(s))^{-1}\, G(s)C(s) \quad (2)$$

the open-loop and closed-loop transfer matrices, respectively. Consider the Nyquist plot of the diagonal elements $l_{mm}(\mathrm{j}\omega) = g_{mm}(\mathrm{j}\omega)c_m(\mathrm{j}\omega), m = 1, \ldots, n$ of $L(\mathrm{j}\omega)$, with a superimposed circle of the radius

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$$\rho_m(\omega) = \sum_{\substack{k=1,\\k\neq m}}^{n} |g_{km}(\mathrm{j}\omega)c_m(\mathrm{j}\omega)|. \tag{3}$$

This circle is referred to as the Gershgorin circle. The whole band composed of the circles for all $\omega > 0$ is called the Gershgorin band. Stability of the closed loop can be tested by the following theorem.

Theorem 1 [Direct Nyquist Array (DNA) (Rosenbrock (1970); Maciejowski (1989))] Let the Gershgorin bands centered on the diagonal elements $l_{mm}(\mathrm{j}\omega)$ of $L(\mathrm{j}\omega), m = 1,\dots,n$ exclude the point $(-1+\mathrm{j}0)$ (such a transfer matrix $L(s)$ is called column diagonally dominant). Let the i-th Gershgorin band encircles the point $(-1+\mathrm{j}0)$ N_i times anticlockwise. Then, the closed loop transfer matrix $H(s)$ is stable if and only if

$$\sum_{i=1}^{n} N_i = p_0 \tag{4}$$

where p_0 is the number of unstable poles of $L(s)$.

Since most practical processes are open-loop stable, $p_0 = 0$ is assumed throughout this paper. In that case the closed loop transfer matrix $H(s)$ with column diagonally dominant open loop transfer matrix $L(s)$ is stable if and only if the Nyquist plots of $l_{mm}(\mathrm{j}\omega)$ do not encircle the point $(-1+\mathrm{j}0)$ for all $m = 1,\dots,n$.

3. DECENTRALIZED PI CONTROL BASED ON SHAPING THE GERSHGORIN BANDS

The goal is to shape the Nyquist plots of diagonal open-loop transfer functions such that it circumferences a circle with radius Q centered in the point $(-1+0\mathrm{j})$ in the complex plane (see Fig. 2).

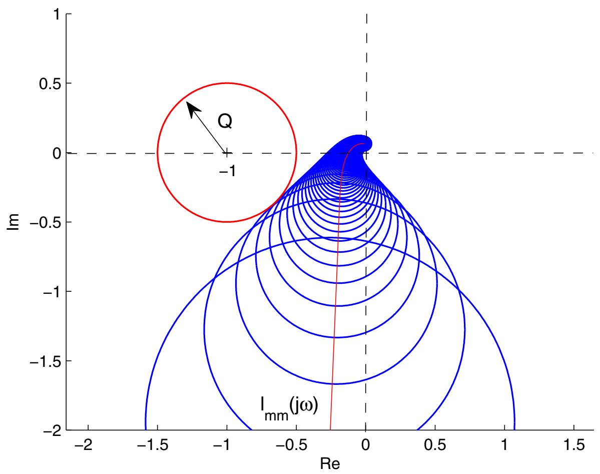

Fig. 2. Gershgorin band and Q-circle

The condition that the Gershgorin band touches the Q-circle can be expressed as

$$|1 + l_{mm}(\mathrm{j}\omega)| \geq Q + \rho_m(\omega) \quad \forall\omega, m \tag{5}$$

with the equality sign holding for just one $\omega = \omega_{sm}$ for each $m = 1,\dots,n$.

If we introduce functions $d_m(\omega)$ by

$$d_m(\omega) = |1 + l_{mm}(\mathrm{j}\omega)|^2 - (Q + \rho_m(\omega))^2, m = 1,\dots,n$$

necessary conditions for the inequality (5) to be met are

$$d_m(\omega) = 0 \tag{6}$$
$$\frac{\partial d_m(\omega)}{\partial\omega} = d'(\omega) = 0, \quad m = 1,\dots,n. \tag{7}$$

Let us write each element of the plant transfer matrix $g_{km}(\mathrm{j}\omega)$ (possibly involving a time delay) as

$$g_{km}(\mathrm{j}\omega) = a_{km}(\omega) + \mathrm{j}b_{km}(\omega) = r_{km}(\omega)\mathrm{e}^{\mathrm{j}\phi_{km}(\omega)} \tag{8}$$

and use a decentralized PI controller

$$C(s) = \mathrm{diag}\{c_1(s),\dots,c_n(s)\}$$
$$c_m(s) = k_{\mathrm{P}m} + \frac{k_{\mathrm{I}m}}{s},\ m = 1,\dots,n, \tag{9}$$

to control the plant.

