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Disturbance Rejection in Parameter-varying Web-winding Systems

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Abstract: We propose a control scheme to reject disturbances in web tension that are caused by reel eccentricities in parameter-varying web-winding systems. When the web winds from one reel to the other, the nominal reel radii vary. In addition to this, the reels are not perfectly circular and the eccentricities introduce disturbances to the tension output. If the web is thin enough, in a certain time span, the nominal reel radii can be considered as constants and the reel eccentricities are periodic. In our design, the entire web winding process in which the whole pack of web winds from a full source reel to an empty take-up reel is investigated at a certain number of operating points. At each operating point, an adaptive controller that synthesizes the inverse input whose system response cancels the disturbances is developed. Gain-scheduling control is then used between different operating points. The scheme is simulated on a decoupled tension loop of an example reel-to-reel web-winding system that represents a digital tape system.

Keywords: Parameter-varying systems; Disturbance rejection; Adaptive control; Data storage systems.

1. INTRODUCTION

Reel-to-reel web-winding systems are common in industries that manufacture, fabricate, and transport materials such as paper, metal, and film. One of the main objectives in these applications is to increase as much as possible the web transport velocity while simultaneously regulating the tension. One of the challenges here is that the system is usually time-varying as the radii of the reels change when the web winds. The variations in the radii are composed of two components including the nominal reel radii change and the reel eccentricities. The nominal reel radii change is due to the transport of the web from the source reel to the take-up reel. The reel eccentricities are a result of imperfect manufacturing processes that produce reels with non-perfectly circular cross sections. In some transport applications where the web passes guide posts, the time-varying radii of these posts are only due to eccentricities of the posts. The reel eccentricities are usually unknown and not addressed in current existing control schemes. They introduce disturbances to both the web velocity and web tension.

This study aims at developing controllers to reject reel eccentricity disturbances in the tension in parameter-varying web-winding systems. A few studies have examined disturbance rejection in web-winding systems. Koc et al. (2002) proposes robust controllers that address nominal radii variations in a web-winding system. Edwards et al. (1987) reviews reel eccentricity controls for strip rolling mills when the eccentricities are identified. Xu et al. (2002) adapts the magnitude of the input disturbance to a linear time-invariant (LTI) web-winding system assuming the frequency of the disturbance is constant and known. In Zhong et al. (2005), the disturbances caused by the reel eccentricities to tape tension are considered as either input or output disturbances with time-varying frequencies to a LTI tape system and adaptive control algorithms are designed to take into account the time-varying frequency of the disturbances.

The research described in this paper focuses on designing control schemes that account for the radii variations including both the nominal change and the reel eccentricities. We first investigate the impact on the output tension from the time-varying radii to conclude that the tension error caused by reel eccentricities can be canceled by a compensation input. We then propose a continuous-time domain control algorithm to synthesize the compensation input that attenuates the tension error caused by reel eccentricities. This approach extends the ideas in Bodson et al. (2001) and Zhong et al. (2005) to synthesize an inverse input to cancel the effects of the reel eccentricities. Fundamentally, this algorithm is based on the internal model principle (IMP), which states that a system can perfectly track the reference with zero steady-state error if a model of the disturbance generator is included internally in the closed-loop system (Francis and Wonham(1976)). Under the condition that the web is thin enough, within a certain time span, the nominal change can be considered negligible and tension error caused by reel eccentricities is periodic. We investigate a certain number of operating points in the entire process in which the whole pack of web winds from a full reel to an empty reel. At each operating point, an adaptive controller is designed. A gain-scheduling method is used between different operating points.

This paper is organized as follows. In Section 2, we first review the popular lumped parameter model of a magnetic tape system, an example reel-to-reel web-winding system. Then we discuss the time-varying reel radii and introduce the time-varying tension loop used in this paper. We also analyze the disturbances introduced by the varying parameters to the tension to conclude that the disturbances can be canceled by a properly synthesized compensation input. Section 3 describes the adaptive controller that synthesizes a time-varying compensation input to attenuate

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the tension error, and simulation results are shown in Section 4. Finally, Section 5 provides conclusions and discusses future work.

