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The air-breathing hypersonic vehicle adaptive backstepping control design based on the dynamic surface

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Abstract: Since a linear dynamic system parameter uncertainties and un-modeled dynamics in Air-breathing hypersonic vehicle (ABHV), extended it to a nonlinear dynamics system including the linear parameter uncertain term in control matrix. An adaptive tuning function is used to compensate uncertainty impact for system and the robust term is designed to solve the issue of approximation error. The projection operator function is introduced to avoid possible controller singularity problem. In adaptive inverse design progress, a dynamic surface method is used and the first order filter is introduced. The Lyapunov stability theorem guaranteed error is uniformly bounded. The simulation shows the effectiveness of the algorithm.

Keywords: air-breathing hypersonic vehicle, adaptive, backstepping, uncertainty

1. INTRODUCTION

Air-breathing hypersonic vehicle (ABHV) generally refers to aircraft which scramjet powered and flight within the atmosphere with greater than five times the speed of sound. ABHV are intended to be a reliable and cost-effective technology for access to space(Wu Hongxin 2009, Fidan B 2003). In addition, the ABHV can also effectively improve survival ability because it can make the air defence system difficult to intercept.(Chavez, F 1994). Because ABHV adopts a scramjet engine, airframe/propulsion integration design technology, so that there is coupling interference between the propulsion system and the aerodynamic surfaces(Bilimoria, K 1995). And external disturbances and unknown factors in the process of its flight is very significant, which caused the ABHV has a complex and variable aerodynamic characteristics(Chavez, F 1999). ABHV are more sensitive to angle of attack, flight attitude and dynamic pressure, and all these factors had to bring great challenges to design its control system.

These classes of uncertain nonlinear systems are suitable for using backstepping method for the design of the controller. Through the design of the virtual control, the non matching uncertainties of system can be good compensated, to ensure the stability and reliability of the control system(Taeyoung L 2001). The backstepping design method can deal with non matched uncertainties. The Backstepping method has been widely applied in various engineering fields, and it is also a common method for aircraft control system design. Literature (Kojic 1999)using backstepping solves the time-scale separation hypothesis problem which usually faced in feedback linearization theory applied in aircraft control system, which can reflect more real aircraft dynamic. Literature (Krstic M 1992)designed adaptive backstepping flight control system for fighter aircraft, which used neural network to compensate system error caused by some uncertainty factors such as aerodynamic parameter error, to keep the plane flight steadily. Literature(Li Y H 2004)used adaptive backstepping method to design control system for re-entry hypersonic vehicle. In view of the “term expansion” problem in backstepping design progress, the literature (Swaroop D 1997) roposed dynamic surface control (DSC) method firstly.

This article adopts the dynamic surface method to design the controller of ABHV. The numerical simulation proved that through the used of regulating function, projection operator and continuous robust function, the dynamic surface adaptive inverse control could compensate various uncertainties including linear parametric part and un-modeled dynamics which could meet during the flight progress effectively.

2. VEHICLE MODELLING

The NASA Langley Research Center developed hypersonic vehicle Winged-Cone is the main object of hypersonic vehicle research at present, and the full set of aerodynamic parameters have been published.

When ABHV Winged-Cone flight in near space in cruise status, its uncertain model is expressed as follows:

$$\dot{x}_1 = f_{10}(x_1) + \varphi_1(x_1)\psi + g_1(x_1)x_2 + w_1(x_1)u + \Delta_1(x_1,t) \quad (1)$$

$$\begin{aligned}\dot{x}_2 &= f_{20}(x_1,x_2) + \varphi_2(x_1,x_2)\psi + \\ &\left[g_{20}(x_1) + \phi(x_1)\theta\right]u + \Delta_2(x_1,x_2,t)\end{aligned} \quad (2)$$

$$y = x_1 \quad (3)$$

Where $x_1 = [\alpha \quad \beta \quad \sigma]^T$ are angle of attack, sideslip angle and bank angle, respectively; $x_2 = [p \quad q \quad r]^T$ are roll rate, pitch rate and yaw rate, respectively; $u = [\delta_e \quad \delta_a \quad \delta_r]^T$ are control input; $y = x_1 = [\alpha \quad \beta \quad \sigma]^T$ are control output. Assume that the state of the system variables and the output can be measured. $f_{10}(x_1)$, $f_{20}(x_1,x_2)$, $g_{20}(x_1)$ are nominal part of nonlinear function $f_1(x_1)$, $f_2(x_1,x_2)$,and $g_2(x_1)$; $\varphi_1(x_1)\psi$, $\varphi_2(x_1,x_2)\psi$ and $\phi(x_1)\theta$ are parametric linear part of nonlinear function $f_1(x_1)$, $f_2(x_1,x_2)$,and $g_2(x_1)$,which as the main representative of matched uncertainties caused by the impact of changes in aerodynamic parameters; ψ,θ is the unknown parameter vector; The mismatched uncertainties which is caused by the body elastic deformation in hypersonic flight condition is represented by $\Delta_1(x_1,t)$ and $\Delta_2(x_1,x_2,t)$ .The remaining parameters refer to references(Shaughnessy J 1990) and(Shahriar K 2005), and function expressions refer to reference(Shahriar K 2005).

