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Improved Control of Distributed Parameter Systems with Time-Varying Delay Based on Mobile Actuator-Sensor Networks ⋆

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Abstract: This paper investigates the control problem of distributed parameter systems (DPS) with time-varying delay by employing mobile actuator-sensor networks. It is assumed that each agent in the networks has a sensor device which can measure spatial state, and an actuator device that can dispense control signals to spatially distributed process and can communicate with its neighbors. To better control the DPS with time-varying delay, the strategy how to navigate the agents is considered. By constructing Lyapunov functionals and using inequality analysis, criterias for stability of the DPS with time-varying delays are derived. Meanwhile, the guidance scheme of every agent with augmented vehicle dynamics is derived. Simulation results show that such mobile actuator-sensor networks can improve the control performance of the DPS with time delay.

1. INTRODUCTION

The mobile actuator-sensor networks have many innate advantages compared to wireless sensor networks on the improved performance, monitoring and efficient control of processes in practical applications, and have received increasing attention, see for example Akyildiz and Kasimoglu (2004), Akkaya and Senel (2009), Tricaud and Chen (2009, 2010). The networked actuators which are usually attached to mobile agents (terrain robots, underwater vehicles, UAVs) can dispense relevant control signal on practical systems to improve certain control objects with the help of networked sensors. In particular, the control or estimation of distributed parameter systems (DPS) using mobile actuator-sensor networks has gained research attention from scholars in many areas (Uciński, 2004; Zeng & Ayalew, 2010; Chao & Chen, 2012; Tricaud & Chen, 2012). For instance, in Chao, Chen and Ren (2006) moving actuators were used to control the spatially distributed process with the help of the static sensor networks using central voronoi tessellations. Zeng and Ayalew (2010) investigated the estimation and coordinated control using one mobile radiant actuator. Recently, the control and estimation problem of 1D DPS were investigated in Demetriou (2010) by assuming that every mobile agent was massless and inertialess. The controller design problem for 1D DPS and adaptive control problem of 2D DPS were also investigated in Demetriou (2011, 2012), where the vehicle dynamics for each mobile agent was augmented in the mobile actuator-sensor networks. However, most works for the control and/or estimation of DPS focus on the systems without time delays.

On the other hand, the time delays often appear in many chemical systems, biological systems, electrical engineering systems and mechanical applications(Sen, Ghosh & Ray, 2010). Therefore, time delays are considered in order to better reflect the reality in the distributed parameter systems. However, delays may destabilize the systems, so the stability and control probelm for distributed parameter systems with time delays have been intensively studied in the recent years (Luo, Xia, Liu & Deng, 2009; Fridman & Orlov, 2009; Fridman, Nicaise & Valein, 2010; Tai & Lun, 2012). For example, the exponential stability of DPS with time-varying delays was investigated in Fridman and Orlov (2009), and several sufficient conditions for exponential stabilization were given by using different Lyapunov functions and linear matrix inequality. However, all the above results only considered the stability of delayed DPS from theory and didn't answer how to effectively improve the performance of the control systems in the guarantee of the stability of distributed parameter systems with time-varying delay from the practical application.

Inspired by the above works, this paper discusses not only the stability but also the improved control for distributed parameter systems with time-varying delay using mobile actuator-sensor networks. By constructing a new Lyapunov functional and using inequality analysis, the stability conditions of DPS with time-varying delay are derived, and a stable guidance scheme for every mobile agent is provided.

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2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the distributed parameter system with time-varying delay given by

$$\frac{\partial w(t,x)}{\partial t} = a_1 \frac{\partial^2 w(t,x)}{\partial x^2} + a_2 w(t,x) + a_3 w(t-\tau(t),x) + \sum_{i=1}^{n} b(x,\theta_i(t))u_i(t),$$

$$y(t) := \begin{bmatrix} y_1(t;\theta_1(t)) \\ \vdots \\ y_n(t;\theta_n(t)) \end{bmatrix} = \begin{bmatrix} \int_0^l c(x;\theta_1(t))w(t,x)dx \\ \vdots \\ \int_0^l c(x;\theta_n(t))w(t,x)dx \end{bmatrix}, \quad (1)$$