2. TIME-VARYING WEB-WINDING SYSTEMS

Digital tape systems are one of the applications of reel-to-reel web-winding systems. The prototypical lumped parameter model of a two-reel tape system is illustrated in Fig. 1 (Franklin et al. (1998), Mathur and Messner (1998), Lu (2002), and Baumgart and Pao (2007)). The tape winds from the source reel (the left one) to the take-up reel. $J_i(t)$, $r_i(t)$, and $\omega_i(t)$ are the inertia, radius, and angular velocity of each reel, respectively, for $i = 1, 2$. The unsupported tape between the two tangential points on the reels is modeled by a parallel dashpot and spring with damping coefficient D and spring constant K.

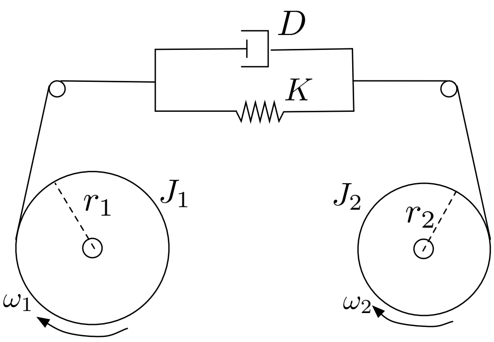

Fig. 1. The magnetic tape system: an example reel-to-reel web-winding system.

Each reel is driven by a DC motor whose motor friction viscosity coefficient is β_i and the torque constant is K_{ti}. The current applied to each motor is $u_i(t)$. When the tape winds, both the reel radii r_i and the rotating inertia of the reels J_i change. Other parameters of the web-winding system such as D and K might also vary (Baumgart and Pao (2007)). In this study, we focus on the variations in tension directly caused by the time-varying radii and consider other parameters constant.

Define $T(t)$ as the tape tension and $V_i(t)$ as the tangential velocity of the tape at each reel. Choose the state $X = [T(t), V_1(t), V_2(t)]^\top$ and the input $U = [u_1(t), u_2(t)]^\top$. Defining $\eta(t) = \frac{r_1^2(t)}{J_1(t)} + \frac{r_2^2(t)}{J_2(t)}$, the tape system can then be described by the state-space equation (Baumgart and Pao (2007))

$$\dot{X}(t) = \mathbf{A}(t)X(t) + \mathbf{B}(t)U(t),$$

with

$$\mathbf{A}(t) = \begin{bmatrix} -D\eta(t) & -K+D\frac{\beta_1}{J_1(t)} & K-D\frac{\beta_2}{J_2(t)} \\ \frac{r_1^2(t)}{J_1(t)} & -\frac{\beta_1}{J_1(t)} & 0 \\ -\frac{r_2^2(t)}{J_2(t)} & 0 & -\frac{\beta_2}{J_2(t)} \end{bmatrix}$$

and

$$\mathbf{B}(t) = \begin{bmatrix} -D\frac{r_1(t)K_{t_1}}{J_1(t)} & D\frac{r_2(t)K_{t_2}}{J_2(t)} \\ \frac{r_1(t)K_{t_1}}{J_1(t)} & 0 \\ 0 & \frac{r_2(t)K_{t_2}}{J_2(t)} \end{bmatrix}.$$

The parameters of the tape system considered in the simulations to be presented are listed in Table 1.

Table 1. Tape system parameters considered.

Parameter	Description	Value
K_t	Motor torque constant	0.0189 N m/Amp
D	Dashpot constant	0.9 N sec/m
$r(t)$	Radius of one reel	0.014 m − 0.028 m
r_m	Mid-pack radius	0.0229 m
J	Inertia of the reel	3.05e-5 kg m^2
K	Spring constant	600 N/m
β	Motor viscosity coefficient	5.9828e-5 N m sec/rad
ϵ	Tape thickness	7.7e–6 m