Put forward the following hypothesis:

Assumption1: Ignore the influence caused by rudder deflection to the aerodynamic force, that is $w_1(x_1) = 0$. Assumption2: $g_{20}(x_1)$ is bounded and invertible; Assumption3(Zhou Li 2008): $g_1(x_1)$ is non-singular and norm bounded, that is exists a positive constant g_{1h} and g_{1l} , $g_{1l} \le \|g_1(x_1)\| \le g_{1h}$.
Assumption 4(Khalil H K 1996): There is an unknown positive constant p_i and known non-negative smooth function $\delta_i(x,t)$, $\|\Delta_i(x,t)\| \le p_i\delta_i(x,t), i = 1,2$ .Where $\|\bullet\|$ represents Euclid norm for a vector or 2-norm for a matrix.

3 ADAPTIVE BACKSTEPPING DYNAMIC SURFACE CONTROL SYSTM DESIGN AND STABILITY ANALYSIS

Define the error surface: $z_1 = x_1 - x_{1d}$, $z_2 = x_2 - x_{2d}$, where $x_{1d} = [\alpha_d \quad \beta_d \quad \sigma_d]^T$ is desired command of system. Differentiating z_1 with respect to time, we can get:

$$\dot{z}_1 = f_{10}(x_1) + \varphi_1(x_1)\psi + g_1(x_1)x_2 + \Delta_1(x_1,t) - \dot{x}_{1d} \quad (4)$$

Defined $\hat{\psi}$, $\hat{\theta}$, $\hat{p}_i$ are the estimated value of parameters ψ , θ , p_i , respectively. So, we can define the parameter error as follows: $\tilde{\psi} = \psi - \hat{\psi}$, $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{p}_i = p_i - \hat{p}_i$.

Designing the virtual controller $\bar{x}_{2d}$ as follows:

$$\bar{x}_{2d} = \frac{1}{g_1(x_1)}\left[-f_{10}(x_1) - \varphi_1(x_1)\hat{\psi} - k_1 z_1 - v_1 + \dot{x}_{1d}\right], k_1 > 0 \quad (5)$$

Where v_1 is robust item. Substituting equation (5) into the equation (4) yield:

$$\dot{z}_1 = -k_1 z_1 + \varphi_1(x_1)\tilde{\psi} + g_1(x_1)(x_2 - \bar{x}_{2d}) + \Delta_1(x_1,t) - v_1 \quad (6)$$

Let us consider the following Lyapunov candidate function:

$$V_1 = \frac{1}{2}z_1^T z_1 + \frac{1}{2}\tilde{\psi}^T \Gamma_\psi \tilde{\psi} + \frac{1}{2r_1}\tilde{p}_1^2 \quad (7)$$

Where Γ_1 is positive definite symmetric matrix, and $r_1 > 0$ which should be designed.

Differentiating V_1 with respect to time, we can get:

$$\begin{aligned}\dot{V}_1 &= -k_1\|z_1\|^2 + z_1^T\varphi_1(x_1)\tilde{\psi} + z_1^T g_1(x_1)(x_2 - \bar{x}_{2d}) \\ &+ z_1^T\left[\Delta_1(x_1,t) - v_1\right] - \tilde{\psi}^T\Gamma_1\dot{\hat{\psi}} - \frac{1}{r_1}\tilde{p}_1\dot{\hat{p}}_1\end{aligned} \quad (8)$$