where $(t,x) \in \mathbf{R}^+ \times \Omega$, $\Omega = [0,l]$, $a_1 > 0$, a_2 and a_3 are constants, $\tau(t)$ denotes the time-varying delay satisfying $\tau(t) \geq 0$ and $\dot{\tau}(t) < \eta < 1$. $w(t,x)$ denotes the state; $b(x;\theta_i(t))$ and $c(x;\theta_i(t))$ represent the spatial distribution of the i^{th} actuator and the i^{th} sensor, respectively; $u_i(t)$ represents the control signal of the i^{th} actuator; $\theta_i(t)$ represents the position of the i^{th} mobile agent which is affixed with an actuator and a sensor. The initial boundary value conditions are given by

$$w(t,0) = w(t,l) = 0, \quad (2)$$

$$w(t,x) = \varphi(t,x), (t,x) \in [-\tau, 0] \times \Omega, \quad (3)$$

where $\varphi(t,x)$ is the suitable smooth function.

Assumption 1. (Demetriou, 2010) In order to simplify stability analysis, it is assumed that the spatial distributions of the i^{th} actuator and the i^{th} sensor in the i^{th} mobile agent are the same and given by

$$b(x;\theta_i(t)) = c(x;\theta_i(t)) = \begin{cases} 1 & \text{if } \ x \in [\theta_i(t)-\varepsilon, \ \theta_i(t)+\varepsilon] \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

which can also be written by $b(x;\theta_i(t)) = c(x;\theta_i(t)) = H(\theta_i(t)-\varepsilon) - H(\theta_i(t)+\varepsilon)$, where $H(\theta_i(t)-\varepsilon)$ and $H(\theta_i(t)+\varepsilon)$ are two Heaviside step functions.

Assumption 1 implies that the i^{th} mobile agent (sensor) can provide the spatially averaged observation in the sense that

$$y_i(t) = \int_0^l c(x;\theta_i(t))w(t,x)dx \approx 2\varepsilon \times \frac{w(t,\theta_i(t)-\varepsilon) + w(t,\theta_i(t)+\varepsilon)}{2}, i = 1,2,\cdots,n. \quad (5)$$

Assumption 2. The motion equation of the i^{th} mobile agent is given by

$$m_i \ddot{\theta}_i(t) = f_i(t), \quad i = 1,\cdots,n,$$

$$\theta_i(0) = \theta_{i0}, \quad \dot{\theta}_i(0) = 0, \quad (6)$$

where $\theta_i(t)$ denotes the position of the i^{th} mobile actuator-sensor agent within the spatial domain $\Omega = [0,l]$, and $f_i(t)$ is its associated control force.

Here, we consider not only the stability of distributed parameter control systems with time-varying delay (1) but also how to design the control force $f_i(t)$ such that the state $w(t,x)$ converges to zero faster than the fixed actuators and sensors.

Assumption 3. Given an agent i, its neighbor set is $N_i = \{j | \ |\theta_i - \theta_j| < R\}$. it is assumed that the i^{th} agent can transmit information $(\theta_i, \dot{\theta}_i, y_i)$ to its neighbor agents and can receive information $(\theta_j, \dot{\theta}_j, y_j), j \in N_i$, from its neighbor agents.

To control the distributed parameter system with time-varying delay and ensure the state $w(t,x)$ converges to zero, how to choose both the control signals $u_i(t), i = 1,2,\cdots,n$, and the agent positions $\theta_i(t), i = 1,2,\cdots,n$, is crucial. For the need of minimal design complexity, the static output feedback controller is considered and given by $u_i(t) = -\sum_{j\in N(i)} r_{ij} y_j(t), i = 1,2,\cdots,n$, where $r_{ij} > 0$ is the feedback gain, and $y_j(t), j = 1,2,\cdots,n$ are the averaged measurements given by (5).

3. STABILITY ANALYSIS AND GUIDANCE OF MOBILE AGENTS

In this section, the stability criteria for the distributed parameter system with time-varying delay and the guidance law for every agent are derived.