2.1 *Variations in Reel Radii*

The variations in the radii of a reel-to-reel web-winding system consist of two components: (a) the nominal change as a result of web winding and (b) the reel eccentricities due to non-perfectly circular reels, also known as reel runout. The nominal reel radii are often known while the reel eccentricities are generally unknown. The time-varying radii only due to nominal changes are denoted as $r_{ni}(t)$:

$$r_{ni}(t) = r_i(t_0) \mp \int_{t_0}^{t} \frac{\epsilon\omega_i(\tau)}{2\pi} d\tau$$

where ϵ is the tape thickness. The reel eccentricities can be represented as variations in reel radii relative to the nominal radii of the reels. Denoting reel eccentricities as $r_{ri}(t)$, the radius of the source reel $r_1(t)$ is

$$\begin{aligned} r_1(t) &= r_{n1}(t) + r_{r1}(t) \\ &= r_1(t_0) - \int_{t_0}^{t} \frac{\epsilon\omega_1(\tau)}{2\pi} d\tau + r_{r1}(t) \end{aligned}$$

and the radius of the take-up reel $r_2(t)$ is

$$r_2(t) = r_2(t_0) + \int_{t_0}^{t} \frac{\epsilon\omega_2(\tau)}{2\pi} d\tau + r_{r2}(t).$$

If ϵ is small enough compared to the initial radii and the reel runout, the nominal change in reel radii can be considered as negligible during a certain time span. In practice, in each reel revolution, radii variations due to reel eccentricities are more significant than those caused by the nominal radii change.

To address the variations in reel radii, we model the tension loop as a time-varying system. When designing controllers to attenuate the disturbances caused by the unknown reel eccentricities, we approximate the tension loop as a piecewise time-invariant system where the nominal radii is assumed to be constant. Hence the reel eccentricities affect the tension loop in a periodic manner at the reel rotating frequency.

2.2 *Time-varying Tension Loop in a Tape System*

Using the decoupling method in Lu (2002), the tension loop is perfectly decoupled from the velocity loop at mid-pack where the radii of both reels are equal. The transfer function of the decoupled tension loop from either input current to output tension is

$$G_T = \frac{2K_t D r_m (s + \frac{K}{D})}{Js^2 + (\beta + 2Dr_m^2)s + 2Kr_m^2}, \tag{1}$$

where r_m is the reel radius at mid-pack. We will account for the time-varying dynamics in a piecewise constant fashion, where the dynamics are considered fixed within multiple ranges of nominal radius.

The generic block diagram of the system under investigation is shown in Fig. 2. T_d is the desired tension value and y is the tension output. The controller C_T is a notch controller designed for mid-pack and also guarantees system stability. C_A is an adaptive controller that generates the compensation input u_a to take into account the tension disturbances caused by the time-varying reel radii.

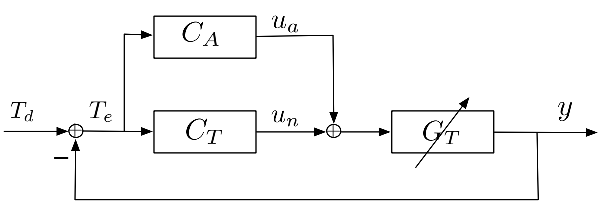

Fig. 2. The time-varying tension control loop.

To simulate the parameter-varying tension loop, the transfer function G_T in Equation (1) is realized using the observability canonical state-space form in which T is the first state and the system matrices $A(t)$ and $B(t)$ are functions of $r(t)$:

$$\begin{cases} \dot{X}_T(t) = A(t)X_T(t) + B(t)u(t) \\ Y_T(t) = CX_T(t) \end{cases} \tag{2}$$

where

$$A(t) = \begin{bmatrix} -\frac{\beta+2Dr(t)^2}{J} & 1 \\ -\frac{2Kr(t)^2}{J} & 0 \end{bmatrix} = \begin{bmatrix} -1.95 + 5.87 \times 10^4 r(t)^2 & 1 \\ -3.9 \times 10^7 r(t)^2 & 0 \end{bmatrix},$$

$$B(t) = \begin{bmatrix} \frac{2K_t Dr(t)}{J} \\ \frac{2K_t Kr(t)}{J} \end{bmatrix} = \begin{bmatrix} 1.11 \times 10^3 r(t) \\ 7.39 \times 10^5 r(t) \end{bmatrix}.$$