If the control can not guarantee that $\hat{\psi}$, $\hat{\theta}$ maintained at a given bounded closed set, the parameter drift and control singularity problem will occur. In order to avoid this kind of phenomenon, here we use the projection operator developed in literature(Khalil H K 1996). Let the parameter vector constraint region as follows:

$$\begin{aligned}\Pi_{\psi_0} &\triangleq \left\{\hat{\psi} \,\middle|\, \|\hat{\psi}^T\hat{\psi}\|^2 \le \beta_\psi\right\} \\ \Pi_\psi &\triangleq \left\{\hat{\psi} \,\middle|\, \|\hat{\psi}^T\hat{\psi}_n\|^2 \le \beta_\psi + \delta_\psi\right\}\end{aligned} \quad (9)$$

$$\Pi_{\theta_0} \triangleq \left\{ \hat{\theta} \middle| \left\| \hat{\theta}^T \hat{\theta} \right\|^2 \leq \beta_\theta \right\}$$
$$\Pi_\theta \triangleq \left\{ \hat{\theta} \middle| \left\| \hat{\theta}^T \hat{\theta}_n \right\|^2 \leq \beta_\theta + \delta_\theta \right\} \quad (10)$$

In above equation, $\beta_\psi > 0$, $\delta_\psi > 0$, $\beta_\theta > 0$, $\delta_\theta > 0$ .Using the following parameters adaptive law as:

$$\dot{\hat{\psi}} = proj\left(\hat{\psi}, \phi_\psi\right) \quad (11)$$

$$\dot{\hat{\theta}} = proj\left(\hat{\theta}, \phi_\theta\right) \quad (12)$$

Projection operator is defined as:

$$proj\left(\hat{M}, \phi_M\right) = \begin{cases} \phi_M - \dfrac{\left(\left\|\hat{M}\right\|^2 - \beta_M\right)\phi_M^T \hat{M}}{\psi_M \left\|\hat{M}\right\|^2}\left\|\hat{M}\right\| \\ f \left\|\hat{M}\right\|^2 > \beta_M, \quad and\ also\ \phi_M^T \hat{M} > 0 \\ \phi_M, \quad others \end{cases} \quad (13)$$

Where, $M = \psi, \theta$, $\phi_\psi = \Gamma_\psi^{-1} z_i^T \varphi_i, i = 1,2$, $\phi_\theta = \Gamma_\theta^{-1} z_i^T u \phi_2$ .From literature (Khalil H K 1996), we can get :

$$\begin{aligned} &\tilde{\psi}^T \Gamma_\psi \dot{\hat{\psi}} - \tilde{\psi}^T z_i^T \varphi_i \leq 0 \\ &\hat{\psi} \in \Omega_\psi, t \geq 0, \hat{\psi}(0) \in \Omega_{\psi_0} \end{aligned} \quad (14)$$

$$\begin{aligned} &\tilde{\theta}^T \Gamma_\theta \dot{\hat{\theta}} - \tilde{\theta}^T z_i^T \phi_2 \leq 0 \\ &\hat{\theta} \in \Omega_\theta, t \geq 0, \hat{\theta}(0) \in \Omega_{\theta_0} \end{aligned} \quad (15)$$

Let $\tau_1 = \varphi_1^T(x_1) z_1$, select adaptive law is:

$$\begin{aligned} \dot{\tilde{\psi}} &= -\dot{\hat{\psi}} = -proj\left(\hat{\psi}, \Gamma_1^{-1} \tau_1\right) \\ &= -proj\left(\hat{\psi}, \Gamma_1^{-1} \varphi_1^T(x_1) z_1\right) \end{aligned} \quad (16)$$

$\dot{\hat{p}}_1 = r_1 \varepsilon_1 \left\|z_1\right\|^2 \delta_1^2$, where $\varepsilon_1 > 0$ is constant which should be designed and $v_1 = z_1 \hat{p}_1^2 \delta_1^2$ is robust item. Substituting equation (14)-(16) into the equation (8) yield as:

$$\begin{aligned} \dot{V}_1 = &-k_1 \left\|z_1\right\|^2 + z_1^T g_1(x_1)(x_2 - \overline{x}_{2d}) \\ &+ z_1^T \left[\Delta_1(x_1, t) - v_1\right] - \tilde{p}_1 \varepsilon_1 \left\|z_1\right\|^2 \delta_1^2 \end{aligned} \quad (17)$$

Application of Cauchy-Schwarz inequality, we can get:

$$\left\|z_1 \delta_1\right\| \leq \varepsilon_1 \left\|z_1\right\|^2 \delta_1^2 + \frac{1}{4\varepsilon_1} \quad (18)$$