Let us regard (1) as an evolution equation in a Hilbert space. Similar to the assumptions in Demetriou (2010), let $\mathcal{W}$ be a Hilbert space which is equipped with the inner product $\langle\cdot,\cdot\rangle$ and induced norm $|\cdot|$. Let $\mathcal{V}$ be a reflexive Banach space with norm denoted by $\|\cdot\|$. It is assumed that $\mathcal{V}$ is embedded densely and continuously in $\mathcal{W}$. Let $\mathcal{V}^*$ denote the conjugate dual of $\mathcal{V}$ with induced norm $\|\cdot\|_*$. It follows $\mathcal{V} \hookrightarrow \mathcal{W} \hookrightarrow \mathcal{V}^*$ with both embedding dense and continuously, and we have $|\phi| \leq c\|\phi\|$, $\phi \in \mathcal{V}$, for some $c > 0$.

In this work, $\mathcal{W} = L_2(\Omega)$ is the state space and $w(t,\cdot) = \{w(t,x), 0 \leq x \leq l\}$ denotes the state of the distributed parameter system with time-varying delay (1). The Sobolev space $\mathcal{V}$ is given by $\mathcal{V} = H_0^1 = \{\psi \in H^1(\Omega)|\psi(0) = \psi(l) = 0\}$ and its conjugate dual space $\mathcal{V}^*$ is $H^{-1}(\Omega)$. The system's elliptic operator A_1 is given in Dautray and Lions (2000)

$$A_1\phi = \frac{d}{dx}(a_1 \frac{d\phi}{dx}), \ a_1 > 0, \ \phi \in \text{Dom}(A_1),$$

with $\text{Dom}(A_1) = \{\psi \in L_2(\Omega) \ |\psi, \psi' \text{ abs. continuous, } \psi'' \in L_2(\Omega) \text{ and } \psi(0) = \psi(l) = 0\}$. Since $a_1 > 0$, the operator A_1 is bounded and symmetry, and $-A_1$ is coercive in Friedman (1964). Define bounded operators $A_2 = a_2$ and $A_3 = a_3$. The n input operators are given by

$$B_i(\theta_i)u_i(t) = b(x;\theta_i)u_i(t), \ i = 1,\cdots,n$$

and their matrix operator is given by

$$B(\theta)u(t) = \ [B_1(\theta_1) \ \cdots \ B_n(\theta_n)] \begin{bmatrix} u_1(t) \\ \vdots \\ u_n(t) \end{bmatrix}.$$

Since the i^{th} actuator and i^{th} sensor are affixed onboard the i^{th} mobile agent in the sense that the actuator and sensor are collocated from Assumption 1, we have $C_i = B_i^*$ (Curtain and Zwart, 1995) and therefore

$$y_i(t) = \mathcal{C}_i w(t) = B_i^* w(t) = \int_0^l b(x;\theta_i) w(t,x)dx, \ i = 1, \cdots, n.$$

The control law $u_i(t) = -\sum_{j\in N(i)} r_{ij} y_j(t), i = 1, 2, \cdots, n$ can be expressed in a compact form

$$u(t) = -\Gamma y(t)$$

where $u(t) = [u_1(t), \cdots, u_n(t)]^T$, $\Gamma = [r_{ij}]_{n\times n} > 0$, $y(t) = [y_1(t), \cdots, y_n(t)]^T$. The gain matrix $\Gamma = [r_{ij}]_{n\times n} > 0$ means Γ is symmetric positive definite, where $r_{ij} \geq 0$. Hereafter, $r_{ij} > 0$ if and only if the i^{th} agent can exchange information with the j^{th} agent. Thus the closed loop system of (1) is given by

$$\begin{aligned}\dot{w}(t) &= A_1 w(t) + A_2 w(t) + A_3 w(t-\tau(t)) + B(\theta(t))u(t)\\ &= A_1 w(t) + A_2 w(t) + A_3 w(t-\tau(t)) - B(\theta(t))\Gamma y(t)\\ &= A_1 w(t) + A_2 w(t) + A_3 w(t-\tau(t))\\ &\quad -B(\theta(t))\Gamma B^*(\theta(t))w(t). \end{aligned} \tag{7}$$

To further derive the stability conditions on the distributed parameter control system with time-varying delay and the guidance scheme of mobile agents, the following lemmas are needed.