Denote

$$R_m(\omega) = \sum_{\substack{k=1,\\k\neq m}}^{n} |g_{km}(\mathrm{j}\omega)|,\ m = 1,\dots,n. \tag{10}$$

After a straightforward computation one obtains

$$\begin{aligned} d_m(\omega) = {} & 1 - Q^2 + 2\left(a_{mm}(\omega)k_{\mathrm{P}m} + \frac{b_{mm}(\omega)k_{\mathrm{I}m}}{\omega}\right) \\ & + (r_{mm}^2(\omega) - R_m^2(\omega))\left(k_{\mathrm{P}m}^2 + \frac{k_{\mathrm{I}m}^2}{\omega^2}\right) \\ & - 2QR_m(\omega)\sqrt{k_{\mathrm{P}m}^2 + \frac{k_{\mathrm{I}m}^2}{\omega^2}} \end{aligned} \tag{11}$$

and

$$\begin{aligned} d'_m(\omega) = {} & 2\left(a'_{mm}(\omega)k_{\mathrm{P}m} + k_{\mathrm{I}m}\frac{b'_{mm}(\omega)\omega - b_{mm}(\omega)}{\omega^2}\right) \\ & + 2(r_{mm}(\omega)r'_{mm}(\omega) - R_m(\omega)R'_m(\omega))\left(k_{\mathrm{P}m}^2 + \frac{k_{\mathrm{I}m}^2}{\omega^2}\right) \\ & - 2\left(r_{mm}^2(\omega) - R_m^2(\omega)\right)\frac{k_{\mathrm{I}m}^2}{\omega^3} - 2QR'_m(\omega)\sqrt{k_{\mathrm{P}m}^2 + \frac{k_{\mathrm{I}m}^2}{\omega^2}} \\ & + 2QR_m(\omega)\left(k_{\mathrm{P}m}^2 + \frac{k_{\mathrm{I}m}^2}{\omega^2}\right)^{-\frac{1}{2}}\frac{k_{\mathrm{I}m}^2}{\omega^3}. \end{aligned} \tag{12}$$

The equations (6) and (7) are solved numerically with respect to $k_{\mathrm{P}m} > 0, k_{\mathrm{I}m} > 0$ for each ω from a suitably chosen frequency range. The solutions form a curve in the $k_{\mathrm{P}} - k_{\mathrm{I}}$ plane for each individual controller $c_m(s)$, $m = 1,\dots,n$.

According to the DNA theorem, in order to achieve closed-loop stability the frequency plots of the open-loop diagonal elements $l_{mm}(\mathrm{j}\omega)$ should not encircle the critical point $(-1+\mathrm{j}0)$. Solving the equation

$$\begin{aligned} l_{mm}(\mathrm{j}\omega) &= g_{mm}(\mathrm{j}\omega)c_m(\mathrm{j}\omega) \\ &= (a_{mm}(\omega) + \mathrm{j}b_{mm}(\omega))\left(k_{\mathrm{P}m} + \frac{k_{\mathrm{I}m}}{\mathrm{j}\omega}\right) = -1 \end{aligned}$$

for the real and the imaginary part separately one obtains the stability region in the $k_{\mathrm{P}} - k_{\mathrm{I}}$ plane which is delimited by

$$k_{\mathrm{P}m} = -\frac{a_{mm}(\omega)}{r_{mm}^2(\omega)},$$
$$k_{\mathrm{I}m} = -\omega\frac{b_{mm}(\omega)}{r_{mm}^2(\omega)} \qquad (13)$$

plotted for $0 \le \omega < \infty$.

All the PI controllers stabilizing the closed loop and guaranteeing that the minimal distance of the Gershgorin bands from the critical point is Q lie in the $k_{\mathrm{P}} - k_{\mathrm{I}}$ plane on the curve determined above inside the stability region (13).

From all these controllers we will choose those with maximum integral part for each loop since they minimize the sum of integral errors and with reasonable damping they generally produce the best performance, see Shafiei and Shenton (1997).