So, equation (17) will be change as follows:

$$\begin{aligned} \dot{V}_1 = &-k_1 \left\|z_1\right\|^2 + z_1^T g_1(x_1)(x_2 - \overline{x}_{2d}) \\ &+ z_1^T \left[\Delta_1(x_1, t) - v_1\right] - \tilde{p}_1 \varepsilon_1 \left\|z_1\right\|^2 \delta_1^2 \\ \leq &-k_1 \left\|z_1\right\|^2 + z_1^T g_1(x_1)(x_2 - \overline{x}_{2d}) + \\ &\left\|z_1 \delta_1\right\| p_1 - z_1^T v_1 - \tilde{p}_1 \varepsilon_1 \left\|z_1\right\|^2 \delta_1^2 \\ \leq &-k_1 \left\|z_1\right\|^2 + z_1^T g_1(x_1)(x_2 - \overline{x}_{2d}) - \\ &\delta_1^2 \left\|z_1\right\|^2 \left(\hat{p}_1 - \frac{\varepsilon_1}{2}\right)^2 + \frac{p_1}{4\varepsilon_1} \\ \leq &-k_1 \left\|z_1\right\|^2 + z_1^T g_1(x_1)(x_2 - \overline{x}_{2d}) + \frac{p_1}{4\varepsilon_1} \end{aligned} \quad (19)$$

Differentiating z_2 with respect to time, we can get:

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{x}_{2d} \\ &= f_{20}(x_1, x_2) + \varphi_2(x_1, x_2)\psi + \\ &\left[g_2(x_1) + \phi(x_1)\theta\right]u + \\ &\Delta_2(x_1, x_2, t) - \dot{x}_{2d} \end{aligned} \quad (20)$$

If we calculate the $\dot{x}_{2d}$ as follows:

$$\begin{aligned} \dot{x}_{2d} = &\frac{\partial x_{2d}}{\partial x_1}\left[f_{10}(x_1) + \varphi_1(x_1)\psi + g_1(x_1)x_2 + \Delta_1(x_1, t)\right] \\ &+ \frac{\partial x_{2d}}{\partial y_r}\dot{y}_r + \frac{\partial x_{2d}}{\partial \dot{y}_r}\ddot{y}_r - \frac{\partial x_{2d}}{\partial \hat{\psi}}\Gamma_1^{-1}\varphi_1^T(x_1)z_1 \end{aligned} \quad (21)$$

It is easy to see, the calculation progress of $\dot{x}_{2d}$ is very complicated. In this paper, we use dynamic surface technology to calculate $\dot{x}_{2d}$.

$$\kappa \dot{x}_{2d} + x_{2d} = \overline{x}_{2d}, x_{2d}(0) = \overline{x}_{2d}(0) \quad (22)$$

$\kappa > 0$ is parameter should be design. Defend filtering error variable as follows:

$$y_1 = x_{2d} - \bar{x}_{2d} \tag{23}$$

The Lyapunov candidate function changed as follows:

$$V_1 = \frac{1}{2} z_1^T z_1 + \frac{1}{2}\tilde{\psi}^T \Gamma_1 \tilde{\psi} + \frac{1}{2r_1}\tilde{p}_1^2 + \frac{1}{2} y_1^T y_1$$

So the equation (6) and (19) will be change as follows:

$$\begin{aligned} \dot{z}_1 &= f_{10}(x_1) + \varphi_1(x_1)\psi + g_1(x_1)x_2 + \Delta_1(x_1,t) - \dot{x}_{1d} \\ &= f_{10}(x_1) + \varphi_1(x_1)\psi + g_1(x_1)(z_2 + y_1 + \bar{x}_{2d}) \\ &+\Delta_1(x_1,t) - \dot{x}_{1d} = -k_1 z_1 + \varphi_1(x_1)\tilde{\psi} + g_1(x_1) z_2 \\ &+ g_1(x_1) y_1 - v_1 + \Delta_1(x_1,t) \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{V}_1 &= -k_1\|z_1\|^2 + z_1^T g_1(x_1) z_2 + z_1^T g_1(x_1) y_1 \\ &+ z_1^T\left[\Delta_1(x_1,t) - v_1\right] - \tilde{p}_1\varepsilon_1\|z_1\|^2\delta_1^2 + y_1^T\dot{y}_1 \\ &\le -k_1\|z_1\|^2 + z_1^T g_1(x_1) z_2 + z_1^T g_1(x_1) y_1 + \\ &\|z_1\delta_1\| p_1 - z_1^T v_1 - \tilde{p}_1\varepsilon_1\|z_1\|^2\delta_1^2 + y_1^T\dot{y}_1 \\ &\le -k_1\|z_1\|^2 + z_1^T g_1(x_1) z_2 + z_1^T g_1(x_1) y_1 - \\ &\delta_1^2\|z_1\|^2\left(\hat{p}_1 - \frac{\varepsilon_1}{2}\right)^2 + \frac{p_1}{4\varepsilon_1} + y_1^T\dot{y}_1 \le -k_1\|z_1\|^2 + \\ &z_1^T g_1(x_1) z_2 + z_1^T g_1(x_1) y_1 + \frac{p_1}{4\varepsilon_1} + y_1^T\dot{y}_1 \end{aligned} \tag{25}$$

