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A MPC for Start-up Phase Tension and Looper Control in Hot Strip Finishing Mills Using Continuation Approach ⋆

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Abstract: TThis paper gives a design method for a model predictive control (MPC) approach based on a unified performance index throughout the start-up phase tension and looper control which consists of the non-contact and contact modes in order to suppress the deviation of the strip tension while the looper contacts with the strip as quickly as possible. We will formulate the control problem by using a MPC for a piecewise affine (PWA) system with the terminal condition and an unknown terminal time. However, in order to realize the feedback control using a receding horizon strategy, we have to solve nonlinear equations in an on-line manner as precisely as possible. Therefore, the paper gives a method using a continuation method for solving the nonlinear equations efficiently. The efficiency of the proposed method is shown through numerical simulations.

1. INTRODUCTION

In the hot strip finishing mill, several passes of rolling are executed by tandem rolling with 6 or 7 successive stands in the presence of interstand tension to achieve the required reduction, final qualities and tolerances. The looper implemented between each pair of adjacent stands fulfills an important role in tension control. In the start-up phase, the looper is raised above the passline just after the leading end of the strip passes through the downstream stand so that the looper comes into contact with the strip and eventualy forms a loop of the stored strip between the stands.

Several advanced multivariable control schemes have been applied to tension and looper control. Among them are interaction decoupling Kotera and Watanabe [1981], optimal control Seki *et al.* [1991], $\mathcal{H}_\infty$ control Imanari *et al.* [1997] and decentralized control Asano *et al.* [2000]. All of them are, however, intended for feedback control after the start-up phase. On the other hand, both tension and looper angle control in the start-up phase is normally performed in an ad hoc manner; a constant value is given as the looper motor torque reference and the feedback control does not start until the looper comes into contact with the strip.

In such start-up phase tension and looper control in hot strip finishing mills, a hybrid system approach has been proposed Asano *et al.* [2005], Imura *et al.* [2004]. In this research, the transient behaviour of the tension and looper angle in the start-up phase is modeled by a piecewise affine (PWA) system with a sequential mode transition, and a hybrid optimal control approach is applied. Although it shows that the deviation of the strip tension is supressed efficiently, it has drawbacks of heavy computational load because it has to solve a quadratic programming problem with constraints repeatedly in order to search the optimal mode switching time. Hence, it is required to reduce the calculating time of control law so that on-line implementation could be realized.

This paper gives a design method for a model predictive control (MPC) approach by using a unified performance index throughout the start-up phase tension and looper control which consists of the non-contact and contact modes in order to suppress the deviation of the strip tension while the looper contacts with the strip as quickly as possible. We will formulate the control problem by using a MPC for a piecewise affine (PWA) system with the terminal condition and an unknown terminal time. However, in order to realize the feedback control using a receding horizon strategy, we have to solve nonlinear equations in an on-line manner as precisely as possible. Therefore, the paper gives a method using a continuation method for solving the nonlinear equations efficiently. The efficiency of the proposed method is shown through numerical simulations.

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2. TENSION AND LOOPER CONTROL SYSTEM MODEL

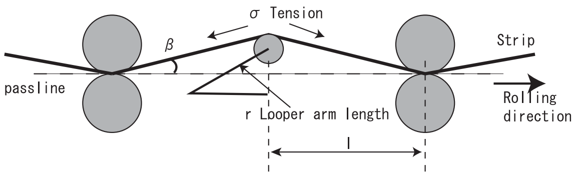

Fig. 1. Looper geometry

J	Looper inertia
θ	Looper angle
σ	Interstand tension
q	Looper torque
q_{ref}	Looper torque reference
D	Looper damping constant
T_{ACR}	Time constant of looper motor ACR
h	Strip thickness
b	Strip width
β	Strip angle with passline
ρ	Strip density
g	Gravitational constant
l	Half of length beween stands
r	Looper arm length
W_L	Looper weight
r_L	Distance between axis and center of gravity of looper
θ_G	Offset angle between center of gravity of looper and looper angle
E	Young's modulus of strip
f	Forward slip
L	Interstand strip length
V_R	Roll velocity
V_{Rref}	Roll velocity reference
T_{ASR}	Time constant of mill motor ASR

Table 1. Nomenclature in the Tension and Looper Control System Model

Consider the looper and one pair of adjacent stands in the hot strip finishing mills shown in Fig. 1. The nomenclature in the tension and looper control system model is given in Table. 1.