4. CONTROL OF WOOD-BERRY DISTILLATION COLUMN MODEL

The pilot-scale distillation column model created by Wood and Berry (1973) and used to separate a methanol–water mixture serves as a typical benchmark example for the verification of MIMO systems control design algorithms. Its transfer matrix description that relates between the input reflux flow rate R and the steam flow rate S to the reboiler, the output overhead X_{D} and bottoms X_{B} mole fractions of methanol, respectively, and the feed flow rate F as a disturbance variable is given by

$$\begin{bmatrix} X_{\mathrm{D}}(s) \\ X_{\mathrm{B}}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8\mathrm{e}^{-s}}{16.7s+1} & \dfrac{-18.9\mathrm{e}^{-3s}}{21s+1} \\ \dfrac{6.6\mathrm{e}^{-7s}}{10.9s+1} & \dfrac{-19.4\mathrm{e}^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8\mathrm{e}^{-8.1s}}{14.9s+1} \\ \dfrac{4.9\mathrm{e}^{-3.4s}}{13.2s+1} \end{bmatrix} F(s). \qquad (14)$$

The curves in the $k_{\mathrm{P}} - k_{\mathrm{I}}$ plane for the values of $Q = 0, 0.1, 0.3$ and 0.5 for both loops are depicted in Fig. 3 and Fig. 4. The parameters of the chosen PI controllers are summarized in Table 1. The Gershgorin bands for $Q = 0.3$ are shown in Fig. 5 and Fig. 6.

Q	$k_{\mathrm{P}1}$	$k_{\mathrm{I}1}$	$k_{\mathrm{P}2}$	$k_{\mathrm{I}2}$
0	0.7214	0.1248	-0.1514	-0.0186
0.1	0.6268	0.0892	-0.1362	-0.0147
0.3	0.4362	0.0409	-0.1048	-0.0087
0.5	0.2506	0.0161	-0.0675	-0.0046

Table 1. Decentralized PI controllers parameters

The closed-loop responses to unit step changes in the setpoints for X_{D} at $t = 0$ and X_{B} at $t = 150\,\mathrm{min}$ and

