Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Decentralized Detection with Censoring Sensors over a Packet-dropping Network

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Abstract: In a "censoring" or "send/no-send" approach to decentralized detection, sensors only transmit "informative" observations to the fusion center, which is able to significantly reduce energy consumption particularly when one hypothesis is more likely. The canonical "censoring" decentralized detection, however, assumes the communication channels between sensors and the fusion center are perfect, which is not quite realistic. We consider the problem of decentralized detection with censoring sensors over networks where packet dropout may occur. A sensor decides whether to use a high or low transmission power to communicate with the fusion center. With a general energy constraint, we prove that, to minimize the probability of error, the transmitting region of likelihood ratio associated with low power level is a single interval, and we derive necessary conditions for the lower and upper thresholds of this interval. For the special case that the available energy is sufficiently small, we show that the intervals have zero lower thresholds and can be determined independently for each sensor. A numerical example is provided to illustrate the main results.

1. INTRODUCTION

In a typical decentralized detection network, a set of spatially deployed sensors are used to collect information and transmit a summary of their observations, via wireless channels, to a fusion center, which determines the nature of a phenomenon. Due to the sensors' limited battery power and constrained bandwidth resource, only partial information is available at the fusion center, which results in degenerate detection performance compared with the centralized counterpart. The nice inherent properties of wireless sensor networks, such as low cost, flexility and robustness, still attracts considerable interests in performing decentralized detection.

Canonical decentralized detection problems assume that a sensor maps its local observations to different quantized levels; quantization effect and corresponding optimal local decision rules for sensors and fusion rules at the fusion center's side is studied. Viswanathan and Varshney [1997], Blum et al. [1997] and references therein provided a review of literatures on this topic. Recently, effect of unreliable wireless communication channels is studied. Chamberland and Veeravalli [2003] analyzed asymptotic detection performance with constrained capacity of wireless channels and showed that having identical binary decision sensors is asymptotically optimal as the number of observations per sensor goes to infinity, if sensors have i.i.d. exponential observations. Chamberland and Veeravalli [2004] studied the same problem but with total energy constraint and showed that when the sensors have i.i.d. observations, having identical sensors is asymptotically optimal.

Unlike the canonical decentralized detection, "censoring" decentralized detection assumes that sensors transmit "real-valued" instead of quantized summary of observations to the fusion center when local information is regarded as "informative". It is reasonable in the sense that the transmitted packet in most packet-based networks contains quite large space for the data and quantization effect can be neglected. The idea of censoring detection is first proposed in Rago et al. [1996] and lately studied in Appadwedula et al. [2002, 2005, 2008], Tay et al. [2007]. Rago et al. [1996] proved that with limited communication rate, to minimize the probability of error, transmission occurs only when the likelihood ratio of one sensor's observation does not fall in one certain single interval. It also showed that this interval has zero lower bound if the available communicate rate is severe. Appadwedula et al. [2002, 2005, 2008] considered the uncertainty in the distribution of the observations and studied censoring strategies for composite testing problems. Asymptotic performance of censoring strategies for both Neyman-Pearson and Bayesian formulations is studied in Tay et al. [2007].

Most existing censoring detection literatures assume reliable communication channels between the sensors and the fusion center, which is not quite realistic. Packet dropout

⋆ This work is supported by an HKUST Caltech Partnership FP004.

is common in wireless communications and can be caused by many factors, such as fading channels, interference and low signal-to-noise ratio (SNR). In this paper, it is assumed that packets may be dropped during transmissions. The sensors have limited energy and can choose to use high or low power level to communicate with the fusion center. We assume that high power level leads to higher packet arrival rate compared with low power level being used. This assumption is reasonable as modern wireless sensor nodes can choose its own transmission power levels (Dargie and Poellabauer [2010]), and higher power level means higher SNR, which leads to higher packet arrival rate.

The main contributions of this paper are summarized as follows:

(1) We consider censoring decentralized detection over packet-dropping networks with energy constraint of each sensor. To the best of our knowledge, this problem formulation is novel.
(2) We prove that, with energy constraint, the optimal transmitting region associated with low transmission power level is a single interval (Theorem 3.1). Two necessary conditions of the lower and upper thresholds of this optimal interval (Theorem 3.2) is given. We also prove that, if the available energy is sufficiently small, this optimal interval for each sensor has zero lower threshold (Proposition 3.4).