Lemma 1. The self-adjoint operators $B(\theta(t))\Gamma B^*(\theta(t))$, $-A_1$ and $-A_1 + B(\theta(t))\Gamma B^*(\theta(t))$ are positive definite.

Proof. Since $\Gamma = \Gamma^T > 0$,

$$\begin{aligned}&\langle B(\theta(t))\Gamma B^*(\theta(t))w(t), w(t)\rangle\\ &= \langle \Gamma B^*(\theta(t))w(t), B^*(\theta(t))w(t)\rangle\\ &\geq \lambda_{\min}(\Gamma)|B^*(\theta(t))w(t)|^2 > 0.\end{aligned}$$

The proof of the positive definite of $-A_1$ and $-A_1 + B(\theta(t))\Gamma B^*(\theta(t))$ is similar to Demetriou (2010).

Lemma 2. $|(-A_1 + B(\theta(t))\Gamma B^*(\theta(t)))w(t)| \geq \lambda_0 |w(t)|$, where $\lambda_0 > 0$ is the minimum eigenvalue of $-A_1 + B(\theta(t))\Gamma B^*(\theta(t))$.

Proof. Let $A_c = -A_1 + B(\theta(t))\Gamma B^*(\theta(t))$, then

$$\begin{aligned}&|(-A_1 + B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2\\ &= \langle A_c w(t), A_c w(t)\rangle\\ &= \langle w(t), A_c^* A_c w(t)\rangle = \langle w(t), A_c A_c w(t)\rangle\\ &\geq \lambda_{\min}(A_c A_c)\langle w(t), w(t)\rangle = \lambda_{\min}^2(A_c)\langle w(t), w(t)\rangle\\ &= \lambda_0^2 |w(t)|^2.\end{aligned}$$

From Lemma 1, $-A_1 + B(\theta(t))\Gamma B^*(\theta(t))$ is positive, so $\lambda_0 > 0$.

Our main result is given in the following theorem.

Theorem 1. Consider the distributed parameter system (1) with time-varying delay satisfying Assumptions 1-3. The proposed guidance law for each agent renders the system (1) stable and improves the control performance in the sense of that the system state $w(t,x)$ converges to zero fast, If there exist positive constants p and q, such that

$$-p\lambda_0^2 + 2pa_2^2 + q < 0 \quad \text{and} \quad 2pa_3^2 - q(1-\eta) < 0.$$

The guidance law for each agent is given by (6), where

$$\begin{aligned}f_i = &- p(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon))(\sum_{j\in N_i} r_{ij} y_j(t))\\ &- k_i\theta_i(t) - d_i\dot{\theta}_i(t), i = 1, 2, \cdots, n,\end{aligned} \tag{8}$$

where $k_i \geq 0$ and $d_i \geq 0$.

Proof. Choose a Lyapunov functional candidate $V(t)$ as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

where

$$\begin{aligned}V_1(t) &= -p\langle w(t), A_1 w(t)\rangle,\\ V_2(t) &= p\langle w(t), B(\theta(t))\Gamma B^*(\theta(t))w(t)\rangle,\\ V_3(t) &= q\int_{t-\tau(t)}^t \langle w(s), w(s)\rangle ds,\\ V_4(t) &= \sum_{i=1}^n (m_i(\dot{\theta}_i(t))^2 + k_i(\theta_i(t))^2),\end{aligned}$$

and $p > 0, q > 0, k_i > 0$.

