Collision-free vehicle formation control using graph Laplacian and edge-tension function

Arshad Mahmood and Yoonsoo Kim*

Department of Aerospace and System Engineering,
Research Center for Aircraft Parts Technology,
Gyeongsang National University,
Jinju 660-701, Republic of Korea
(e-mail: arshad resignation; yoonsoo@gnu.ac.kr).

Abstract: This paper is concerned with collision-free vehicle formation control (FC) when the communication between vehicles is model by a graph. Unlike previous FC works (dealing with either non-trivial vehicle dynamics with no consideration of collision avoidance (CA) or trivial first-order vehicle dynamics with consideration of CA), this paper discusses non-trivial (second-order) vehicle FC with consideration of CA. This collision-free vehicle FC is done by manipulating entries of the graph Laplacian and by constructing a proper edge-tension function. Theoretical and numerical evidences are provided to show that the proposed control law effectively address both CA and FC.

Keywords: Formation control (FC); Collision avoidance (CA); Weighted graph Laplacian; Edge-tension function.

1. INTRODUCTION

Recently there is a tremendous surge of interest among researchers in formation control (FC) of autonomous vehicles due to its broad applications in military and civil areas. For example, a group of autonomous vehicles can be used for air traffic control, surveillance, firefighting, exploration, cleaning up oil spills and rich spatial awareness by distributing in a suitable formation (Martin et al. [2001], Bender. [1991]). Therefore, it may often be required that multiple vehicles move along a pre-defined trajectory while maintaining a desired formation. Moving in formation has many advantages as it can reduce the system cost, increase the robustness and efficiency of the system while providing redundancy and reconfiguration ability (Serrani. [2003], Daniel et al. [2004] and Stilwell et al. [2000]). Many control approaches, for example, a range-based method (Cao et al. [2011]) and a virtual structure approach (Ren. [2003]) have been used to achieve a desired formation. Also, Dashkovskiy et al. [2008] presents a framework of ISS (input-to-state stability) and a small-gain theorem which can be used for effective vehicle FC. One noticeable work is Lafferriere et al. [2005] in which a decentralized control scheme was proposed to achieve formation. Although these existing works propose sound FC schemes for non-trivial vehicles with second-order dynamics, they do not take into account the practically important issue of collision avoidance (CA) in their control designs.

CA is an old topic and has attracted many researchers, especially in aerospace engineering. See Keviczky et al. [2013], Mastellone et al. [2008], Lalish et al. [2008a], Lalish et al. [2008b], Sabattini et al. [2011], Yan et al. [2011], Falconi et al. [2011], Sabattini et al. [2011], Kan et al. [2012], Ammir. [2012] for some recent works in this direction. Yet, these existing works still have various limitations that have to be removed for the present application. Some of the existing works such as Keviczky et al. [2008] use the Receding Horizon Control (RHC) scheme which often requires solving a computationally intense optimization problem and thus has the limitation for real-time applications. Besides, this RHC scheme requires complicated emergency controllers and their invariant sets to define protection zones for CA when RHC problems become infeasible. Michael et al. [2011] also involves solving an optimization problem for CA and has a similar feasibility issue. The works such as Lalish et al. [2008a], Lalish et al. [2008b] are not specifically targeted for formation flying and do require all-to-all communication to collect all vehicles’ states. Mastellone et al. [2008] and Sabattini et al. [2011] present interesting research outputs closely related to the topic of present interest, but they require that a pre-defined reference state (or its accurate estimate) must be known to all vehicles to achieve the objective. In particular, Sabattini et al. [2011] is purely for ground robot applications as it requires the robots to stop and form a desired formation. Note that many aerospace applications do not allow vehicles (e.g. fixed-wing airplanes) to stop; instead, the vehicles are expected to move together in a desired formation with zero relative velocities. Other recent works such as Yan et al. [2011], Falconi et al. [2011], Kan et al. [2012] and Ammir. [2012] also discuss the topic of present interest, but all are restricted to the first-order vehicle dynamics.

* Corresponding author
In this work, the existing works are improved in the following manner. First, an edge-tension function is utilized for collision-free FC, thereby avoiding solving complex optimizations (unlike the RHC approaches). Note that the edge-tension function is originally proposed in Jiand [2007], and successfully used in Falconi et al. [2011] for the collision-free FC of first-order systems. In this paper, this edge-tension function is further exploited for the collision-free control of second-order systems. Second, the control structure to be proposed in this paper is similar to the state feedback control law proposed in Lafferrire et al. [2005], and so the control law is simple, easy-to-implement and may allow vehicles to move in a desired formation with zero relative velocities.

The rest of the paper is organized as follows. In §2, multi-vehicle dynamics is described and the simple control law proposed in Lafferrire et al. [2005] is stated as an example of FC (while not considering CA yet). Then, the clear problem statement is given along the definition of an edge-tension function to be useful for collision-free FC. In §3, our control law is proposed and proven to guarantee stability and desired performance (collision-free flying to a desired formation). This proposed control law is initially for a complete network topology and is subsequently extended for a star (leader-following) network topology. The extended work for a star (leader-following) network topology shall be presented in a journal version of this paper, though. Numerical examples are then provided in §4 to demonstrate the developments in the preceding sections, and concluding remarks follow in §5.