The remainder of this paper is organized as follows. In Section 2, the mathematical model of the considered problem is given. After showing the main results in Section 3, a numerical example is provided in Section 4. Some concluding remarks are provided in the end.

2. PROBLEM SETUP

We are concerned with binary hypothesis testing problem, i.e., H_0 (null) and H_1 (target present), with parallel sensor network topology, as depicted in Fig. 1. Let x_i, $i = 1, 2, \dots, N$ denote the ith sensor's observation. Each sensor node evaluates a local output $g_i(x_i)$ based on its observation and sends the corresponding value to a fusion center. The fusion center will make a final decision ϕ_0 about the state, combining the data collected from all the sensors and its own observation x_0.

Quantization effect is mainly studied in the canonical distributed detection setting, where g_i maps x_i to one of the D_i levels. A special case is that a sensor node just sends its local decision 0 or 1 to the fusion center. On the contrary, for the "send/no-send" censoring detection problem, a sensor node sends the real-valued function $g_i(x_i)$ (likelihood ratio in most cases) to the fusion center when the observation x_i is considered as "informative", otherwise no transmission occurs and the observation is discarded.

In the spirit of censoring detection, we introduce packet dropout in the communication channel between the sensors and the fusion center. Specifically, sensor i has two communication power levels: Δ_i and δ_i. If Δ_i is used for transmission, the arrival rate is $\lambda_{i,1}$, while the arrival rate is $\lambda_{i,2}$ $(0 \le \lambda_{i,2} < \lambda_{i,1} \le 1)$ when δ_i is used. In this case, we define the sensor decision rule as

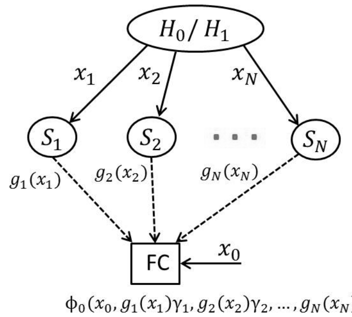

Fig. 1. Topology of censoring decentralized detection with packet dropout

$$I_i(x_i) = \begin{cases} 1, \text{ if } l(x_i) \in R_i, \\ 0, \text{ if } l(x_i) \in R_i^c, \end{cases} \tag{1}$$

where $l(x_i) = \frac{p_1^i(x_i)}{p_0^i(x_i)}$ is the likelihood ratio and $p_j^i(\cdot), i = 1, 2, \dots, N$, $j = 0, 1$, denotes the conditional probability density function (pdf) of the observation for sensor i under the hypothesis j. Sensor i will send the likelihood ratio $l(x_i)$ to the fusion center using power Δ_i if $I_i(x_i)$ takes 1, and δ_i if $I_i(x_i)$ equals zero. Intuitively, in order to achieve a better performance, "informative" data should be transmitted with higher power level. To this end, the set R_i should include likelihood ratio value $l(x_i)$ that can provide more confidence ($l(x_i)$ is too small or too large) to the decision maker for choosing one hypothesis.

As every sensor node has its own battery, it is natural to consider average energy constraint of each sensor independently as

$$\begin{aligned} &\pi_0 \left[\Pr(I_i(x_i) = 1|H_0)\Delta_i + \Pr(I_i(x_i) = 0|H_0)\delta_i\right] \\ &\quad + \pi_1 \left[\Pr(I_i(x_i) = 1|H_1)\Delta_i + \Pr(I_i(x_i) = 0|H_1)\delta_i\right] \le \epsilon_i, \end{aligned}$$

where π_0 and π_1 are the *a priori* probability of the hypothesis H_0 and H_1, respectively; $\Pr(\cdot|\cdot)$ represents the conditional probability and ϵ_i denotes the average energy per transmit available for sensor i. Censoring detection performs well particularly when one of the hypotheses is much more likely compared with the other one. In this paper, we focus on the case that hypothesis H_0 is more probable, i.e., $\pi_0 \gg \pi_1$. It is the case in many applications, such as, fire alarm in a forest or occurrence of anomalies in a friendly environment. Thus, the above energy constraint can be simplified as

$$\Pr(I_i(x_i) = 1|H_0)\Delta_i + \Pr(I_i(x_i) = 0|H_0)\delta_i \le \epsilon_i. \tag{2}$$

We make the following two assumptions.

Assumption 2.1. (Conditional Independence). Observations taken by the sensors and the fusion center are mutually statistically independent, and are also statistically independent taken at different times, under each hypothesis.

