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Extended Kalman Filter for Identification of Nonlinear Earthquake Responses of Bridges

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Abstract: Identification of the nonlinear hysteretic behavior of a reinforced concrete (RC) bridge pier subjected to earthquake loads is carried out based on acceleration measurements of the earthquake motion and bridge responses. The modified Takeda model is employed to represent the hysteretic behavior of the RC pier with a small number of parameters, in which the nonlinear behavior is described by various rules of loading and reloading rather than analytical expressions. The sequential modified extended Kalman filter algorithm is proposed to identify the unknown nonlinear parameters and the state vector separately in two steps, so that the size of the problem for each identification procedure may be reduced and possible numerical problems may be avoided.

1. INTRODUCTION

For the health monitoring of civil infrastructures, it is important to identify the nonlinear behavior related to structural damage. Various system identification techniques are available for the identification of nonlinear structural dynamic systems (Yun and Shinozuka 1980, Hoshiya and Saito 1984, Lee and Yun, 1991; Loh and Chung, 1993; Yoshida and Sato , 2002; Yang et al. 2006).

The forces induced on a bridge structure with reinforced concrete (RC) piers during major earthquakes may exceed the yield capacity of some piers and cause large inelastic deformations and damages in the piers as depicted in Fig. 1. The modified Takeda model (Roufaiel and Meyer, 1987) can effectively reproduce such complex nonlinear hysteretic behavior of RC members with a limited number of parameters.

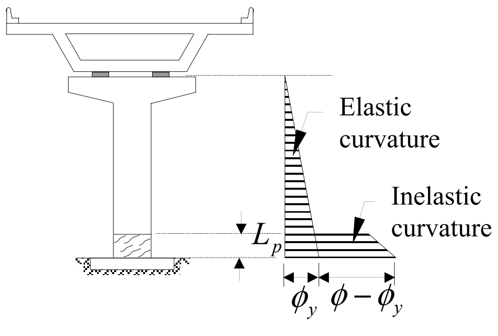


Fig. 1. Inelastic deformation of a RC bridge pier

In this study, identification of the nonlinear hysteric behavior of a RC bridge pier subjected to earthquake loads is carried out. Only the acceleration measurements of the input earthquake motion and bridge responses are utilized, which are the easiest quantities in dynamic measurements, particularly for bridges with long-spans.

A tow-step approach so called the sequential modified extended Kalman filter (SMEKF) algorithm with mode superposition with a modal sorting technique is proposed to identify the unknown parameters and the state vector separately in two steps, so that the size of the problem for each identification procedure may be reduced and possible numerical problems may be avoided. Example analyses are carried out for a continuous bridge model with a RC pier subjected to earthquake loads in the longitudinal and transverse directions.

2. NONLINEAR HYSTERIC BEHAVIOR OF RC

In this study, the modified Takeda model with axial force effect is employed for identification of the nonlinear hysteretic behavior of a RC bridge pier subjected to earthquake excitation.

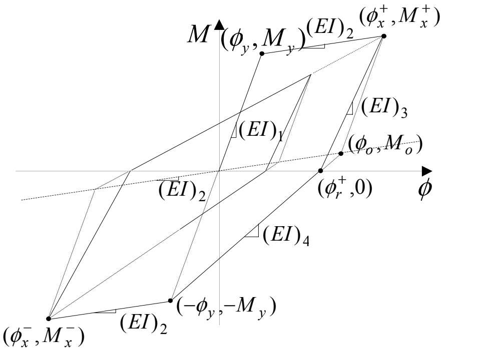
2.1 Moment-Curvature Curve for Cyclic Loading by the Modified Takeda Model

Under load reversals, the stiffness of a RC section may experience degradation due to the cracking of the concrete and slip of the reinforcing bar. In the modified Takeda model, four different kinds of braches may exist in the hysteresis of the moment-curvature ($M-\phi$) relationship as in Fig. 2, and each branch is defined as

- *Elastic loading and unloading*; $(EI)_1 = (EI)_e$, where $(EI)_e$ is the elastic stiffness of the RC member.
- *Inelastic loading*; $(EI)_2 = \alpha (EI)_e$ after yield point (ϕ_y, M_y) , where ϕ_y and M_y are the yield curvature

and yield moment, and α is the ratio of the post yield stiffness to the elastic stiffness

- *Inelastic unloading*; $(EI)_3$, where two points (ϕ_o, M_o) and $(\phi_r^+, 0)$ are defined in Fig. 2.
- *Inelastic reloading*; $(EI)_4$, which may be determined as in Fig. 2.