Next, calculating the derivative of $V(t)$ along with the trajectory of system (7) yields

$$\dot{V}_1(t) = -p\langle \dot{w}(t), A_1 w(t)\rangle - p\langle w(t), A_1\dot{w}(t)\rangle, \tag{9}$$

$$\begin{aligned}\dot{V}_2(t) = &\, p\langle \dot{w}(t), B(\theta(t))\Gamma B^*(\theta(t))w(t)\rangle\\ &+p\langle w(t), B(\theta(t))\Gamma B^*(\theta(t))\dot{w}(t)\rangle\\ &+p\langle w(t), \frac{d}{dt}(B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle,\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_3(t) = &\, q\langle w(t), w(t)\rangle\\ &-q(1-\dot{\tau}(t))\langle w(t-\tau(t)), w(t-\tau(t))\rangle,\end{aligned} \tag{11}$$

$$\dot{V}_4(t) = \sum_{i=1}^n (2m_i\dot{\theta}_i(t)\ddot{\theta}_i(t) + 2k_i\theta_i(t)\dot{\theta}_i(t)).$$

Using the fact that both A and $B(\theta(t))\Gamma B^*(\theta(t))$ are self-adjoint, then

$$\begin{aligned}\dot{V}_1(t) + \dot{V}_2(t) = &- 2p\langle \dot{w}(t), A_1 w(t)\rangle\\ &+ 2p\langle \dot{w}(t), B(\theta(t))\Gamma B^*(\theta(t))w(t)\rangle\\ &+ p\langle w(t), \frac{d}{dt}(B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle.\end{aligned} \tag{12}$$

For the first two parts of (12), using the fact $\pm 2\langle x, y\rangle \leq \langle x, x\rangle + \langle y, y\rangle$, we obtain

$$\begin{aligned}&-2p\langle \dot{w}(t), A_1 w(t)\rangle + 2p\langle \dot{w}(t), B(\theta(t))\Gamma B^*(\theta(t))w(t)\rangle\\ &= -2p\langle \dot{w}(t), (A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle\\ &= -2p\langle (A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t) + A_2 w(t)\\ &\quad +A_3 w(t-\tau(t)), (A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle\\ &= -2p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2\\ &\quad -2p\langle A_2 w(t) + A_3 w(t-\tau(t)),\\ &\quad (A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle\\ &\leq -2p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2\\ &\quad +p\langle A_2 w(t) + A_3 w(t-\tau(t)), A_2 w(t) + A_3 w(t-\tau(t))\rangle\\ &\quad +p\langle (A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t),\end{aligned}$$

$$
\begin{aligned}
&(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)\rangle \\
&= -p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2 + p\langle A_2 w(t), A_2 w(t)\rangle \\
&\quad + p\langle A_3 w(t-\tau(t)), A_3 w(t-\tau(t))\rangle \\
&\quad + 2p\langle A_2 w(t), A_3 w(t-\tau(t))\rangle \\
&\le -p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2 + p\langle A_2 w(t), A_2 w(t)\rangle \\
&\quad + p\langle A_3 w(t-\tau(t)), A_3 w(t-\tau(t))\rangle + p\langle A_2 w(t), A_2 w(t)\rangle \\
&\quad + p\langle A_3 w(t-\tau(t)), A_3 w(t-\tau(t))\rangle \\
&= -p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2 + 2p\langle A_2 w(t), A_2 w(t)\rangle \\
&\quad + 2p\langle A_3 w(t-\tau(t)), A_3 w(t-\tau(t))\rangle. \qquad (13)
\end{aligned}
$$

For the last part of (12), we have

$$
\begin{aligned}
&p\langle w(t), \frac{d(B(\theta(t))\Gamma B^*(\theta(t)))}{dt} w(t)\rangle \\
&= 2p\langle w(t), \dot{\theta}(t)\frac{\partial B(\theta(t))}{\partial \theta(t)}\Gamma B^*(\theta(t))w(t)\rangle \\
&= 2p \begin{bmatrix} \int_0^l \dot{\theta}_1(t)\frac{\partial B_1}{\partial \theta_1(t)} w(t,x)dx \\ \vdots \\ \int_0^l \dot{\theta}_n(t)\frac{\partial B_n}{\partial \theta_n(t)} w(t,x)dx \end{bmatrix}^T \Gamma y(t). \qquad (14)
\end{aligned}
$$