The looper dynamics are described by the following equations:

$$J\ddot{\theta} = q - \delta\{K_\sigma(\theta)\sigma + K_s(\theta)\} - K_L(\theta) - D\dot{\theta} \tag{1}$$

$$\dot{q} = -\frac{1}{T_{ACR}}(q - q_{ref}) \tag{2}$$

where K_σ, K_s and K_L denote the looper load torque by the tension, strip weight and looper weight, respectively, and are given as follows:

$$K_\sigma(\theta) \triangleq 2bhr\cos\theta\sin\beta \tag{3}$$

$$K_s(\theta) \triangleq 2\rho hbg\frac{1}{\cos\beta}r\cos\theta \tag{4}$$

$$K_L(\theta) \triangleq W_L g r_L \cos(\theta + \theta_G) \tag{5}$$

δ is a 0-1 variable which denotes the two modes: $\delta = 1$ in the contact mode (C-mode) and $\delta = 0$ in the non-contact mode (N-mode). The mode transition rule is given as follows:

$$\delta = \begin{cases} 0 & \text{if } \theta \le \theta_{\min} \\ 1 & \text{if } \theta \ge \theta_{\min} \end{cases} \tag{6}$$

where $\theta_{\min}$ is the looper angle when the looper is raised to the passline.

The tension dynamics are governed by the following equations:

$$\dot{\sigma} = \frac{E}{2l}\left\{-\{1 + f(\sigma)\}V_R + \frac{\partial L}{\partial\theta}\dot{\theta}\right\} \tag{7}$$

$$\dot{V}_R = -\frac{1}{T_{ASR}}(V_R - V_{Rref}) \tag{8}$$

The looper angular velocity and the tension at the transition from the N-mode to the C-mode are assumed as follows;

$$\dot{\theta}(t) = \varepsilon_1\dot{\theta}(t_-), \text{if N-mode} \to \text{C-mode} \tag{9}$$

$$\sigma(t) = \sigma(t_-) + \varepsilon_2\dot{\theta}(t_-), \text{if N-mode} \to \text{C-mode} \tag{10}$$

where ε_1 and ε_2 are each an appropriately estimated constant, $\dot{\theta}(t_-) \triangleq \lim_{\tau\uparrow t}\dot{\theta}(\tau)$ and $\sigma(t_-) \triangleq \lim_{\tau\uparrow t}\sigma(\tau)$.

3. MPC FOR TENSION AND LOOPER CONTROL IN THE START-UP PHASE

3.1 Piecewise Affine Model

This subsection introduces a PWA model which represents the tension and looper control in the start-up phase from the initial state in the N-mode to the final state in the C-mode shown in Fig. 2.

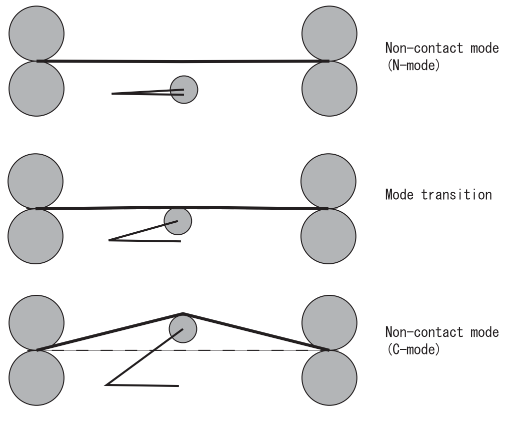

Fig. 2. Control modes

The first, we derive linearized model around an operating point of C-mode. The operating point of C-mode is described by $(\theta_c, 0, \sigma_c, q_c, V_{Rc})$, which are satisfied with

$$q_c = q_{refc} = K_\sigma(\theta_c)\sigma + K_s(\theta_c) + K_L(\theta_c) \tag{11}$$

$$V_{Rc} = V_{Rfefc} \tag{12}$$

Then, the following equations are derived by linearizing Eqs.(1)-(10) with $\delta = 1$:

$$J\ddot{\bar{\theta}} = \bar{q} - K_\sigma(\theta_c)\bar{\sigma} - K(\theta_c,\sigma_c)\bar{\theta} - D\dot{\bar{\theta}} \tag{13}$$

$$\dot{\bar{\sigma}} = F_1(\sigma_c)\bar{V}_R + F_2(\sigma_c, V_{Rc})\bar{\sigma} + F_3(\theta_c)\dot{\bar{\theta}} \tag{14}$$