and

$$C = [\,1 \;\; 0\,].$$

2.3 Tension Error Caused by Reel Eccentricities

To investigate the effects of reel eccentricities on the tension, we apply a sinusoidal reel runout $r_r(t)$ to the tension loop in Fig. 2. The desired tension T_d is set to be 1 N and $C_A = 0$. The runout is

$$r_r(t) = d_m \cos(\alpha(t)),$$

where

$$\alpha(t) = \int_{t_0}^{t} \omega(\tau) d\tau.$$

Here, $\omega(\tau)$ is the time-varying reel rotating frequency and $\alpha(t)$ is the angular rotation of the reel at time t. The magnitude of the runout d_m is 2.54×10^{-5} m. While the radius decreases, the rotating frequency of the reel increases to maintain a constant tangential velocity. The frequency of the tension error is the same as the runout frequency. The magnitude of the tension error depends on the varying radius, as shown in Fig. 3. Other forms of reel eccentricities can be written as the sum of a series of sinusoidal reel runouts if needed.

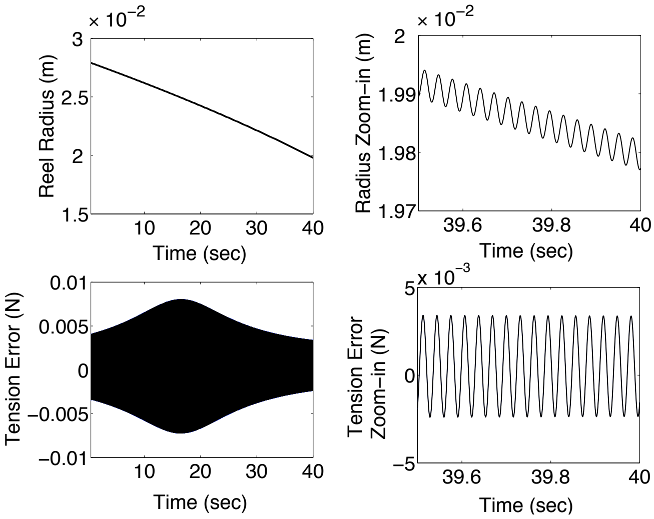

Fig. 3. The tension error caused by sinusoidal reel eccentricities is also sinusoidal at the same frequency as the runout. The magnitude of the error varies as a function of the reel radius.

3. ADAPTIVE CONTROL

In the following development, the fixed controller C_T is a notch controller designed for mid-pack (Lu (2002)):

$$C_T = g \frac{s^2 + \frac{(\beta + 2Dr_m^2)}{J} s + \frac{2Kr_m^2}{J}}{s(s + p_0)},$$

where the gain g is 0.5 and p_0 is 420.

A time-varying feedforward control signal u_a can be designed to attenuate the error in the tension output. Ideally, at any time, the system response to $u_a(t)$ should be the inverse of the tension error. In Zhong et al. (2005), a time-varying adaptive controller is designed to attenuate input disturbances with time-varying frequencies to a LTI system. We extend the algorithm here to account for the time-varying nature of the plant. The adaptive controller C_A is constructed as in Fig. 4.

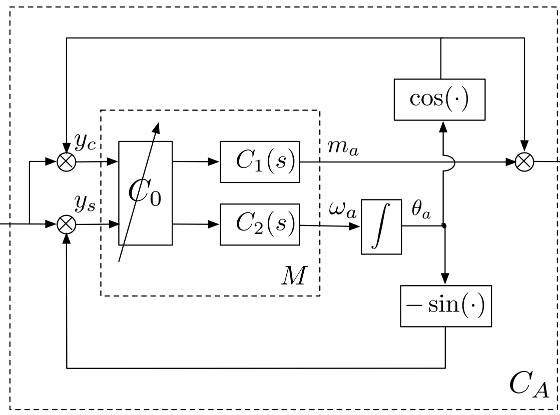

Fig. 4. C_A is a time-varying controller that adapts the frequency ω_a and magnitude m_a to synthesize an input u_a whose plant response ideally is the inverse of the tension error caused by reel eccentricities.