2. VEHICLE DYNAMICS AND PROBLEM FORMULATION

Consider N vehicles as vertices of a graph, with an edge set determined by the relative positions between the respective vehicles. Specifically, let $\mathcal{G}$ denote the set of graphs of order N with vertex set $V = \{1, 2, \ldots, N\}$ and edge set $E = \{e_{ij} : i = 1, 2, \ldots, N-1, j = 2, \ldots, N; i < j\}$, and the edge weight $w_{ij}$ assigned to each edge $e_{ij}$ is a function of the distance $l_{ij}$ between the two vehicles $i$ and $j$. It is assumed in this paper that $w_{ij} = l_{ij}$, and $e_{ij} \in E$ implies $e_{ji} \in E$. The weighted graph Laplacian matrix $L_w$ is defined as below:

$$L_w(x) = \begin{cases} \sum_{k \neq i} w_{ik} & \text{for } i = j; \\ -w_{ij} & \text{for } i \neq j. \end{cases}$$

The dynamics of each vehicle is described by the following equation:

$$\ddot{x}_i = A_{veh}x_i + B_{veh}u_i, \quad i = 1, 2, \ldots, N,$$

where a constant $a_{22}$ needs not to be zero (unlike $a_{22} = 0$ in Lafferrire et al. [2005]). Assuming that vehicle dynamics along each axis is decoupled, the dynamics of each vehicle can be written as

$$\ddot{x}_i = A_{veh}x_i + B_{veh}u_i, \quad i = 1, 2, \ldots, N,$$

where the entries of $x_i = [p_i^T, v_i^T]^T \in \mathbb{R}^{2n}$ represent $n$ configuration variables for vehicle $i$ and their derivatives; $u_i$ represents the control inputs; and

$$A_{veh} = I_n \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B_{veh} = I_n \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (2)$$

Here, $L_w$ is an identity matrix with dimension $n$ and $\otimes$ the Kronecker product. Note that in Lafferrire et al. [2005] the following simple static feedback control law

$$u = -FL(x - h) \quad (3)$$

with constant matrices $F$ and $L$, is proposed to let $N$ second-order vehicles change their positions and velocities to achieve a desired formation described by $h$. Here, $u$ and $x$ are the vectors of $u_i$’s and $x_i$’s, and $L = L_G \otimes I_{2n}$ ($L_G$ is the standard Laplacian matrix of a connected undirected inter-vehicle communication network topology $\mathcal{G}$ - see Kim et al. [2010] for details).

As mentioned earlier, the FC law in (3) may allow collisions between vehicles in the course of flying to a desired formation. In this paper, a new control law similar to (3) is proposed to guarantee both FC and CA. To this end, some preliminary definitions are in order.

Definition 1. (CA) Let $\delta$ be a safety distance (minimum separation) between each pair of vehicles. If the distance between each pair of vehicles is greater than $\delta$, it is said that collision is avoided. i.e. $l_{ij} > (dV_{ij} / dlij) = \delta \forall i, j \in V$. Furthermore, the collision-free realization of $\mathcal{G}$ is defined as

$$\mathcal{G}_\delta = \{ x \in \mathbb{R}^{nN} : l_{ij} > \delta, \forall e_{ij} \in E \}. \quad (4)$$

Definition 2. (Edge-tension function) For given positive constants $\delta$, $\alpha_{ij}$ and $K_{ij}$

$$V_{ij}(l_{ij}) = \alpha_{ij} \left\{ \coth \left( \frac{l_{ij} - \delta}{K_{ij}} \right) + \frac{l_{ij}}{K_{ij}} \right\} - V_{ij}^{\text{min}}. \quad (5)$$

Here, $V_{ij}^{\text{min}}$ is a positive constant which renders the minimum of $V_{ij}$ zero. The edge-tension function $V_{ij}$, as depicted in Fig. 1 with some $\delta$, $\alpha_{ij}$ and $K_{ij}$, is a differentiable nonnegative function of $l_{ij}$ such that

1. $V_{ij} \rightarrow 0$ as $l_{ij} \rightarrow \delta$.
2. $V_{ij}$ attains a unique minimum at $l_{ij} = d_{ij}$, where

$$d_{ij} = \delta + \frac{K_{ij}}{2} \log (3 + 2\sqrt{2}). \quad (6)$$

It should be noted that the edge-tension function (as well as its derivative) becomes infinite as $\delta$ reaches zero, so it may look an odd choice. However, this edge-tension function shall be used in such a way that no edge length reaches $\delta$ (where $dV_{ij} / dl_{ij} = \infty$) and every edge length $l_{ij}$
converges to its desired one $d_{ij}$ (where $V_{ij} = 0$). Clearly, $K_{ij}$ in (6) can be used to design $d_{ij}$ for a given $\delta$. The total tension energy of a graph $\mathcal{G}$ is defined as

$$ V = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} V_{ij} $$

(7)

where $\mathcal{N}_i$ denotes the set of neighbouring vehicles who can talk to vehicle $i$. We are now ready to present our FC laws such that each pair of vehicles $i, j$ achieve a pre-defined desired distance $d_{ij}$ with zero relative velocities while guaranteeing CA.