$$\dot{\bar{q}} = -\frac{1}{T_{ACR}}\left(\bar{q} - \bar{q}_{ref}\right) \tag{15}$$

$$\dot{\bar{V}}_R = -\frac{1}{T_{ASR}}\left(\bar{V}_R - \bar{V}_{Rref}\right) \tag{16}$$

$$\dot{\bar{\theta}}(t) = \varepsilon_1\dot{\bar{\theta}}(t_-), \text{if N-mode} \to \text{C-mode} \tag{17}$$

$$\bar{\sigma}(t) = \bar{\sigma}(t_-) + \varepsilon_2\dot{\bar{\theta}}(t_-), \text{if N-mode} \to \text{C-mode} \tag{18}$$

where

$$K(\theta_c,\sigma_c) \triangleq \sigma_c \left.\frac{\partial K_\sigma}{\partial\theta}\right|_{\theta=\theta_c} + \left.\frac{\partial K_s}{\partial\theta}\right|_{\theta=\theta_c} + \left.\frac{\partial K_L}{\partial\theta}\right|_{\theta=\theta_c} \tag{19}$$

$$F_1(\sigma_c) \triangleq -\frac{E}{2l}\{1+f(\sigma_c)\} \tag{20}$$

$$F_2(\sigma_c, V_{Rc}) \triangleq -\frac{EV_{Rc}}{2l}\left.\frac{\partial f}{\partial\sigma}\right|_{\sigma=\sigma_c} \tag{21}$$

$$F_3(\theta_c) \triangleq \frac{E}{2l}\left.\frac{\partial L}{\partial\theta}\right|_{\theta=\theta_c} \tag{22}$$

The next, we derive linearized model around an operating point of N-mode. The operating point of N-mode is described by $(\theta_n, 0, q_n)$, which are satisfied with

$$q_n = q_{refn} = K_L(\theta_n) \tag{23}$$

Then, the following equations are derived by linearizing Eqs.(1)-(10) with $\delta = 0$:

$$J\ddot{\tilde{\theta}} = \tilde{q} - \left.\frac{\partial K_L}{\partial\theta}\right|_{\theta=\theta_n}\tilde{\theta} - D\dot{\tilde{\theta}} \tag{24}$$

$$\dot{\tilde{q}} = -\frac{1}{T_{ACR}}\left(\tilde{q} - \tilde{q}_{ref}\right) \tag{25}$$

Noting that the tension is measured by a tensiometer mounted on the looper, so it is unmeasurable in the N-mode. Hence, the dynamic equations in terms of tension and roll velocity are not included in the N-mode because it is assumed that the roll velocity reference signal V_{Rref} is kept constant in the N-mode.

The paper considers a MPC based on a unified performance index througout the start-up phase tension and looper control. Hence, the linear model in the N-mode is unified based on the coordinate systems of the C-mode, which yields the following representation:

$$J\ddot{\bar{\theta}} = \bar{q} - \left.\frac{\partial K_L}{\partial\theta}\right|_{\theta=\theta_n}\bar{\theta} - D\dot{\bar{\theta}} + \Delta S_\theta \tag{26}$$

$$\dot{\bar{q}} = -\frac{1}{T_{ACR}}\left(\bar{q} - \bar{q}_{ref}\right) \tag{27}$$

where

$$\Delta S_\theta \triangleq q_c - q_n - \left.\frac{\partial K_L}{\partial\theta}\right|_{\theta=\theta_n}(\theta_c - \theta_n) \tag{28}$$

From Eqs.(13)-(18) and Eqs.(26)-(27), PWA models for the tension and looper control in the start-up phase are given as follows:

N-mode:

$$\frac{\partial}{\partial t}\boldsymbol{x}(t) = \boldsymbol{A}_1\boldsymbol{x}(t,\tau) + \boldsymbol{B}_1 u_1(t,\tau) + \boldsymbol{a}, \quad \text{if} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t,\tau) - p_0 \le 0 \tag{29}$$

NC-mode:

$$\boldsymbol{x}(t,0) = \boldsymbol{E}_{nc}\boldsymbol{x}(t_-) + \boldsymbol{e}_{nc}, \quad \text{if} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t_-) - p_0 = 0, \text{ and N-mode} \to \text{C-mode} \tag{30}$$