$$\phi_o = \frac{1}{1-\alpha}\left[\phi_x^+ - \frac{M_x^+}{(EI)_1}\right], M_o = \frac{\alpha}{1-\alpha}[\phi_x^+ (EI)_1 - M_x^+]$$

$$(EI)_3 = \frac{M_x^+}{\phi_x^+ - \phi_r^+}, \quad (EI)_4 = \begin{cases} \dfrac{-M_y - M_o}{-\phi_y - \phi_o} & \text{at the 1st cycle} \\ \dfrac{M_x^- - M_o}{\phi_x^- - \phi_o} & \end{cases}$$

where $\phi_r^+ = \phi_o - \dfrac{M_o}{(EI)_4}$

Fig. 2. Hysteretic moment - curvature behavior of the modified Takeda model

In Fig. 2, (ϕ_x^+, M_x^+) and (ϕ_x^-, M_x^-) are the maximum previous excursions in the positive and negative directions, which are to be updated along with $(EI)_3$ and $(EI)_4$ as the hysteresis proceeds.

2.2 Significant hysteretic behavior of RC member

Stiffness degradation; Under the load reversals well into the inelastic range, the stiffness of a reinforced concrete member decreases due to the cracking in concrete and slip of reinforced bars. As a consequence, a reduction in the overall structural stiffness occurs as in Fig 3(a).

Pinching effect by shear force; To reflect the pinching effect, Roufaiel and Meyer (1987) proposed a modification of the reloading branch as shown in Fig. 3(b). The characteristic point (ϕ_p, M_p) on the original elastic loading curve is determined as

$$\phi_p = \varepsilon\phi_n \;\; ; \;\; M_p = \varepsilon M_n \tag{1}$$

where $\varepsilon = 0$ for $a/d < 1.5$, $\varepsilon = 0.4(a/d) - 0.6$ for $1.5 < (a/d) < 4.0$, $\varepsilon = 1$ for $(a/d) \geq 4.0$; $a =$ the shear span length; $d =$ the effective depth of the section; and (ϕ_n, M_n) is the crossing pint of the reloading curve and the initial elastic loading curve.

Pinching effect by axial force; When RC member is subjected to an axial load, the moment-curvature relation may alter. The following empirical formula was adapted for M^* in Fig. 3(c) as

$$M^* = M_x \cdot \exp\left(\beta \frac{\phi_x}{\phi_y} \frac{P}{|P_0|}\right) \tag{2}$$

where (ϕ_x, M_x) represents the maximum previous excursion in the opposite direction; ϕ_y is the corresponding curvature at yield; P is the axial compressive force; P_o is the nominal compressive strength; and β is the parameter which controls the pinching behavior by the axial force.

Strength deterioration; If a RC member is strained beyond a certain critical level during cyclic loadings, its strength may deteriorate as shown in Fig. 3(d). The following strength drop index is used.

$$\Delta M = \gamma M_y \left(\frac{\phi_x - \phi_y}{\phi_y}\right)^{1.5} \tag{3}$$

where ΔM is the moment capacity reduction in a single load cycle up to curvature ϕ; γ is the strength deterioration parameter.

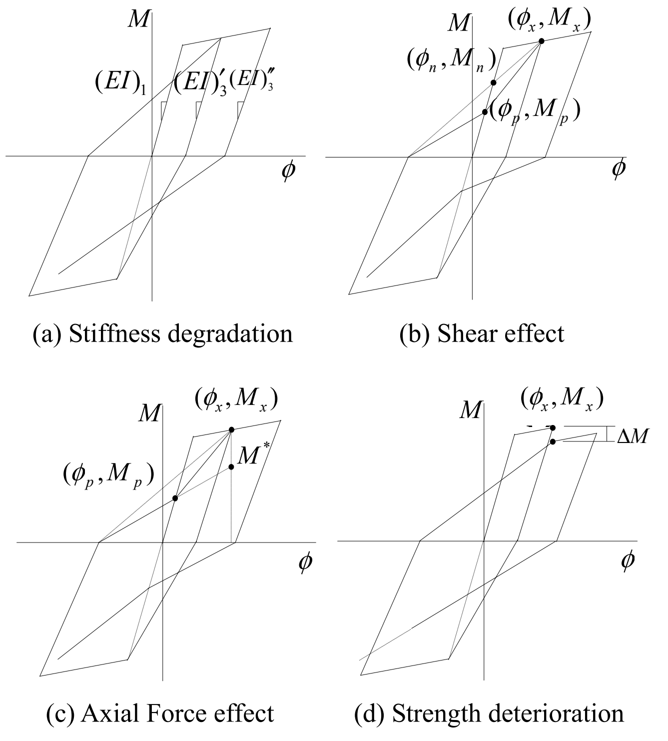


Fig. 3. Significant hysteretic behavior of a RC member

3. NONLINEAR DYNAMIC ANALYSIS BY MODE SUPERPOSITION WITH MODAL SORTING

Nonlinear dynamic analysis is generally carried out by means of a direct step-by-step integration, which involves a large number of degrees of freedom (DOFs) and high computational cost. However, a dynamic formulation with a large number of DOFs may cause serious difficulty in the present nonlinear system identification problem. Hence modal superposition with a modal sorting scheme is employed in approximation in the nonlinear dynamic analysis procedure, so that the size of the identification problem may be reduced and the efficiency and accuracy of the parameter identification may be improved.