Assumption 2.2. (No Point Mass). For all sensors and the fusion center, the distribution function of the likelihood ratio $l(x_i) \in (0, \infty], i = 0, 1, 2, \dots, N$, is continuous and has continuous derivative conditioned on each hypothesis.

Remark 2.3. The above two assumptions are common and standard in decentralized detection formulations and can be found in most literatures, Rago et al. [1996], Viswanathan and Varshney [1997], Appadwedula et al. [2008], to list a few. If the observations are dependent, complexity of the decentralized detection problem will make it less tractable Viswanathan and Varshney [1997], Tsitsiklis and Athans [1985]. The no point mass assumption is technical and can be satisfied for most practical signals.

As there exists packet dropout between the sensors and the fusion center, the fusion center cannot obtain the complete knowledge of the sensors' observations. When a packet is dropped, the fusion center has to estimate the likelihood ratio and get an imprecise one. Let γ_i be the indicator function that takes 1 if the fusion center successfully receives the data from sensor i, and equals zero otherwise. Note that the observations of fusion center are always available, i.e., $\gamma_0 = 1$. We define the censored likelihood at the fusion center as

$$l_{FC}(x_i) = \begin{cases} l(x_i), \text{ if } \gamma_i = 1, \\ \rho_i, \text{ if } \gamma_i = 0, \end{cases} \tag{3}$$

where ρ_i is given by

$$\rho_i = \frac{\Pr(\gamma_i = 0|H_1)}{\Pr(\gamma_i = 0|H_0)}. \tag{4}$$

Note that, unlike the censoring one, if an offline transmission strategy is adopted, nothing can be obtained when a packet is dropped. Another reason why censoring strategy outperforms the offline one is that in a censoring approach, as in the preceding analysis, compared with the counterpart, more "informative" observations are sent using the high power level and hence arrive at the fusion center successfully with a higher probability.

Let

$$P_e = \pi_1 \Pr(\text{decide} H_0|H_1) + \pi_0 \Pr(\text{decide} H_1|H_0) \tag{5}$$

be the probability of detection error. Given the information available at the fusion center, either precise likelihood ratios or estimates ρ_i, the optimal fusion rule that minimizes P_e is a likelihood ratio test. Specifically, whatever the high power level region R_i in (1) is, the optimal fusion rule under the constraint (2) is given by

$$\phi_0 = \begin{cases} 1, \prod_{i=0}^{N} l_{FC}(x_i) \geq \tau, \\ 0, \text{ otherwise}, \end{cases} \tag{6}$$

where $\tau = \frac{\pi_0}{\pi_1}$. This is a quite straightforward conclusion based on the centralized detection theory (Poor [1988]).

Let R_{++} denote the set of positive real numbers. The problem we are facing now is, in order to minimize P_e, how to partition R_{++} into R_i and R_i^c for each sensor.

3. MAIN RESULTS

In this section, we will prove that the optimal transmitting region associated with δ_i is a single interval and show some properties of this interval under different conditions.

Theorem 3.1. With the energy constraint (2), local sensor decision rule (1) and assumptions 2.1 and 2.2, the optimal censoring region R_i^c in (1) that minimizes P_e is a single interval.

Proof: The proof is very long and similar to that of Theorem 1 in Rago et al. [1996], although our problem is quite different. The key idea is to show that for a generic R_i^c that may consist of several intervals, we can always find another single interval R_i^c that has no larger P_e and satisfies the energy constraint at the same time. This proof mainly consists of three steps.

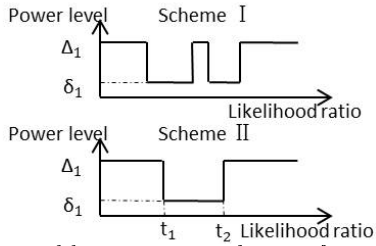

Fig. 2. Two possible censoring schemes for sensor 1

(1) Construct two proper schemes. This part is similar to that in Rago et al. [1996].
(2) Obtain the difference between the two schemes' detection performance Δ_{P_e}.
(3) Determine the sign of the difference. It will be shown that the difference for our problem can be reduced to a multiple of that in Rago et al. [1996].