Examining (14) in detail, we obtain

$$
\begin{aligned}
&\int_0^l \dot{\theta}_i(t)\frac{\partial B_i}{\partial \theta_i(t)} w(t,x)dx \\
&= \dot{\theta}_i(t)\int_0^l \frac{\partial B_i}{\partial \theta_i(t)} w(t,x)dx \\
&= \dot{\theta}_i(t)\int_0^l \frac{\partial (H(\theta_i(t)-\varepsilon) - H(\theta_i(t)+\varepsilon))}{\partial \theta_i(t)} w(t,x)dx \\
&= \dot{\theta}_i(t)\int_0^l (\delta(\theta_i(t)-\varepsilon) - \delta(\theta_i(t)+\varepsilon))w(t,x)dx \\
&= \dot{\theta}_i(t)(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon)). \qquad (15)
\end{aligned}
$$

Thus it follows from (14) and (15),

$$
\begin{aligned}
&p\ \langle w(t), \frac{d(B(\theta(t))\Gamma B^*(\theta(t)))}{dt} w(t)\rangle \\
&= 2p\sum_{i=1}^{n}[\ \dot{\theta}_i(t)(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon)) \\
&\quad \times(\sum_{j\in N_i} r_{ij}y_j(t))\ ]. \qquad (16)
\end{aligned}
$$

Using the condition in (6), we get

$$
\begin{aligned}
\dot{V}_4(t) &= \sum_{i=1}^{n}(2m_i\dot{\theta}_i(t)\ddot{\theta}_i(t) + 2k_i\theta_i(t)\dot{\theta}_i(t)) \\
&= \sum_{i=1}^{n} 2\dot{\theta}_i(t)\ (m_i\ddot{\theta}_i(t) + k_i\theta_i(t)) \\
&= \sum_{i=1}^{n} 2\dot{\theta}_i(t)\ (f_i + k_i\theta_i(t)). \qquad (17)
\end{aligned}
$$

Combining (11), (13), (16) and (17), and using the condition $\dot{\tau}(t) \le \eta < 1$, we get that

$$
\begin{aligned}
\dot{V}(t) \le &-p|(A_1 - B(\theta(t))\Gamma B^*(\theta(t)))w(t)|^2 \\
&+ 2p\langle A_2 w(t), A_2 w(t)\rangle \\
&+ 2p\langle A_3 w(t-\tau(t)), A_3 w(t-\tau(t))\rangle \\
&+ q\langle w(t), w(t)\rangle - q(1-\eta)\langle w(t-\tau(t)), w(t-\tau(t))\rangle \\
&+ 2p\sum_{i=1}^{n}[\ \dot{\theta}_i(t)(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon)) \\
&\quad \times(\sum_{j\in N_i} r_{ij}y_j(t))\ ] \\
&+ 2\sum_{i=1}^{n}\dot{\theta}_i(t)\ (f_i + k_i\theta_i(t)). \qquad (18)
\end{aligned}
$$

From Lemma 2 and using $A_2 = a_2, A_3 = a_3$, we have

$$
\begin{aligned}
\dot{V}(t) \le &(-p\lambda_0^2 + 2pa_2^2 + q)|w(t)|^2 \\
&+(2pa_3^2 - q(1-\eta))|w(t-\tau(t))|^2 \\
&+2\sum_{i=1}^{n}\ \dot{\theta}_i(t)[p(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon)) \\
&\times(\sum_{j\in N_i} r_{ij}y_j(t)) + f_i + k_i\theta_i(t)\ ]. \qquad (19)
\end{aligned}
$$

Considering the control force (8), the derivative of $V(t)$ becomes

$$
\begin{aligned}
\dot{V}(t) \le &(-p\lambda_0^2 + 2pa_2^2 + q)|w(t)|^2 \\
&+(2pa_3^2 - q(1-\eta))|w(t-\tau(t))|^2 \\
&-2\sum_{i=1}^{n}\ d_i(\dot{\theta}_i(t))^2. \qquad (20)
\end{aligned}
$$

If the conditions $-p\lambda_0^2 + 2pa_2^2 + q < 0, 2pa_3^2 - q(1-\eta) < 0$ are hold, then we have $\dot{V}(t) < 0$. Thus, from the standard Lyapunov stability theorems, the system (1) is stable. The proof is completed.