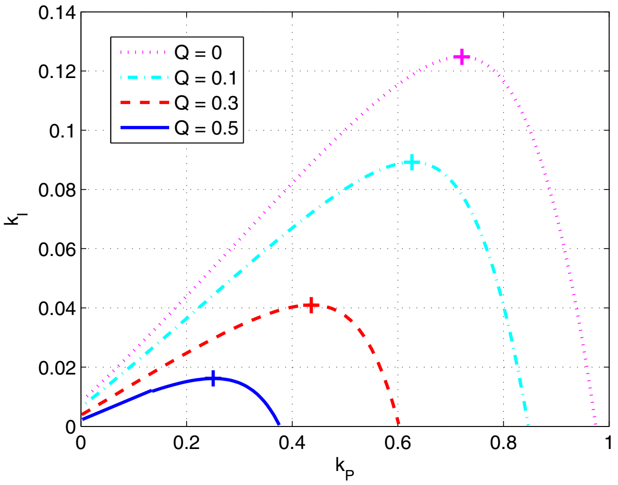

Fig. 3. PI controllers with different minimal distance – loop 1

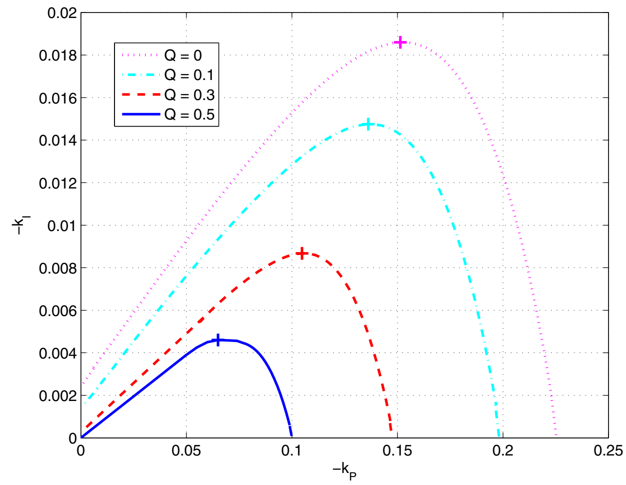

Fig. 4. PI controllers with different minimal distance – loop 2

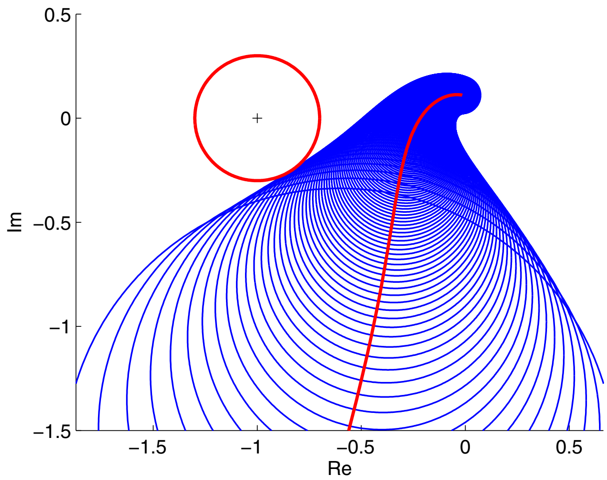

Fig. 5. Gershgorin band for $Q = 0.3$ – loop 1

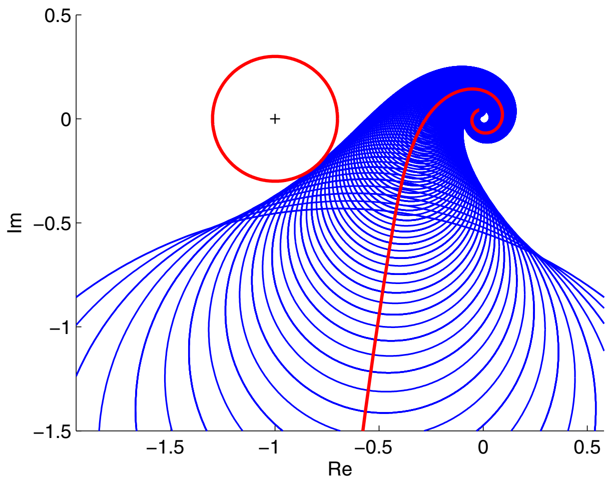

Fig. 6. Gershgorin band for $Q = 0.3$ – loop 2

to a unit step feed flow disturbance at $t = 300$ min for different values of Q are shown in Fig. 7 and Fig. 8. One can see that the value of Q represents a suitable parameter for tuning of the closed-loop responses. For small values of Q the response is too oscillatory with a high overshoot and undershoot whereas for too high values of Q the response is overdamped.

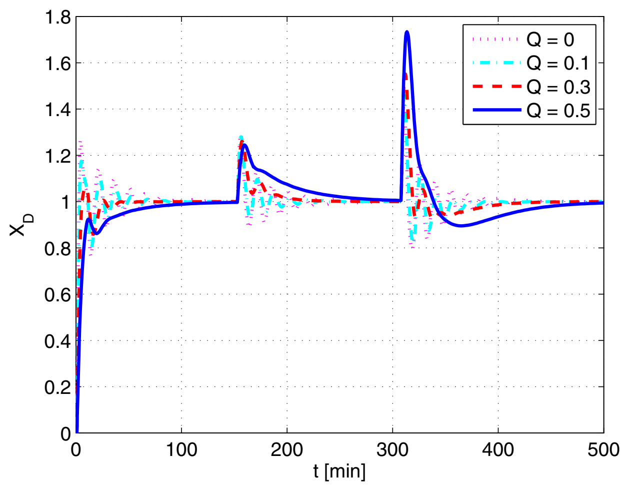

Fig. 7. Closed-loop responses for Wood-Berry distillation column – loop 1

5. CONCLUSION

The paper presents a method for multiloop decentralized PI controller design. The method is applicable to a plant described by a stable square transfer matrix with time delays. The proposed algorithm shapes the Gershgorin bands so that their minimal distance from the critical point is equal to a prescribed value that serves as a tuning parameter. Simulation results performed on a control of the Wood-Berry distillation column model confirm that the chosen parameter is suitable for tuning the closed-loop response.

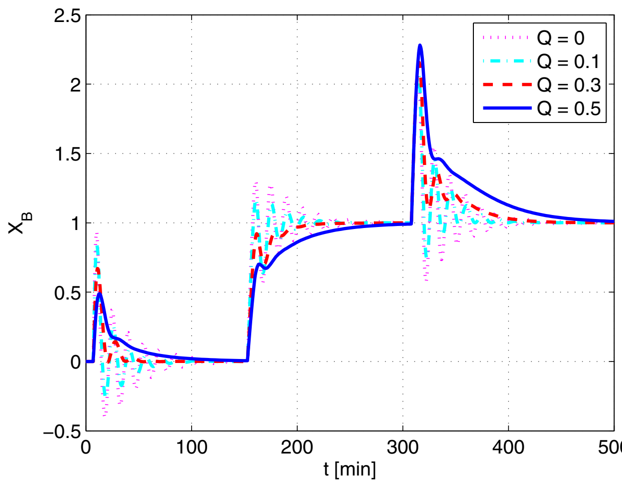

Fig. 8. Closed-loop responses for Wood-Berry distillation column – loop 2

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