C-mode:

$$\frac{\partial}{\partial t}\boldsymbol{x}(t) = \boldsymbol{A}_2\boldsymbol{x}(t,\tau) + \boldsymbol{B}_2\boldsymbol{u}(t,\tau), \quad \text{if} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t,\tau) - p_0 \ge 0 \tag{31}$$

where

$$\boldsymbol{x} \triangleq \left[\bar{\theta}, \dot{\bar{\theta}}, \bar{q}, \bar{\sigma}, \bar{V}_R\right]^{\mathrm{T}}, \quad \boldsymbol{u} \triangleq \left[\bar{q}_{ref}, \bar{V}_{Rref}\right]^{\mathrm{T}}$$

$$\boldsymbol{x_1} \triangleq \left[\bar{\theta}, \dot{\bar{\theta}}, \bar{q}\right]^{\mathrm{T}}, \quad \boldsymbol{x_2} \triangleq \left[\bar{\sigma}, \bar{V}_R\right]^{\mathrm{T}}, \quad u_1 \triangleq \bar{q}_{ref}$$

$$\boldsymbol{A}_1 \triangleq \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}, \quad \boldsymbol{B}_1 \triangleq \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix}, \quad \boldsymbol{a} \triangleq \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix} \tag{32}$$

$$\boldsymbol{A}_2 \triangleq \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ \hline 0 & a'_{42} & 0 & a'_{44} & a'_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{array}\right], \quad \boldsymbol{B}_1 \triangleq \begin{bmatrix} 0 \\ 0 \\ b_1 \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

$$\boldsymbol{B}_2 \triangleq \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix}, \quad \boldsymbol{E}_{nc} \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{34}$$

$$\boldsymbol{e}_{nc} \triangleq \begin{bmatrix} \boldsymbol{e}_{nc1} \\ \boldsymbol{e}_{nc2} \end{bmatrix} \triangleq \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline \sigma_n - \sigma_c \\ 0 \end{array}\right], \tag{35}$$

$$\boldsymbol{c}^{\mathrm{T}} \triangleq [1\ 0\ 0\ 0\ 0], \quad p_0 \triangleq \theta_n - \theta_c \tag{36}$$

$$a_{21} \triangleq -\frac{1}{J}\left.\frac{\partial K_L}{\partial\theta}\right|_{\theta=\theta_n}, \quad a_{22} \triangleq -\frac{1}{J}D, \quad a_{23} \triangleq \frac{1}{J}$$

$$a_{33} \triangleq -\frac{1}{T_{ACR}}, \quad b_1 \triangleq \frac{1}{T_{ACR}}, \quad f \triangleq \frac{1}{J}\Delta S_\theta$$

$$a'_{21} \triangleq -\frac{1}{J}K(\theta_c,\sigma_c), \quad a'_{22} \triangleq -\frac{1}{J}D, \quad a'_{23} \triangleq \frac{1}{J}$$

$$a'_{24} \triangleq -\frac{1}{J}K_\sigma(\theta_c), \quad a'_{42} \triangleq F_3(\theta_c)$$

$$a'_{44} \triangleq F_2(\sigma_c, V_{Rc}), \quad a'_{45} \triangleq F_1(\sigma_c)$$

$$a'_{55} \triangleq -\frac{1}{T_{ASR}}, \quad b_2 \triangleq \frac{1}{T_{ASR}}$$

Here, the initial state is $\boldsymbol{x}_{10} \triangleq [\theta_n - \theta_c, 0, 0]^{\mathrm{T}}$.

3.2 MPC Formulation

Now, we assume that the mode transition from the N-mode to the C-mode occurs sequentially, and once the C-mode starts, it never returns N-mode. Under the assumption, let's consider a MPC for the looper and tension control in the start-up phase which consists of non-contact and contact modes. The performance index is unified one which evaluates the performance from N-mode to the C-mode shown in the next:

$$J_t = \int_0^\infty \left\{ \boldsymbol{x}^{\mathrm{T}}(t,\tau)\boldsymbol{Q}\boldsymbol{x}(t,\tau) + \boldsymbol{u}(t,\tau)^{\mathrm{T}}\boldsymbol{R}\boldsymbol{u}(t,\tau) \right\} \, d\tau$$
$$\longrightarrow \min, \qquad \boldsymbol{Q} \geq 0, \quad \boldsymbol{R} > 0 \tag{37}$$

where, τ is a virtual time for the calulation of optimal control law, and t stands for the real time which starts the looper and tension control at a initial time t_0. From the receding horizon strategy, just $\boldsymbol{u}(t,0)$, actually $\boldsymbol{u}(t,\tau),\ t \leq \tau \leq t+\varepsilon,\ \ \varepsilon > 0$, is applied to the system after the optimal control law is derived.