In Theorem 1, it is assumed that every agent can receive or transmit information of its measurements from/to its immediate neighbors in the sense that the local connectivity of the agents is required. This may be relaxed if any two agents can't exchange information with each other. In this case, the non-interacting controllers are given by $u_i(t) = -r_iy_i(t), r_i > 0, i = 1, \cdots, n$. From Theorem 1, it is easy to get the following corollary.

Corollary 1. Consider the distributed parameter system (1) with time-varying delay satisfying Assumptions 1-2. Every agent only takes information from its own sensor and the control law for the i^{th} actuator is given by $u_i(t) = -r_iy_i(t), i = 1, 2, \cdots, n$. The proposed guidance law for each agent renders the system (1) stable and improves the control performance in the sense of that the system state $w(t,x)$ converges to zero fast, if there exist positive constants p and q, such that

$$-p\lambda_0^2 + 2pa_2^2 + q < 0 \quad and \quad 2pa_3^2 - q(1-\eta) < 0.$$

The guidance law for each agent is given by (6), where

$$
\begin{aligned}
f_i = &-pr_i\varepsilon(w^2(t,\theta_i(t)-\varepsilon) - w^2(t,\theta_i(t)+\varepsilon)) \\
&- k_i\theta_i(t) - d_i\dot{\theta}_i(t), i = 1, 2, \cdots, n,
\end{aligned}
$$

where $k_i \ge 0$ and $d_i \ge 0$.

When $\tau(t) \equiv \tau$ (τ is a constant), Theorem 1 will reduce to the following Corollary 2.

Corollary 2. Consider the distributed parameter system (1) with $\tau(t) \equiv \tau$ satisfying Assumptions 1-3. The guidance law for each agent renders the system (1) stable and enhances the performance in the sense of that the system state $w(t,x)$ converges to zero fast, if there exist positive constants p and q, such that

$$-p\lambda_0^2 + 2pa_2^2 + q < 0 \quad and \quad 2pa_3^2 - q < 0.$$

The guidance law for each agent is given by (6), where

$$f_i = -p(w(t,\theta_i(t)-\varepsilon) - w(t,\theta_i(t)+\varepsilon))(\sum_{j \in N_i} r_{ij} y_j(t)) - k_i\theta_i(t) - d_i\dot{\theta}_i(t), i = 1, 2, \cdots, n, \quad (21)$$

where $k_i \geq 0$ and $d_i \geq 0$.

Remark 1. It should be noted that the control force $f_i(t)$ in Theorem 1 for the i^{th} agent depends on not only its location and velocity, but also the difference of the measurements $w(t, x_i^a - \varepsilon) - w(t, x_i^a + \varepsilon)$ and the output readings of its neighbor agents $y_j(t), j \in N_i$, in the sense that the mobile agents and the spatial process interact with each other.

4. SIMULATIONS

Consider the system (1) with the initial condition $\varphi(t,x) = sin(\pi x)e^{-7x^2}(t+2)$ where $x \in \Omega = [0,1], t \in [-\tau, 0], \tau = 2$. The coefficients are $a_1 = 0.003$, $a_2 = 0.01$ and $a_3 = 0.01$. For simplicity, the closed loop system is simulated for 20 seconds with two mobile actuator-sensor agents whose parameters are chosen as $m_i = 1, p = 1, k_i = 0.002, d_i = 0.5, i = 1,2$ and $\varepsilon = 0.035$. The gain matrix Γ was taken as

$$\begin{bmatrix} 20 & 10 \\ 10 & 20 \end{bmatrix}.$$

The initial conditions for the two mobile agents are chosen as $\theta_1(0) = 0.2$, $\dot{\theta}_1(0) = 0$, $\theta_2(0) = 0.8$, $\dot{\theta}_2(0) = 0$. The positions for the two fixed agents are 0.2 and 0.8.