3. CONTROL LAW DESIGN

This section begins with the following theorem that allow $N$ vehicles to converge to formation without collision.

**Theorem 3.** Suppose two undirected network topologies of $N$ vehicles are given as $\mathcal{G}_i, \mathcal{G}_f$ whose realizations belong to $\mathcal{G}_f$, and set $K_{ij}$ in (6) such that $V_{ij}$ attains its minimum at $l_{ij} = d_{ij}$, where $d_{ij}$ is the desired relative distance between agents $i$ and $j$ of $\mathcal{G}_f$. Then, the control law $u_i$ ($i = 1, \cdots, N$) for $i$th vehicle

$$ u_i = -\frac{1}{N} \sum_{j \in \mathcal{N}_i} \left\{ (1 + a_{22})v_{ij} + \nabla_p V_{ij} \right\} $$

(8)

drives the vehicles from the initial configuration $\mathcal{G}_i$ to the desired formation $\mathcal{G}_f$ without collision, provided that all vehicles remain connected to each other at all times. Here, $\mathcal{N}_i$ is the set of natural numbers from 1 to $N$ except $i$, $v_{ij}$ is the relative velocity between vehicles $i$ and $j$, and $\nabla_p V_{ij}$ is the gradient of $V_{ij}$ with respect to $p_i$.

**Proof.** The proof shall be presented in a journal version of this paper.

It is interesting to note that the proposed controller (8) is similar to (3) found in Lafferriere et al. [2005]. To see this, first note that $\nabla_p V_{ij} = v_{ij} (p_i - p_j)$, where

$$ w_{ij} = \alpha_{ij} \left\{ -\cosh \left( \frac{l_{ij} - \delta}{K_{ij}} \right) + 1 \right\} \frac{1}{l_{ij} K_{ij}}.$$

The control law $u_i$ can then be written as:

$$ u_i = -\sum_{j \in \mathcal{N}_i} \left\{ (p_i - p_j) \frac{w_{ij}}{N} + \frac{1 + a_{22}}{N} (v_i - v_j) \right\}, $$

or in matrix form

$$ u_i = -I_N \otimes [f_1, f_2] \left( L_w \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + L_G \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) x. $$

(9)

where $[f_1, f_2] = [1/N, (1 + a_{22})/N]$ and $x$ is the stack vector of positions and velocities of $N$ vehicles. Since the motion along each axis is independent for vehicle $i$, (9) can be written as:

$$ u_i = -\left( I_N \otimes I_n \otimes [f_1, f_2] \right) \times \left( L_w \otimes I_n \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + L_G \otimes I_n \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right) x$$

$$= -FL_{wg}x.$$

Note that the desired formation vector $h$ in (3) is now embedded in $w_{ij}$ in the form of $d_{ij}$. The aforementioned formation control law can be extended in a way to accommodate a star (leader-following) network topology. This extension, however, shall be presented in a journal version of this paper.

4. NUMERICAL EXAMPLE

To test the proposed control (8) in a simulation environment, five vehicles were initially lined up (marked with ‘x’) in Fig. 2-(a) and are required to form a pentagon formation defined by $h = [0; 1] \otimes h_p = [0; 1] \otimes [h_1; h_2; h_3; h_4; h_5]^T$, where for some chosen $d_{ij}$ $\begin{bmatrix} h_1 \cdots h_5 \end{bmatrix} = \begin{bmatrix} [0; 0] \cdots [0; 1] \end{bmatrix}$ and $h_5 = [d_{12}; \cdots; d_{15}]$. Once the desired inter-vehicle distances are fixed, $K_{ij}$ can be chosen based on (6). Also, $a_{22} = 0.1$, $\delta = 2.0$ and $\alpha_{ij} = 1.0$.

Fig. 2 shows the simulation results when (8) is used for formation control. Fig. 2-(a) shows that five vehicles start from the line formation (×) and form the required pentagon formation (○) in the end. Fig. 2-(b) shows the control effort required to form the desired formation. Note that as the vehicles achieve the desired formation, the control effort becomes zero. Fig. 2-(c) shows relative distances between each pair of vehicles, along with the minimum separation line of $\delta = 2.0$. Fig. 3 shows the same formation reconfiguration scenario when (3) in Lafferriere et al. [2005] is used to achieve the desired formation. In this case, Fig. 2-(c) clearly shows that some vehicles violate the minimum separation constraint.

In the second scenario, vehicles are required to form a circular formation from an initial horizontal configuration, where vector $h_p$ is given by $\theta=2\pi/N$ and $h_{i+1} = [d_{12}\cos(i\theta); d_{12}\sin(i\theta)]$ ($i = 0, \cdots, N - 1$). Fig. 4 shows that vehicles achieve the circular formation while maintaining the distances above the pre-defined safety distance of $\delta = 1.0$.

1 $[a; b]$ denotes a column vector of $a$ and $b$; this is a Matlab-like notation.
REFERENCES


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