Now, noting that the MPC control law is equivalent to a linear optimal regulator after the C-mode, and the optimal value of the perfomance index during the C-mode, could be evaluated by using the state variables just when the C-mode starts, the MPC for the start-up phase looper and tension control could be reduced as the MPC in the N-mode which is formulated in the following way.

$$J_t = \int_0^{t_s} \left\{ \boldsymbol{x}_1^{\mathrm{T}}(t,\tau)\boldsymbol{Q}_1\boldsymbol{x}_1(t,\tau) + r_1 u_1(t,\tau)^2 \right\} \, d\tau$$
$$+ \begin{bmatrix} \bar{\boldsymbol{x}}_1(t,t_s) \\ \bar{\boldsymbol{x}}_2(t,t_s) \end{bmatrix}^{\mathrm{T}} \boldsymbol{P} \begin{bmatrix} \bar{\boldsymbol{x}}_1(t,t_s) \\ \bar{\boldsymbol{x}}_2(t,t_s) \end{bmatrix} \longrightarrow \min, \tag{38}$$
$$\boldsymbol{Q}_1 \geq 0, \quad r_1 > 0$$
$$\text{s.t.} \quad \frac{\partial}{\partial t}\boldsymbol{x}_1(t,\tau) = \boldsymbol{A}_1\boldsymbol{x}_1(t,\tau) + \boldsymbol{B}_1 u_1(t,\tau) + \boldsymbol{a} \tag{39}$$
$$\bar{\boldsymbol{c}}^{\mathrm{T}}\boldsymbol{x}_1(t_s) - p_0 = 0, \quad \bar{\boldsymbol{c}}^{\mathrm{T}} \triangleq [1,\ 0,\ 0] \tag{40}$$
$$\begin{bmatrix} \bar{\boldsymbol{x}}_1(t_s) \\ \bar{\boldsymbol{x}}_2(t_s) \end{bmatrix} = \boldsymbol{E}_{nc} \begin{bmatrix} \boldsymbol{x}_1(t_s) \\ \boldsymbol{x}_2(t_s) \end{bmatrix} + \boldsymbol{e}_{nc} \tag{41}$$

where $\boldsymbol{P}$ is a positive definite matrix of Ricatti equation for the state space equation in the C-mode, which gives the optimal value of performance index after C-mode.

t_s is a switching time, which is unkonwn beforehand. Eq.(40) is the contact condition at the switching time t_s. Eq.(40) represents the state jump at NC-mode.

4. MPC USING A CONTINUATION METHOD

From the necessary condition of optimal control problem with the terminal condition and unknown terminal time, the necessary conditions of the control law in the N-mode are given by

$$\frac{\partial}{\partial \tau}\boldsymbol{\lambda}(t,\tau) = -H_{\boldsymbol{x}_1}^{\mathrm{T}}, \quad H_{u_1} = 0 \tag{42}$$
$$\frac{\partial}{\partial \tau}\boldsymbol{x}_1(t,\tau) = \boldsymbol{A}_1\boldsymbol{x}_1(t,\tau) + \boldsymbol{B}_1 u_1(t,\tau) + \boldsymbol{a} \tag{43}$$
$$\boldsymbol{x}_1(t_0,0) = \boldsymbol{x}_{10}, \quad \bar{\boldsymbol{c}}^{\mathrm{T}}\boldsymbol{x}_1(t,t_s) - p_0 = 0 \tag{44}$$
$$\boldsymbol{\lambda}(t,t_s) = \boldsymbol{K}\boldsymbol{x}_1(t,t_s) + \boldsymbol{L} + \boldsymbol{\nu}(t)\bar{\boldsymbol{c}} \tag{45}$$
$$[H]_{\tau=0} = 0, \tag{46}$$

where, the Hamiltonian H is defined as

$$\begin{aligned} H \;=\; & \boldsymbol{x}_1^{\mathrm{T}}(\tau)\boldsymbol{Q}_1\boldsymbol{x}_1(\tau) + r_1 u_1(\tau)^2 \\ & + \boldsymbol{\lambda}^{\mathrm{T}}(\tau)\left(\boldsymbol{A}_1\boldsymbol{x}_1(\tau) + \boldsymbol{B}_1 u_1(\tau) + \boldsymbol{a}\right) \end{aligned} \tag{47}$$