The system state for the uncontrolled case and the controlled case with two mobile agents which are affixed with actuators and sensors are depicted in Fig. 1 and Fig. 2, respectively. It can be seen that the distributed parameter system with time delay is well controlled by the mobile actuator-sensor networks.

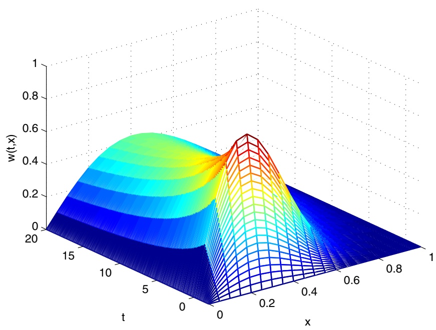

Fig. 1. The state of system (1) without control.

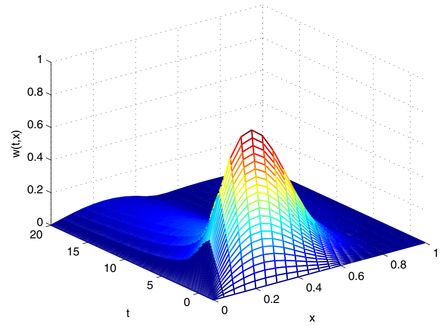

Fig. 2. The state of system (1) with two mobile actuators.

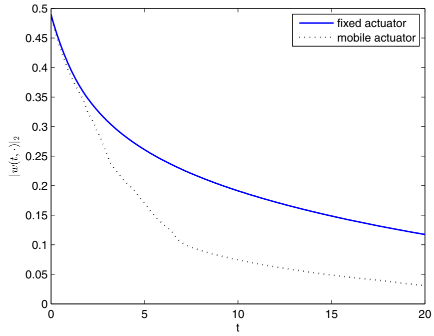

Fig. 3. Evolution of state L_2 norm.

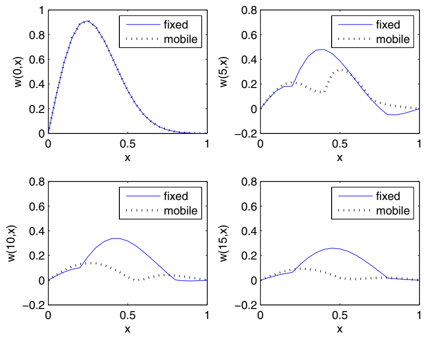

Fig. 4. Spatial distribution of $w(t,x)$ at $t = 0, 5, 10, 15$.

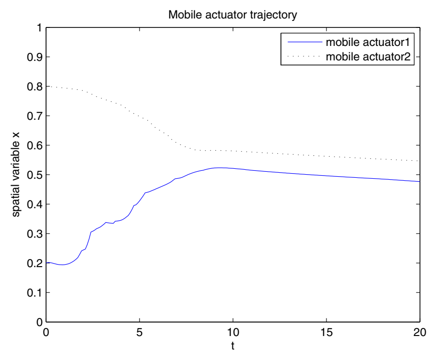

Fig. 5. Actuator trajectory.

Fig. 3 depicts the evolution of state $L_2(0,1)$ for the closed loop system with two fixed actuators and two mobile actuators. As we can see, the state L_2 norm for the mobile actuator-sensor networks converges to zero much faster than that for the fixed actuator-sensor networks, so the effects of the mobile actuator-sensor networks on controlling distributed parameter systems with time delay are encouraging. Fig. 4 depicts the spatial distribution of the closed loop state at four time instances. It is evident that the state at different time instances converges to zero much faster when the agents are allowed to move. Finally, the trajectory for the two mobile actuators is depicted in Fig. 5.

5. CONCLUSION

In this paper, the performance enhancement of distributed parameter systems with time-varying delay using mobile actuator-sensor networks has been presented. We have also provided an analytical expression for the control force of each mobile agent and the stabilization conditions for the closed loop systems with time-varying delays. The simulation results have revealed that mobile actuator-sensor networks could better enhance the performance on the process state.

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