Differentiating y_1 with respect to time, we can get: $\dot{y}_1 = -y_2/\kappa - \dot{\bar{x}}_{2d}$, and propose the following hypothesis: Assumption 5: Assuming that the desired trajectory tracking x_{1d} sufficiently smooth and that $\dot{x}_{1d}$ and $\ddot{x}_{1d}$ for bounded function, which belongs to the compact set $\boldsymbol{D}_r \subset \boldsymbol{R}^3$ .In addition, assuming the error vector z_1, z_2, y_1, $\tilde{\psi}$, $\tilde{\theta}$, $\tilde{p}_i$ belongs to a compact set $\boldsymbol{\Omega}_s \subset \boldsymbol{R}^6$.

By assuming 5 that there exist a bounded continuous function η,

$$\left|\dot{\bar{x}}_{2d}\right| \le \eta(z_1, z_2, y_1, \tilde{\psi}, x_{1d}, \dot{x}_{1d}, \ddot{x}_{1d}) \tag{26}$$

So we can get :

$$\begin{aligned} y_1^T\dot{y}_1 &= y_1^T\cdot\left(-y_1/\kappa - \dot{\bar{x}}_{2d}\right) \\ &= -\|y_1\|^2/\kappa - y_1^T\dot{\bar{x}}_{2d} \\ &\le -\|y_1\|^2/\kappa + \left|y_1^T\eta\right| \end{aligned} \tag{27}$$

From assuming 3 and Young inequation $ab < a^2/2 + b^2/2$, we can get that:

$$z_1^T g_1(x_1) y_1 \le g_{1h}\cdot\|z_1\|^2 + g_{1h}\cdot\|y_1\|^2/4 \tag{28}$$

$$\left|y_1^T\eta\right| \le \|y_1\|^2/2 + \eta^2/2 \tag{29}$$

Therefore, equation (25) changed as follows:

$$\begin{aligned} \dot{V}_1 &\le -k_1\|z_1\|^2 + z_1^T g_1(x_1) z_2 + z_1^T g_1(x_1) y_1 + \frac{p_1}{4\varepsilon_1} + y_1^T\dot{y}_1 \\ &\le k_1\|z_1\|^2 + z_1^T g_1(x_1) z_2 + \frac{p_1}{4\varepsilon_1} + g_{1h}\cdot\|z_1\|^2 + g_{1h}\cdot\|y_1\|^2/4 \\ &-\|y_1^2\|/\kappa + \|y_1\|^2/2 + \eta^2/2 \end{aligned} \tag{30}$$

Defined the Lyapunov candidate function as follows:

$$\begin{aligned} V_2 &= \frac{1}{2} z_1^T z_1 + \frac{1}{2}\tilde{\psi}^T\Gamma_\psi\tilde{\psi} + \frac{1}{2r_1}\tilde{p}_1^2 + \frac{1}{2} y_1^T y_1 + \\ &\frac{1}{2} z_2^T z_2 + \frac{1}{2}\tilde{\theta}^T\Gamma_\theta\tilde{\theta} + \frac{1}{2r_2}\tilde{p}_2^2 \end{aligned} \tag{31}$$

Differentiating V_2 with respect to time, we can get:

$$\begin{aligned} \dot{V}_2 &= z_1^T\dot{z}_1 + z_2^T\dot{z}_2 + \tilde{\psi}^T\Gamma_\psi\dot{\tilde{\psi}} + \tilde{\theta}^T\Gamma_\theta\dot{\tilde{\theta}} \\ &+ \frac{1}{r_1}\tilde{p}_1\dot{\tilde{p}}_1 + \frac{1}{r_2}\tilde{p}_2\dot{\tilde{p}}_2 + y_1^T\dot{y}_1 \\ &= z_1^T(-k_1 z_1 + \varphi_1(x_1)\tilde{\psi} + g_1(x_1) z_2 + g_1(x_1) y_1 + \Delta_1(x_1,t) - v_1) \\ &+ z_2^T\{f_{20}(x_1,x_2) + \varphi_2(x_1,x_2)\psi + \left[g_2(x_1) + \phi(x_1)\theta\right]u + \\ &\Delta_2(x_1,x_2,t) - \dot{x}_{2d}\} + +\tilde{\psi}^T\Gamma_\psi\dot{\tilde{\psi}} + \tilde{\theta}^T\Gamma_\theta\dot{\tilde{\theta}} + \frac{1}{r_1}\tilde{p}_1\dot{\tilde{p}}_1 \\ &+ \frac{1}{r_2}\tilde{p}_2\dot{\tilde{p}}_2 + y_1^T\dot{y}_1 \end{aligned} \tag{32}$$