Without loss of generality, we focus on sensor 1 and R_1^c. We consider the well defined probability space $(\Omega, \mathcal{A}, \mathcal{P})$, where Ω is the sample space, $\mathcal{A}$ is a $\sigma-$algebra and $\mathcal{P}$ is the associated probability measure. As R_1^c is a set in $\mathcal{A}$, according to measure theory, it must be a countably many union of disjoint intervals. For simplicity, as depicted in Fig. 2, $R_1^{c^I}$ for scheme I consists of two intervals, while scheme II has a single interval $R_1^{c^{II}} = [t_{1,1}, t_{1,2}]$. Note that censoring regions of the other sensors for the two schemes are identical. For convenience, we will write $t_{1,1}, t_{1,2}, \lambda_{1,1}, \lambda_{1,2}, p_0^1(\cdot), p_1^1(\cdot)$ and $l(x_1)$ as $t_1, t_2, \lambda_1, \lambda_2, p_0(\cdot), p_1(\cdot)$ and l_1, respectively. To make the average energy consumption in the two schemes remain unchanged, $R_1^{c^{II}} = [t_1, t_2]$ must satisfy the following two equations:

$$\int_{R_1^{c^{II}}} p_0(l_1) dl_1 = \int_{R_1^{c^I}} p_0(l_1) dl_1, \tag{7}$$

$$\int_{R_1^{c^{II}}} p_1(l_1) dl_1 = \int_{R_1^{c^I}} p_1(l_1) dl_1. \tag{8}$$

Note that for any $R_1^{c^I}$, under Assumption 2.2, there always exists such a unique $R_1^{c^{II}}$. The estimate of dropping likelihood ratio is given by

$$\rho_1^j = \frac{(1-\lambda_1) + (\lambda_1 - \lambda_2) \int_{R_1^{c^j}} p_1(l_1) dl_1}{(1-\lambda_1) + (\lambda_1 - \lambda_2) \int_{R_1^{c^j}} p_0(l_1) dl_1}, \tag{9}$$

where $j = I, II$ corresponds to scheme I and II, respectively. One notes that ρ_1^j for the two schemes are also the same, hence we will use the notation ρ_1 in the remainder of the proof.

Let $\boldsymbol{x}_1 = [x_0, x_2, \ldots, x_N]$ and $u = l_{FC}(\boldsymbol{x}_1)$. As the two schemes differs only in the censoring region of sensor 1, u is identical. Under conditional independence assumption, the probability of error associated with each hypothesis can be obtained as

$$P_e^j = \pi_1 \left[1 - \int dF_1(u) \int_{l_1 : l_{FC}^j(x_1) = \tau/u}^{\infty} p_0(l_1)(l_1 - \frac{\tau}{u}) dl_1 \right], \tag{10}$$

where $F_i(\cdot), i = 0, 1,$ denotes the distribution function conditioned on hypothesis i.

From (10), the difference between the probability of error for the two schemes is given by

$$\begin{aligned}\Delta_{P_e} &= P_e^{II} - P_e^{I}\\ &= \pi_1 \int_0^\infty dF_1(u) \Bigg\{ \lambda_1 \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^I} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad - \lambda_1 \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^{II}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad + \lambda_2 \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^{c^I}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad - \lambda_2 \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^{c^{II}}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1 \Bigg\}\\ &+ \pi_1 \int_{\frac{\tau}{\rho_1}}^\infty dF_1(u) \Bigg\{ (1-\lambda_1) \int_{R_1^I} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad - (1-\lambda_1) \int_{R_1^{II}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad + (1-\lambda_2) \int_{R_1^{c^I}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\quad - (1-\lambda_2) \int_{R_1^{c^{II}}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1 \Bigg\}. \quad (11)\end{aligned}$$

Since

$$\int_{R_1^{c^I}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1 = \int_{R_1^{c^{II}}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1,$$

we can rewrite (11) as

$$\begin{aligned}\frac{\Delta_{P_e}}{\lambda_1-\lambda_2} &= \pi_1 \int_0^\infty dF_1(u)\\ &\quad * \Bigg\{ \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^I} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\qquad - \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^{II}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1 \Bigg\}\\ &= \pi_1 \int_0^\infty dF_1(u)\\ &\quad * \Bigg\{ \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^I \bigcap R_1^{c^{II}}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1\\ &\qquad - \int_{l_1:(l_1>\frac{\tau}{u})\bigcap R_1^{c^I} \bigcap R_1^{II}} p_0(l_1)(l_1-\frac{\tau}{u})dl_1 \Bigg\}. \quad (12)\end{aligned}$$

It is proved in Rago et al. [1996] that the RHS of (12) is non-positive. Using the fact that $\lambda_1 > \lambda_2$, we have $\Delta_{P_e} \le 0$, and the theorem is proved. ■

The single interval censoring region can significantly facilitate the local processing of sensors, especially when the likelihood ratio is monotone in observations. In general, determining the optimal lower and upper thresholds of these intervals for all the sensors is a joint optimization involving $2N$ variables, the computation load of which may be an issue. To alleviate this, we derive two necessary conditions of the lower and upper thresholds for each sensor independently.