The idea of this algorithm is to generate the compensation input u_a by adapting the magnitude m_a and the frequency ω_a from the error signal T_e. The adaptation rule is an extension to an algorithm that rejects a sinusoidal disturbance with constant frequency ω_0 to a LTI system P (Bodson et al.(2001)). Let m_a, ω_a, and θ_a denote the estimates of magnitude, frequency, and phase angle of the compensation input, respectively. Bodson et al. (2001) maps the vector $[y_c \; y_s]^\top$ into the parameters $[m_a \; \omega_a]^\top$ with a constant operator M defined as

$$M = \text{diag}(C_1(s), C_2(s))Q^{-1}.$$

Here, $C_1(s)$ and $C_2(s)$ are two fixed compensators chosen to guarantee the stability and the convergence speed of the adaptation loops; the matrix Q is

$$Q = \frac{1}{2}\left[\begin{array}{cc} P_R(j\omega) & -P_I(j\omega) \\ P_I(j\omega) & P_R(j\omega) \end{array}\right]\Bigg|_{\omega=\omega_0},$$

where $P_R(j\omega) \doteq \text{Re}(P(j\omega))$ and $P_I(j\omega) \doteq \text{Im}(P(j\omega))$.

In the tape tension loop shown in Fig. 2, the frequency of the disturbance in the output is an integer multiple of the reel rotating frequency. The dynamics from the plant input current $u_n + u_a$ to the tension output y, i.e., the plant input sensitivity dynamics (Astrom and Murray (2008)), are parameter-varying instead of LTI because of the time-varying G_T. So the matrix Q is a function of the reel radius $r(t)$ and the time-varying frequency of the tension error caused by the reel runout. More specifically in this study, the tape tangential velocity is set to 4 m/sec. When the reel is full ($r = 0.028$ m), the rotating frequency is 143 rad/sec. For an empty reel ($r = 0.014$ m), the rotating frequency is 286 rad/sec. The magnitude and phase of the closed tension loop in Fig. 2 for different reel radii and hence different rotating frequencies is illustrated in Fig. 5.

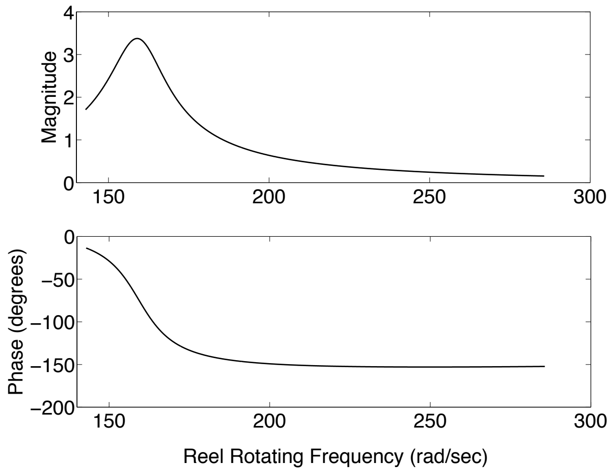

Fig. 5. Magnitude and phase of the plant input sensitivity dynamics as a function of the time-varying reel radius evaluated at the corresponding rotating frequency.

In the adaptation, the mapping operator M needs to address the time-varying plant input sensitivity dynamics at corresponding frequencies. Since the reel eccentricities are unknown, we use $r_n(t)$ for $r(t)$ in G_T to compute the approximate plant input sensitivity function S. During an entire winding process in which the whole pack of tape winds from a full source reel to an empty take-up reel, S is evaluated at a total number of L operating points. Here, L is determined based on the variations in the magnitude of the frequency response of the plant input sensitivity dynamics to corresponding disturbance frequencies. At each operating point, a radius value is chosen to represent a certain reel radius range and the plant input sensitivity dynamics is denoted as $S(\ell)$. The frequency response of $S(\ell)$ at the corresponding rotating frequency is computed and used in the adaptive controller.