Design parameters for the adaptive law as follows:

$$\dot{\tilde{\psi}} = -\dot{\hat{\psi}} = -\operatorname{proj}\left[\boldsymbol{\Gamma}_\psi^{-1}\varphi_1^T(x_1) z_1 + \boldsymbol{\Gamma}_\psi^{-1}\varphi_2^T(x_1,x_2) z_2\right] \tag{33}$$

$$\dot{\hat{p}}_2 = r_2\varepsilon_2\|z_2\|^2\delta_2^2 \tag{34}$$

Where $\varepsilon_2 > 0$ is parameter which should be designed.

Designed controller u as followed:

$$u=-\frac{1}{g_2(x_1)+\phi(x_1)\hat{\theta}}\Big[g_1^T(x_1)z_1+ f_{20}(x_1,x_2)+k_2z_2+\varphi_2\hat{\psi}+v_2-\dot{x}_{2d}\Big] \tag{35}$$

$$\begin{aligned}\dot{\hat{\theta}}&=\operatorname{proj}\Big\{\hat{\theta},-\frac{\boldsymbol{\Gamma}_\theta^{-1}\phi^T(x_1)z_2}{g_2(x_1)+\phi(x_1)\hat{\theta}}\Big[g_1^T(x_1)z_1+ f_{20}(x_1,x_2)+k_2z_2+\varphi_2\hat{\psi}+v_2-\dot{x}_{2d}\Big]\Big\} \\ &=\operatorname{proj}\Big\{\hat{\theta},\boldsymbol{\Gamma}_\theta^{-1}\phi^T(x_1)\cdot(z_2^Tu)\Big\}\end{aligned} \tag{36}$$

And $v_2=z_2\hat{p}_2^2\delta_2^2$ is robust item .

Substituting equation (27)-(29), (34)-(36) into the equation (32) yield as:

$$\begin{aligned}\dot{V}_2&\le-k_1\|z_1\|^2-k_2\|z_2\|^2+g_{1h}\|z_1\|^2+\frac{g_{1h}\|y_1\|^2}{4}-\frac{\|y_1\|^2}{\kappa} \\ &+\frac{\|y_1\|^2}{2}+\frac{\eta^2}{2}+\Big\{z_1^T\big(\Delta_1(x_1,t)-v_1\big)-\tilde{p}_1\varepsilon_1\|z_1\|^2\delta_1^2\Big\} \\ &+\Big\{z_2^T\big(\Delta_2(x_1,x_2,t)-v_2\big)-\tilde{p}_2\varepsilon_2\|z_2\|^2\delta_2^2\Big\}\end{aligned} \tag{37}$$

Application of Cauchy-Schwarz inequality, we can get:

$$\|z_2\delta_2\|\le\varepsilon_2\|z_2\|^2\delta_2^2+\frac{1}{4\varepsilon_2} \tag{38}$$

Similar to equation (19) discussed above, we can get

followed result as :

$$\begin{aligned}&z_i^T\big(\Delta_i(x_i,t)-v_i\big)-\tilde{p}_i\varepsilon_i\|z_i\|^2\delta_i^2 \\ &\le-\delta_i^2\|z_i\|^2\Big(\hat{p}_i-\frac{\varepsilon_i}{2}\Big)^2+\frac{p_i}{4\varepsilon_i}\end{aligned} \tag{39}$$

In addition, from the assumption 5, continuous function can η be set to a maximum value M_0 . So The equation (37) was changed as followed:

$$\begin{aligned}\dot{V}_2&\le-k_1\|z_1\|^2-k_2\|z_2\|^2+g_{1h}\|z_1\|^2+\frac{g_{1h}\|y_1\|^2}{4}-\frac{\|y_1\|^2}{\kappa} \\ &+\frac{\|y_1\|^2}{2}+\frac{\eta^2}{2}+\Big\{z_1^T\big(\Delta_1(x_1,t)-v_1\big)-\tilde{p}_1\varepsilon_1\|z_1\|^2\delta_1^2\Big\} \\ &+\Big\{z_2^T\big(\Delta_2(x_1,x_2,t)-v_2\big)-\tilde{p}_2\varepsilon_2\|z_2\|^2\delta_2^2\Big\} \\ &\le-k_1\|z_1\|^2-k_2\|z_2\|^2+g_{1h}\|z_1\|^2+\frac{g_{1h}\|y_1\|^2}{4}-\frac{\|y_1\|^2}{\kappa} \\ &+\frac{\|y_1\|^2}{2}+\frac{\eta^2}{2}-\delta_1^2\|z_1\|^2\Big(\hat{p}_1-\frac{\varepsilon_1}{2}\Big)^2+\frac{p_1}{4\varepsilon_1}-\delta_2^2\|z_2\|^2\Big(\hat{p}_2-\frac{\varepsilon_2}{2}\Big)^2 \\ &+\frac{p_2}{4\varepsilon_2}\le-(k_1-g_{1h})\|z_1\|^2-k_2\|z_2\|^2+\frac{g_{1h}\|y_1\|^2}{4}-\frac{\|y_1\|^2}{\kappa} \\ &+\frac{\|y_1\|^2}{2}+\frac{\eta^2}{2}+\frac{p_1}{4\varepsilon_1}+\frac{p_2}{4\varepsilon_2}\le-(k_1-g_{1h})\|z_1\|^2-k_2\|z_2\|^2 \\ &+\frac{g_{1h}\|y_1\|^2}{4}-\frac{\|y_1\|^2}{\kappa}+\frac{\|y_1\|^2}{2}+\frac{M_0^2}{2}+\frac{p_1}{4\varepsilon_1}+\frac{p_2}{4\varepsilon_2}\end{aligned} \tag{40}$$

When k_1 , k_2 , ε_1 , ε_1 are large enough and κ is to achieve an appropriate small value, it can achieve result as followed:

$$\dot{V}_2\le-kV_2+c \tag{41}$$

Where $k=\min\{2(k_1-g_{1h}),2k_2,2(1/\kappa-g_{1h}/4-1/2)\}$, $c=M_0^2/2+\frac{p_1}{4\varepsilon_1}+\frac{p_2}{4\varepsilon_2}$ are bounded constants.

From equation (41), we can get :

$$V_2(t)\le V_2(0)e^{-kt}+\frac{c}{k}\le c_0,\forall t\ge0 \tag{42}$$

$c_0=V_2(0)+\frac{c}{k}$,from equation (30),we can easily get :

$$\begin{aligned}&\|\tilde{\psi}\|^2\le\frac{2c_0}{\psi_{\min}(\boldsymbol{\Gamma}_\psi)},\|\tilde{\theta}\|^2\le\frac{2c_0}{\psi_{\min}(\boldsymbol{\Gamma}_\theta)}, \\ &\|\tilde{p}_i\|^2\le2r_2c_0,\|y_1\|^2\le2c_0,\|z_i\|^2\le2c_0,i=1,2\end{aligned} \tag{43}$$

Equation (42)-(43) shows that all signals of closed-loop system are bounded.

In summary, we have the following conclusions: consider a multi-input multi-output system (1)-(3), assuming all the condition is satisfied, the use of the design method mentioned above , the virtual control signal and the control signal is in the form of equation (5) and (35), respectively, the adaptive parameter adjustment law is in the form of equation (33) and(36), respectively, the system state tracking error and parameter estimation errors are bounded, and the index number to the system received a neighbourhood of the origin:

$$\boldsymbol{\Omega}_i = \left\{ z_1, \tilde{\psi}_i, \tilde{\theta}_i \middle| \sum_{i=1}^{2} \|z_i\|^2 \le 2c_0, \|\tilde{\psi}\|^2 \le \frac{2c_0}{\psi_{\min}\left(\boldsymbol{\Gamma}_\psi\right)}, \right.$$
$$\left. \|\tilde{\theta}\|^2 \le \frac{2c_0}{\psi_{\min}\left(\boldsymbol{\Gamma}_\theta\right)}, \|y_1\|^2 \le 2c_0 \right\}, i = 1,2 \tag{44}$$

4 NUMERICAL SIMULATION

The initial conditions for the simulation is: V=8Ma , $h = 20000m$, $M = 13000kg$, $p = q = r = 0^o$,the engine thrust is $T = 200KN$ .The desired track guidance commands: $\alpha_d = 2^o, \beta_d = 0^o, \sigma_d = 0.3^o$ and through a low-pass filter $3/(s+4)$ .Set the controller parameter as follows:

$\mathbf{\Gamma}_\psi = diag(2.5, 2.5, 2.5)$, $\mathbf{\Gamma}_\theta = diag(1.8, 1.8, 1.8)$, $k_1 = 14$, $k_2 = 22$, $\kappa = 0.25$, $\varepsilon_1 = 1$, $\varepsilon_2 = 1.5$.