Theorem 3.2. With the energy constraint (2), local sensor decision rule (1) and assumptions 2.1 and 2.2, the optimal censoring region $R_i^c = [t_{i,1}, t_{i,2}]$ in (1) that minimizes P_e must satisfy the following two constraints:

(1) $t_{i,1} \le \rho_i \le t_{i,2}$,
(2) $\int_{t_{i,1}}^{t_{i,2}} p_0^i(l(x_i))dl(x_i) = \frac{\Delta_i - \epsilon_i}{\Delta_i - \delta_i}$.

Proof: Without loss of generality, we focus on sensor 1 and prove the following two statements using the same notation as the ones used in the proof of Theorem 3.1:

(1) $t_1 \le \rho_1 \le t_2$,
(2) $\int_{t_1}^{t_2} p_0(l_1)dl_1 = \frac{\Delta_1-\epsilon_1}{\Delta_1-\delta_1}$.

First, we prove that $t_1 \le \rho_1 \le t_2$ holds if the energy constraint has the form of

$$\Pr(l_1 \in R_1|H_0)\Delta_1 + \Pr(l_1 \in R_1^c|H_0)\delta_1 = \epsilon_1', \quad (13)$$

where $\delta_1 \le \epsilon_1' \le \Delta_1$. The probability of error is given by

$$P_e = \pi_1 \left[1 - \int_{\Omega_1} (l_{FC} - \tau) dF_0(l_{FC})\right], \quad (14)$$

where Ω_1 is the decision region for hypothesis H_1, which is defined by $\{u, l_{FC}(x_1) | u l_{FC}(x_1) \ge \tau\}$. It is appropriate, according to Theorem 3.1, to let $R_1^c = [t_1, t_2]$. Under Assumption 2.1, (14) can be rewritten as

$$\begin{aligned}\frac{P_e}{\pi_1} &= 1 - \lambda_1 \int_0^{t_1} p_0(l_1) \int_{\tau/l_1}^\infty (l_1 u - \tau) dF_0(u) dl_1\\ &- (1-\lambda_1) \int_0^{t_1} p_0(l_1)dl_1 \int_{\tau/\rho_1}^\infty (\rho_1 u - \tau) dF_0(u)\\ &- \lambda_1 \int_{t_2}^\infty p_0(l_1) \int_{\tau/l_1}^\infty (l_1 u - \tau) dF_0(u) dl_1\\ &- (1-\lambda_1) \int_{t_2}^\infty p_0(l_1)dl_1 \int_{\tau/\rho_1}^\infty (\rho_1 u - \tau) dF_0(u)\\ &- \lambda_2 \int_{t_1}^{t_2} p_0(l_1) \int_{\tau/l_1}^\infty (l_1 u - \tau) dF_0(u) dl_1\\ &- (1-\lambda_2) \int_{t_1}^{t_2} p_0(l_1)dl_1 \int_{\tau/\rho_1}^\infty (\rho_1 u - \tau) dF_0(u). \quad (15)\end{aligned}$$

Now we will get the derivative of $\frac{P_e}{\pi_1}$ with respect to t_1, in finding which the derivative of the two dependent variables t_2 and ρ_1 must be considered. As $R_1^c = [t_1, t_2]$, one can rewrite (13) as

$$\left(1 - \int_{t_1}^{t_2} p_0(l_1)dl_1\right)\Delta_1 + \int_{t_1}^{t_2} p_0(l_1)dl_1 \delta_1 = \epsilon_1'. \quad (16)$$

Taking the derivative of (16) w.r.t t_1,

$$\frac{\partial t_2}{\partial t_1} = \frac{p_0(t_1)}{p_0(t_2)}. \quad (17)$$

Substituting $R_1^c = [t_1, t_2]$ into (9),

$$\rho_1 = \frac{(1-\lambda_1) + (\lambda_1-\lambda_2)\int_{t_1}^{t_2} l_1 p_0(l_1)dl_1}{(1-\lambda_1) + (\lambda_1-\lambda_2)\int_{t_1}^{t_2} p_0(l_1)dl_1}. \quad (18)$$