and $\boldsymbol{K}$ and $\boldsymbol{L}$ are defined as

$$\begin{aligned} \boldsymbol{K} &\triangleq 2\left(\boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{E}_1 + 2\boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{E}_2 + \boldsymbol{E}_2^{\mathrm{T}}\boldsymbol{P}_3\boldsymbol{E}_2\right) \\ \boldsymbol{L} &\triangleq 2\left(\boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}_2^{\mathrm{T}} + \boldsymbol{E}_2^{\mathrm{T}}\boldsymbol{P}_3\right)\boldsymbol{E}_3\boldsymbol{x}_2(t_s) \\ &\quad + 2\left(\boldsymbol{E}_2^{\mathrm{T}}\boldsymbol{P}_3 + \boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}_2\right)\boldsymbol{e}_{nc2} \end{aligned} \tag{48}$$

$$\begin{bmatrix} \boldsymbol{E}_1 & \boldsymbol{0} \\ \boldsymbol{E}_2 & \boldsymbol{E}_3 \end{bmatrix} \triangleq \boldsymbol{E}_{nc}, \quad \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{P}_2^{\mathrm{T}} & \boldsymbol{P}_3 \end{bmatrix} \triangleq \boldsymbol{P} \tag{49}$$

From the conditions Eqs.(42)-(46), the MPC control law in the N-mode can be given as

$$u_1(t,0) = -\frac{1}{2r_1}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{\lambda}(t,0) \tag{50}$$

so that the following nonlinear equations in terms of the unknown variables vector, $\boldsymbol{U}(t) = [\boldsymbol{\lambda}(t,0)^{\mathrm{T}}, \nu(t), t_s(t)]^{\mathrm{T}}$ are satisfied.

$$\boldsymbol{F}(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) = \begin{bmatrix} \boldsymbol{F}_1(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \\ F_2(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \\ F_3(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \end{bmatrix} = \boldsymbol{0} \tag{51}$$

$$\begin{aligned} &\boldsymbol{F}_1(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \\ &\triangleq \{\boldsymbol{M}_4(t_s(t)) - \boldsymbol{K}\boldsymbol{M}_2(t_s(t))\}\,\boldsymbol{\lambda}(t,0) \\ &\quad -\bar{\boldsymbol{c}}\nu(t) - \boldsymbol{K}\boldsymbol{W}_1(t_s(t)) + \boldsymbol{W}_2(t_s(t)) - \boldsymbol{L} \end{aligned} \tag{52}$$

$$\begin{aligned} &F_2(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \\ &\triangleq \bar{\boldsymbol{c}}\boldsymbol{M}_2(t_s(t)) - p_0 + \bar{\boldsymbol{c}}^{\mathrm{T}}\boldsymbol{W}_1(t_s(t)) \end{aligned} \tag{53}$$

$$\begin{aligned} &F_3(\boldsymbol{U}(t),\boldsymbol{x}_1(t,0)) \\ &\triangleq \boldsymbol{x}_1^{\mathrm{T}}(t,0)\boldsymbol{Q}_1\boldsymbol{x}_1(t,0) - \frac{1}{4r_1}\boldsymbol{\lambda}^{\mathrm{T}}(t,0)\boldsymbol{B}_1\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{\lambda}(t,0) \\ &\quad + \boldsymbol{\lambda}^{\mathrm{T}}(t,0)\boldsymbol{A}_1\boldsymbol{x}_1(t,0) + \boldsymbol{\lambda}^{\mathrm{T}}(t,0)\boldsymbol{a} \end{aligned} \tag{54}$$

where

$$\boldsymbol{M} \triangleq \begin{bmatrix} \boldsymbol{A}_1 & -\dfrac{1}{2r_1}\boldsymbol{B}_1\boldsymbol{B}_1^{\mathrm{T}} \\ -2\boldsymbol{Q}_1 & -\boldsymbol{A}_1^{\mathrm{T}} \end{bmatrix} \tag{55}$$