Define

$$C_0(\ell) = \frac{1}{2}\left[\begin{array}{cc} S(\ell, j\omega)_R & -S(\ell, j\omega)_I \\ S(\ell, j\omega)_I & S(\ell, j\omega)_R \end{array}\right]\Bigg|_{\omega=\omega_d(t)}, \ell = 1, 2, \cdots, L,$$

where $S(\ell, j\omega)_R \doteq \text{Re}(S(\ell, j\omega))$, $S(\ell, j\omega)_I \doteq \text{Im}(S(\ell, j\omega))$, and $\omega_d(t)$ is the frequency of the tension error. We thus have a series of mapping operators

$$M(\ell) = \text{diag}(C_1(s), C_2(s))C_0(\ell)^{-1}, \ell = 1, 2, \cdots, L.$$

for different operating points.

4. SIMULATION RESULTS

The reel is assumed to be an elliptical shape with eccentricity 0.1 in the following simulations. The tension error caused by the elliptical reel runout is sinusoidal with a magnitude on the order of 10^{-3} N as shown in Fig. 6. The frequency of the tension error and the reel runout is twice the reel rotating frequency as a result of the symmetry of the ellipse. The magnitude and phase of the plant input sensitivity dynamics as a function of the time-varying reel radius evaluated at the corresponding tension error frequency is time-varying as illustrated in Fig. 7.

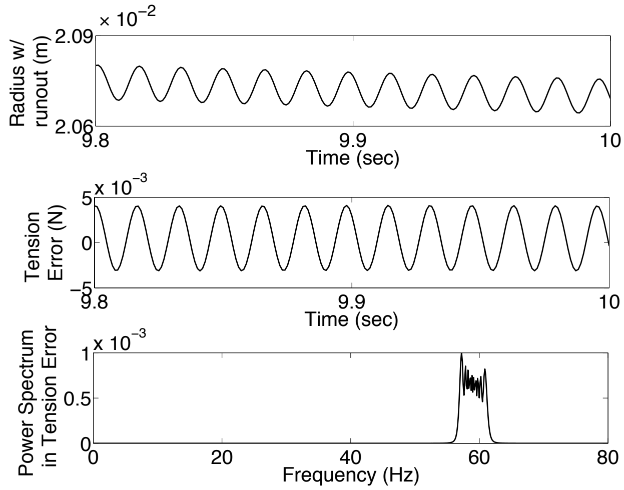

Fig. 6. The tension error caused by elliptical reel runout is on the order of 10^{-3} N and the dominant frequency in the tension error is time-varying.

In the simulation, we choose $C_1(s) = \frac{5}{s}$ and $C_2(s) = \frac{10000(s+10)}{s(s+100)}$ so that the poles of the two adaptation loops are stable and the convergence speed is acceptable. Fig. 8 shows the simulated tension error when the plant input sensitivity dynamics S is evaluated at mid-pack for the entire process. Fig. 9 illustrates the tension error when S is evaluated at two operating points ($\ell = 2$). The first operating point covers the nominal reel radius changing from full reel to mid-pack and the second covers the remaining portion from mid-pack to empty reel. In this initial work, the adaptive controller switches from the first operating point to the second without any special consideration to ensure continuity or smoothness of the plant control input $u_n + u_a$. The reel radius of the first operating point is 0.028

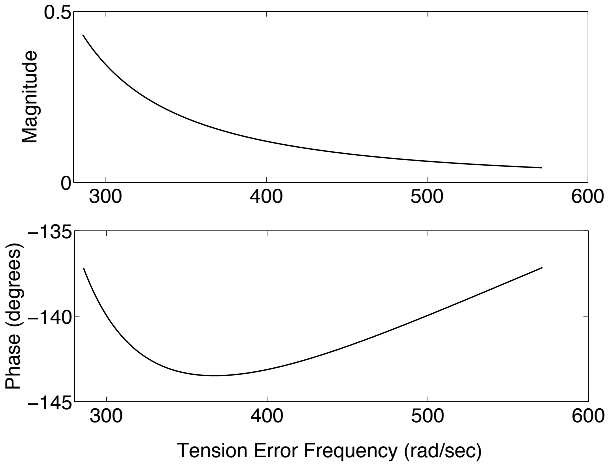

Fig. 7. The magnitude and phase of the plant input sensitivity dynamics evaluated at the corresponding disturbance frequencies when the reel has an elliptical shape.

m and the other is 0.022 m. After convergence, the tension error is attenuated by about 80%. With the gain-scheduling controller, the adaptive controller converges much faster and the transient tension error during convergence is much smaller. More operating points can also be selected.