Taking the derivative of the above equation w.r.t t_1,

$$\frac{\partial \rho_1}{\partial t_1} = \frac{(\lambda_1-\lambda_2)(t_2-t_1)p_0(t_1)}{(1-\lambda_1) + (\lambda_1-\lambda_2)\int_{t_1}^{t_2} p_0(l_1)dl_1}. \quad (19)$$

As observations of the fusion center are not censored, under the Assumption 2.2, $F_0(u)$ is continuous and has continuous derivative w.r.t. u. Then applying the fundamental theorem of calculus, one has

$$\frac{\partial P_e}{\partial t_1} = \pi_1(\lambda_1 - \lambda_2)p_0(t_1) * \Upsilon,$$
$$\Upsilon \equiv \int_{\tau/t_2}^{\tau/t_1} (t_2 u - \tau) dF_0(u) + (t_2 - t_1) \int_{\tau/t_1}^{\infty} u dF_0(u) - (t_2 - t_1) \int_{\tau/\rho_1}^{\infty} u dF_0(u). \tag{20}$$

If $t_1 > \rho_1$, obviously $\Upsilon > 0$, i.e., $\frac{\partial P_e}{\partial t_1} \geq 0$. Hence to minimize P_e, t_1 must decrease until $t_1 \leq \rho_1$. We can rewrite (20) as

$$\begin{aligned}\Upsilon &= \int_{\tau/t_2}^{\tau/t_1} (t_2 u - \tau) dF_0(u) - (t_2 - t_1) \int_{\tau/\rho_1}^{\tau/t_1} u dF_0(u) \\ &\leq \int_{\tau/t_2}^{\tau/t_1} (t_2 - t_1) u dF_0(u) - (t_2 - t_1) \int_{\tau/\rho_1}^{\tau/t_1} u dF_0(u).\end{aligned} \tag{21}$$

If $t_2 < \rho_1$, from (21), we know $\Upsilon < 0$. Hence to minimize P_e, t_1 must increase, so does t_2. Therefore one has $t_2 \geq \rho_1$.

Now we prove the second statement: $\int_{t_1}^{t_2} p_0(l_1) dl_1 = \frac{\Delta_1 - \epsilon_1}{\Delta_1 - \delta_1}$. Note that the energy constraint (2) can be rewritten as $\int_{t_1}^{t_2} p_0(l_1) dl_1 \leq \frac{\Delta_1 - \epsilon_1}{\Delta_1 - \delta_1}$. We need to prove that, to minimize P_e, the available energy should be used up. It suffices to prove that for a fixed t_1, $\frac{\partial P_e}{\partial t_2}$ is positive for all possible t_2. If t_1 is fixed, taking derivative of (18) w.r.t. t_2,

$$\frac{\partial \rho_1}{\partial t_2} = \frac{(\lambda_1 - \lambda_2) p_0(t_2)(t_2 - \rho_1)}{(1 - \lambda_1) + (\lambda_1 - \lambda_2) \int_{t_1}^{t_2} p_0(l_1) dl_1} \tag{22}$$

After taking derivative of (15), one can obtain

$$\frac{\partial P_e}{\partial t_2} = p_0(t_2) \left(\lambda_1 \Upsilon_1 + \lambda_2 \Upsilon_2\right),$$
$$\Upsilon_1 \equiv \int_{\tau/t_2}^{\tau/\rho_1} t_2 u dF_0(u) + \int_{\tau/\rho_1}^{\infty} \tau dF_0(u) - \int_{\tau/t_2}^{\infty} \tau dF_0(u),$$
$$\Upsilon_2 \equiv \int_{\tau/t_1}^{\infty} \tau dF_0(u) + \int_{\tau/t_1}^{\tau/\rho_1} t_2 u dF_0(u) - \int_{\tau/\rho_1}^{\infty} \tau dF_0(u) + \int_{\tau/t_1}^{\infty} (t_2 - t_1) u dF_0(u). \tag{23}$$

As $t_2 \geq \rho_1$, $t_2 u \geq \tau$, if $u \in [\frac{\tau}{t_2}, \frac{\tau}{\rho_1}]$. Hence one can get

$$\Upsilon_1 > \int_{\tau/t_2}^{\tau/\rho_1} \tau dF_0(u) + \int_{\tau/\rho_1}^{\infty} \tau dF_0(u) - \int_{\tau/t_2}^{\infty} \tau dF_0(u) = 0.$$