$$\begin{bmatrix} \boldsymbol{M}_1(t_s(t)) & \boldsymbol{M}_2(t_s(t)) \\ \boldsymbol{M}_3(t_s(t)) & \boldsymbol{M}_4(t_s(t)) \end{bmatrix} \triangleq \exp\left(\boldsymbol{M}t_s(t)\right) \tag{56}$$
$$\boldsymbol{W}_1(t_s(t)) \triangleq \boldsymbol{M}_1(t_s(t))\boldsymbol{x}_1(t,0) + \boldsymbol{a}_1(t_s(t)) \tag{57}$$
$$\boldsymbol{W}_2(t_s(t)) \triangleq \boldsymbol{M}_3(t_s(t))\boldsymbol{x}_1(t,0) + \boldsymbol{a}_2(t_s(t)) \tag{58}$$
$$\begin{bmatrix} \boldsymbol{a}_1(t_s(t)) \\ \boldsymbol{a}_2(t_s(t)) \end{bmatrix} \triangleq \int_0^{t_s(t)} \exp\left(\boldsymbol{M}\tau\right) d\tau \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{0} \end{bmatrix} \tag{59}$$

However, in order to realize the feedback control Eqn. (50) using a receding horizon strategy, we have to solve a nonlinear equations Eqn. (51) on an on-line manner as precisely as possible. Therefore, the paper proposes the method using a continuation method for solving the nonlinear equations efficiently.

These nonlinear equations can be represented as

$$\boldsymbol{F}(\boldsymbol{U}(t), \boldsymbol{x}_1(t,0)) = 0 \tag{60}$$

where $\boldsymbol{U}(t) = [\boldsymbol{\lambda}(t,0),\ \nu(t),\ t_s(t)]^{\mathrm{T}}$. The solution of the equations Eqn. (60) can be traced using the following differential equation.

$$\frac{d}{dt}\boldsymbol{F}(\boldsymbol{U}(t), \boldsymbol{x}_1(t,0)) = -\zeta \boldsymbol{F}(\boldsymbol{U}(t), \boldsymbol{x}_1(t,0)) \tag{61}$$

where $\zeta > 0$. From Eqn. (61), it follows that

$$\frac{d}{dt}\boldsymbol{U}(t) = \boldsymbol{F}_U{}^{-1}\left(-\zeta \boldsymbol{F} - \boldsymbol{F}_{\boldsymbol{x}_1}\frac{\partial}{\partial t}\boldsymbol{x}_1(t,0)\right) \tag{62}$$

Now, $\boldsymbol{F}_U$ and $\boldsymbol{F}_{\boldsymbol{x}_1}$ can be calculated explicitly as follows.

$$\boldsymbol{F}_U = \begin{bmatrix} \boldsymbol{M}_4(t_s(t)) - \boldsymbol{K}\boldsymbol{M}_2(t_s(t)) & -\bar{\boldsymbol{c}} & \boldsymbol{X}_1 \\ \bar{\boldsymbol{c}}^{\mathrm{T}}\boldsymbol{M}_2(t_s(t)) & 0 & \boldsymbol{X}_2 \\ \boldsymbol{X}_3 & 0 & 0 \end{bmatrix} \tag{63}$$

where

$$\begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{bmatrix} \triangleq \begin{bmatrix} -\boldsymbol{K} & \boldsymbol{I} \\ \bar{\boldsymbol{c}}^{\mathrm{T}} & \boldsymbol{0} \end{bmatrix} \exp(\boldsymbol{M} t_s(t)) \times \left\{ \boldsymbol{M} \begin{bmatrix} \boldsymbol{x}_1(t,0) \\ \boldsymbol{\lambda}(t,0) \end{bmatrix} + \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{0} \end{bmatrix} \right\} \tag{64}$$

$$\boldsymbol{X}_3 \triangleq -\frac{1}{2r_1}\boldsymbol{\lambda}(t,0)^{\mathrm{T}}\boldsymbol{B}_1\boldsymbol{B}_1^{\mathrm{T}} + \boldsymbol{x}(t,0)^{\mathrm{T}}\boldsymbol{A}_1^{\mathrm{T}} + \boldsymbol{a}^{\mathrm{T}} \tag{65}$$

$$\boldsymbol{F}_{\boldsymbol{x}_1} = \begin{bmatrix} -\boldsymbol{K}\boldsymbol{M}_1(t_s(t)) + \boldsymbol{M}_3(t_s(t)) \\ \bar{\boldsymbol{c}}\boldsymbol{M}_1(t_s(t)) \\ 2\boldsymbol{x}_1(t,0)^{\mathrm{T}}\boldsymbol{Q}_1 + \boldsymbol{\lambda}(t,0)^{\mathrm{T}}\boldsymbol{A}_1 \end{bmatrix} \tag{66}$$

Since the proposed MPC control law is given in the Eqn. (50) where the $\boldsymbol{\lambda}(t,0)$ is determined by solving the differential equation Eqn. (62). Thus, we can realize the feedback control using a receding horizon strategy.