Simulation results are shown below. Figure 1 to Figure 6 that demonstrated the contrast curve for the system real state and the expected value. In those figures, the dotted line is results of the expected value, the solid line corresponding to the state of the system real output.

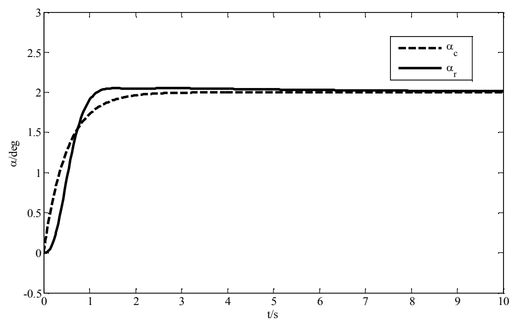

Fig 1 Simulation curve of α

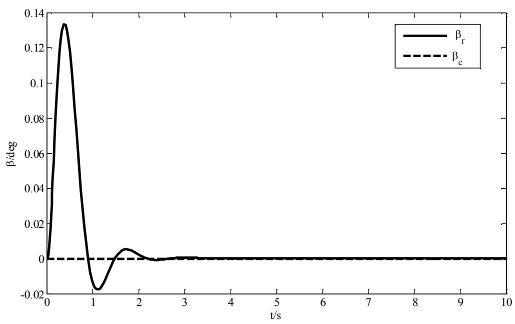

Fig 2 Simulation curve of β

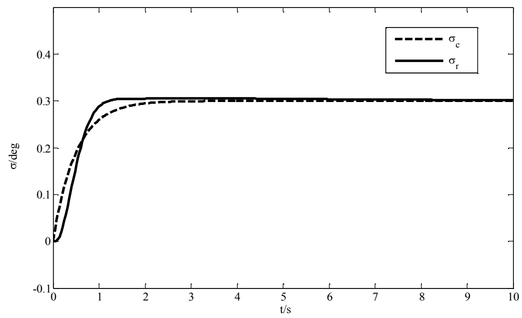

Fig 3 Simulation curve of σ

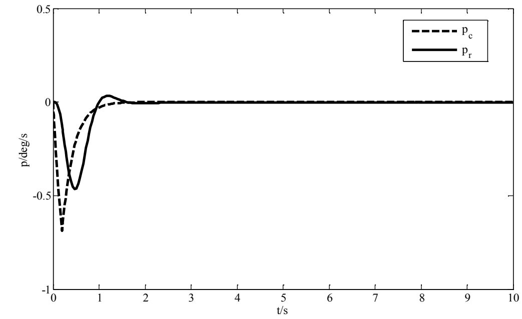

Fig 4 Simulation curve of p

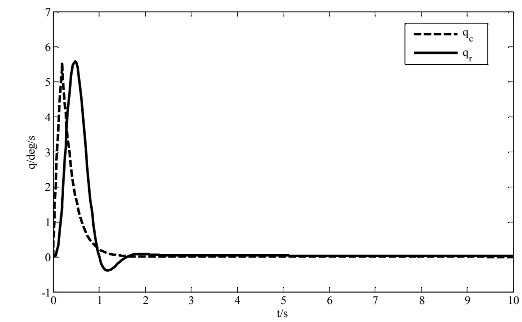

Fig 5 Simulation curve of q

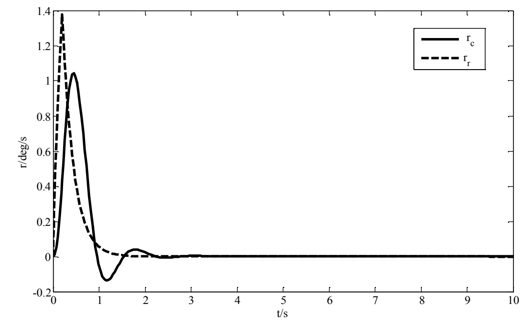

Fig 6 Simulation curve of r

As can be seen from the simulation result, the backstepping-based control design method for ABHV dynamics model has good dynamic quality and tracking performance and also proved the effectiveness of the proposed design method.

5 CONCLUSIONS

Since backstepping technology can not only deal with mismatched uncertainties, but also to deal with nonlinear systems with unknown parameters, therefore the method makes nonlinear control law design becomes systematic and structured.

In this paper, we had considered the linear parameter uncertainties for ABHV and proposed a dynamic surface based technique to design backstepping adaptive control system, and use adaptive function complement, which could use adaptive adjustment function to compensate impact of system uncertainty. By introducing the projection operator, the controller singularity problem that may occur could be avoided. The simulation shows the effectiveness of the algorithm.

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