Similarly,

$$\Upsilon_2 > \int_{\tau/t_1}^{\infty} \tau dF_0(u) + \int_{\tau/t_1}^{\tau/\rho_1} \tau dF_0(u) - \int_{\tau/\rho_1}^{\infty} \tau dF_0(u) = 0.$$

Finally, we get $\frac{\partial P_e}{\partial t_2} > 0$ for any possible t_1 and t_2. The proof is thus complete. ∎

Remark 3.3. Note that there always exists $R_i^c = [t_{i,1}, t_{i,2}]$ such that $t_{i,1} \leq \rho_i \leq t_{i,2}$, for example, $t_{i,1} \leq \rho_i \leq t_{i,2}$ holds for all $t_{i,1} < 1$ and $t_{i,2} > 1$. The second constraint means that better performance can be expected at the cost of more energy consumption, which agrees with our intuition.

Generally, $t_{i,1}$ and $t_{i,2}$ can be obtained only using numerical methods, however, it will be shown later that under some conditions (see Proposition 3.4), the optimal censoring region can be determined for each sensor independently.

Proposition 3.4. With assumptions 2.1 and 2.2 and local sensor decision rule (1), if the available energy ϵ_i in constraint (2) is sufficiently small, the optimal censoring region R_i^c in (1) that minimizes P_e can be uniquely determined as

$$t_{i,1} = 0, \tag{24}$$
$$\int_0^{t_{i,2}} p_0^i(l(x_i)) dl(x_i) = \frac{\Delta_i - \epsilon_i}{\Delta_i - \delta_i}. \tag{25}$$

Proof: Note that it suffices to prove zero lower bounds (24), as (25) is then straightforward according to Theorem 3.2. Without loss of generality, we focus on sensor 1 and prove $t_1 = 0$. To this end, we need to prove $\frac{\partial P_e}{\partial t_1} > 0$, i.e., Υ in (20) is positive for any $t_1 > 0$.

Given a fixed t_1, $\frac{\partial P_e}{\partial t_2}$ is given in (22). According to Theorem 3.2, $t_2 > \rho_1$, hence $\frac{\partial P_e}{\partial t_2} > 0$ for any possible t_2, and one can get

$$\rho_1 \leq \frac{(1 - \lambda_1) + (\lambda_1 - \lambda_2) \int_{t_1}^{\infty} l_1 p_0(l_1) dl_1}{(1 - \lambda_1) + (\lambda_1 - \lambda_2) \int_{t_1}^{\infty} p_0(l_1) dl_1} \equiv \bar{\rho_1}. \tag{26}$$

Thus,

$$\begin{aligned}\Upsilon &= \int_{\tau/t_2}^{\tau/\rho_1} (t_2 u - \tau) dF_0(u) - \int_{\tau/\rho_1}^{\tau/t_1} (\tau - t_1 u) dF_0(u) \\ &\geq \int_{\tau/t_2}^{\tau/\bar{\rho_1}} (t_2 u - \tau) dF_0(u) - \int_{\tau/\bar{\rho_1}}^{\tau/t_1} (\tau - t_1 u) dF_0(u).\end{aligned} \tag{27}$$

As $\bar{\rho_1}$ is independent of t_2, (27) is monotonically increasing w.r.t. t_2. Hence, for any t_1, one can find a sufficiently large $t_2 \geq t_2^\star$ (available energy is sufficiently small) such that $\Upsilon > 0$, i.e., $\frac{\partial P_e}{\partial t_1} > 0$. Hence zero lower bound is proved. Note that $t_2^\star$ is larger and close to $\frac{\int_0^{\tau/t_1} \tau dF_0(u) + t_1 \int_{\tau/\bar{\rho_1}}^{\tau/t_1} u dF_0(u)}{\int_0^{\tau/\bar{\rho_1}} u dF_0(u)}$. ∎

Remark 3.5. The main results, Theorem 2 and Proposition 1, in Rago et al. [1996] are special cases of our theorems with $\lambda_{i,1} = 0, \lambda_{i,2} = 1$ and $\delta_i = 0$.