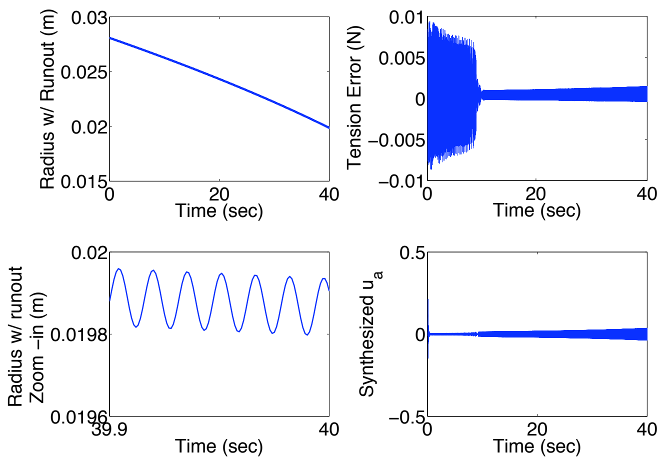

Fig. 8. Simulation results when the plant input sensitivity dynamics S is only evaluated at r_m.

5. CONCLUSIONS, DISCUSSION, AND FUTURE WORK

A control scheme to reject disturbances caused by reel eccentricities in parameter-varying web-winding systems has been discussed. The nominal reel radii of web-winding systems change when the web winds between reels. The unknown reel eccentricities aggravate the time-varying nature of the web-winding system and introduce disturbances in the web tension. To attenuate the disturbances, the control algorithms need to take into account both the nominal variation and the eccentricities in the reel radii.

The outlined scheme extends an algorithm that was originally designed for periodic disturbance rejection in LTI systems. Within a certain time period when the nominal radii change

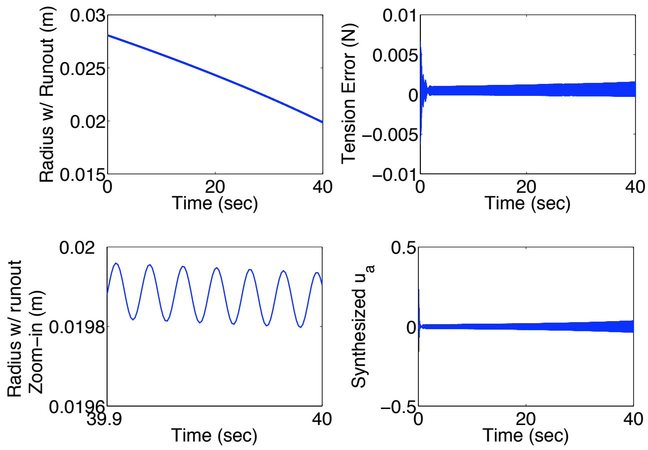

Fig. 9. Simulation results when the plant input sensitivity dynamics S is evaluated at two operating points.

is small and the plant can be considered as invariant, the disturbances caused by reel eccentricities are periodic at the rotation period. When the whole pack of web winds from a full reel to an empty reel, a series of adaptive controllers are designed for a certain number of operating points. Each operating point represents a certain range of the nominal reel radius. The algorithm is applied to a model of a digital tape system and simulation results show that it attenuates the reel eccentricity disturbances in tape tension.

There are a few issues worth further addressing in the future. First, since the reel eccentricities are unknown, the plant model applied in the adaptive controller is an approximation of the actual plant. Thus the computed compensation inputs are not ideally accurate and hence there are residual errors. Second, the algorithm is designed for sinusoidal reel eccentricities. Other forms of reel eccentricities can be written as a sum of harmonics and the algorithm can be extended to account for multiple harmonics. Third, the performance of the adaptive controller relies on how to assign operating points and the transient process when switching between operating points may need special attention. Future work will focus on these issues with the goal of eliminating tension error.

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