5. NUMERICAL SIMULATION

In this section, we will show the efficiency of the proposed method. The system parameters in Eqs.(32)-(36) are the same as ones given in the literature Imura *et al.* [2004]. The control objective in the simulation is to raise the looper angle from the initial horizontal position ($\theta = 0^o$) to the operating point of the C-mode ($\theta = 20^o$) through the passline ($\theta = 10^o$), while keeping the interstand tension a operating point $\sigma_n = \sigma_c = 1.0^6(\mathrm{kg}/(\mathrm{m})^2)$. The weighting matrix $\boldsymbol{Q}_1$ and r of the performance index in the N-mode is given by

$$\boldsymbol{Q}_1 = \mathrm{diag}\,[\,100,\ 10000,\ 0.001\,],\quad r_1 = 0.0005$$

The weighting matrix $\boldsymbol{Q}_2$ and $\boldsymbol{R}_2$ of the performance index in the C-mode is given by

$$\boldsymbol{Q}_2 = \mathrm{diag}\,[\,100,\ 10000,\ 0.001,\ 1000,\ 1\,],$$
$$\boldsymbol{R}_2 = \mathrm{diag}\,[\,0.001,\ 0.001\,]$$

From Fig. 3, we can see that the proposed control law works well to control the looper and tension in the hot strip mill even in the presence of disturbances. Furthermore, the calculation time for deriving the control law in the non-contact mode is around 1.1[msec] on the average even in the presence of disturbances, while it takes around 100[msec] on the average in the case of Imura *et al.* [2004], Asano *et al.* [2005]. Therefore, we can claim that the proposed method improves efficiency of the calculation load, which leads to implement in on-line manner.

The simlation was executed using Workstation Astrike Windows XP Preinstallation Model, Xeon (TM) CPU 3.06GHz 1.00GB RAM, Matlab Ver 7.1.0.246 (R14).

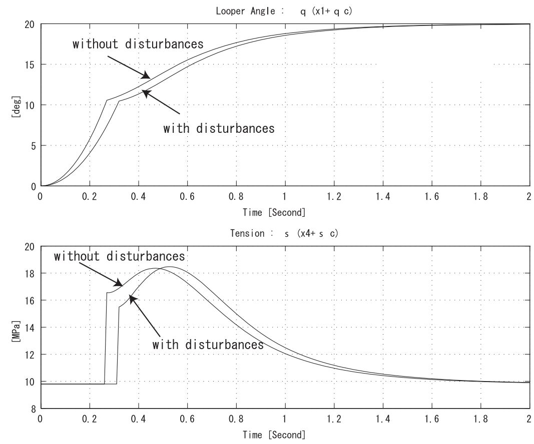

Fig. 3. Looper angle, interstand tension by the proposed MPC

The figure Fig. 4 shows the comparative results the nonlinear function $\boldsymbol{F}$ in Eqn. (60) during in the N-mode between with and without continuation method. From the figure, it follows that the proposed approach using continuation method could obtain even more precise solutions of the nonlinear equations $\boldsymbol{F}$ in Eqn. (60) than ones without using a continuation method.

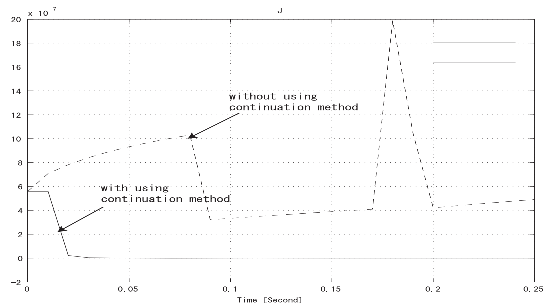

Fig. 4. The norm of nonlinear function F without using a continuation method (dashed line) and with using the continuation method (solid line)

6. CONCLUDING REMARKS

This paper showed the tension and looper control in the hot strip finishing mill based on PWA (piecewise affine) systems with the terminal condition and an unknown terminal time. While the approach in the earlier literatureImura *et al.* [2004], Asano *et al.* [2005] have difficulty to implement in an on-line manner, the proposed method improves efficiency of the calculation load. Although the paper focused on reducing the calculation load, the performance improvement in the contact mode remains for future works.

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