4. SIMULATION RESULTS

In this section, by a numerical example, we show the benefits of using censoring strategies and illustrate the idea of Proposition 3.4. Consider the problem of detecting a mean-shift in Gaussian noises with identical sensors as

$$H_0 : x_i \sim \mathcal{N}(0, 1) \qquad \text{versus} \qquad H_1 : x_i \sim \mathcal{N}(1, 1).$$

The observations of the fusion center and the two sensors are i.i.d distributed.

Let $\pi_0 = 0.9, \lambda_1 = 0.95$ and $r = \frac{\Delta - \epsilon}{\Delta - \delta}$. Performance of two scenarios: $\lambda_2 = 0.3, N = 3$ and $\lambda_2 = 0.5, N = 2$ is simulated. From Fig. 3, we observe that as the available energy increases (i.e., r decreases), the probability P_e decreases, which agrees with our intuition. Note that when the available energy is large enough ($r < 0.6$ for the first scenario and $r < 0.8$ for the second one), P_e almost remains unchanged as r increases. This can be

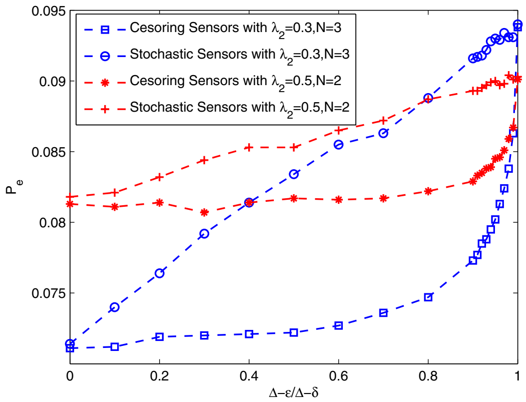

Fig. 3. Error probability as a function of energy ratio $\frac{\Delta-\epsilon}{\Delta-\delta}$ for senors using censoring or stochastic transmitting strategies.

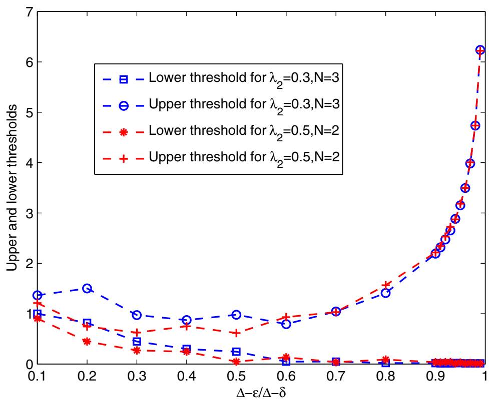

Fig. 4. Lower and upper thresholds as functions of energy ratio $\frac{\Delta-\epsilon}{\Delta-\delta}$.

verified from (18) and (23). When the available energy is sufficiently large, both the lower and upper thresholds approach to 1; from (18) we know that the estimate of dropped likelihood ρ is close to 1 too. $\frac{\partial P_e}{\partial t_2}$ in (23) becomes close to zero when t_1, t_2 and ρ are close to each other.

To show the benefits of adopting censoring strategies, we plot the P_e for the sensors which communicate with the fusion center with a randomly chosen power level. Of course, the sensors must satisfy the same energy constraint, and from (4), we know the estimates of dropping likelihood ratios are all 1. It can be seen that P_e of "stochastic sensors" is much larger than that of "censoring sensors" and is almost linear with r. In the first scenario, P_e "censoring sensors" has 16% decrease compared with "stochastic sensors" at best, while for the second one, the best decrease is 7%. This is because the more number of sensors and bigger difference between λ_1 and λ_2, compared with "stochastic sensors", the greater performance improvement can be obtained by using the censoring strategy.

The lower threshold t_1 as a function of r is shown in Fig. 4. It can be observed that when the available energy is small enough ($r > 0.9$), the lower threshold is zero and lower threshold is quite small when $r > 0.6$.

5. CONCLUSION

Censoring decentralized detection over packet-dropping networks is considered in this paper. The result that the transmitting region associated with low power level is a single interval facilitates local processing of sensors, especially in the case when the likelihood ratio is monotone in the observations. Two necessary conditions about lower and upper thresholds of this interval that minimize the probability of error are derived, which is able to significantly reduce computation load of numerically searching for the optimal censoring regions of the overall system. When the available energy is sufficiently small, the joint optimization problem is decoupled as the lower thresholds are zero and the upper thresholds can be determined independently for each sensor. Future work includes finding the optimal transmitting strategy when sensors have continuous power levels and associated packet arrival rates.

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