3.1 Equation of Motion

If a structure is subjected to a severe earthquake, some weak elements may experience damage and the dynamic response of the structural system becomes nonlinear, which can be generally described by a nonlinear equation of motion as

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) + R(t) = -M\{L\}\ddot{u}_g(t) \tag{4}$$

where M , C and K are the mass, damping, and initial stiffness matrix; $U(t)$, $\dot{U}(t)$, $\ddot{U}(t)$ are the displacement, velocity, and acceleration vectors; $\{L\}$ is the influence vector accounting the direction of the earthquake excitation; $\ddot{u}_g(t)$ is the ground acceleration, and $R(t)$ is the nonlinear residual force vector. If a mode superposition method is used to approximate and to reduce the present nonlinear dynamic problem, and if a diagonal modal damping is assumed, a series of modal equations of motion can be obtained from (4) as

$$\ddot{q}_n(t) + 2\varsigma_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \bar{f}_n(t)\ , n = 1, 2, \cdots, l \tag{5}$$

where $q_n(t), \dot{q}_n(t)$ and $\ddot{q}_n(t)$ are the modal displacement, velocity and acceleration for the *n*-th mode; ζ_n and ω_n are the corresponding damping ratio and natural frequency; and $\bar{f}_n(t)$ is the modal load which includes the nonlinear residual force. Hence, the above modal equations can be solved iteratively at each time by updating the nonlinear residual force.

3.2 Mode Superposition Method with Modal Sorting

When the mode superposition method is applied for the analysis of the dynamic systems, the truncation of modes may cause significant difficulty in obtaining reasonable dynamic response. Therefore, a modal sorting technique is proposed to select the modes with larger contribution to the DOF near the damaged location. The *j*-th modal contribution to the *i*-th DOF Ξ_{ij} under earthquake load nay be evaluated as

$$\Xi_{ij} = \phi_{ij}\Gamma_j S_j \tag{6}$$

where ϕ_{ij} is the *j*-th eigenvector at the *i*-th DOF, Γ_j is the modal participation factor at the *j*-th mode; S_j is the deformation response spectrum of the ground motion at the *j*-th natural period at $\omega = \omega_j$. The modes are sorted by the order of the magnitudes of those modal contribution values for a specific DOF. With the sorted modal vectors, the global displacement vector can be obtained as

$$U(t) = \tilde{\Phi}Q(t) \tag{7}$$

where $\tilde{\Phi}$ is the matrix of the sorted eigen-vectors matrix, and $Q(t)$ is the corresponding modal displacement vector.

4. NONLINEAR PARAMETER IDENTIFICATION OF BRDGES USING EXTENDED KALMAN FILTER

For identifying the hysteretic behavior of a RC bridge pier under severe earthquake loads, the extended Kalman filtering (EKF) technique is utilized in this study.

4.1 State Equation

At first a series of the modal equations of motion in discrete time, (5), are transformed into a general form of a nonlinear discrete state equation at $t = k\Delta t$ as

$$X_{k+1} = G(X_k, \bar{f}_k(X_k); k) + w_k \tag{8}$$

where X_k is the augmented state vector including the modal displacements, velocities, accelerations and the unknown parameters (M_y, α, β and γ as defined in Section 2) as

$$\begin{aligned} X_k = [&q_1(k)\ \dot{q}_1(k)\ \ddot{q}_1(k)\ \cdots\ q_l(k)\ \dot{q}_l(k)\ \ddot{q}_l(k) \\ &M_y(k)\ \alpha(k)\ \beta(k)\ \gamma(k)]^T \end{aligned} \tag{9}$$

where l is the number of modes included; and w_k is a system noise vector with a covariance Q .

In (8)-(9), the state vector includes the acceleration terms in addition to the conventional state vector consisting of displacement and velocity terms. Acceleration records are easier to measure than displacement and velocity, particularly for bridges with long-spans, hence only acceleration measurements are utilized in the present identification problem. Then the observation equation can be written as

$$Y_k = h(X_k; k) + v_k \tag{10}$$

where Y_k is the acceleration measurement vector, which contains the relative accelerations at the selected nodes to the ground motion, and v_k is the observation noise vector with a covariance R_k.

4.2 Sequential Modified Extended Kalman Filter

In this study, two steps approach so called the sequential modified extended Kalman filter (SMEKF) is proposed. At first the state vector consisting of only the system responses is estimated based on the concurrent estimates of the parameters using the EKF. Then the unknown parameters are identified based on the estimated state using the sequential prediction error method (Lee and Yun 1991, Yun et al. 1997). The SMEKF procedure is summarized in Fig. 4.

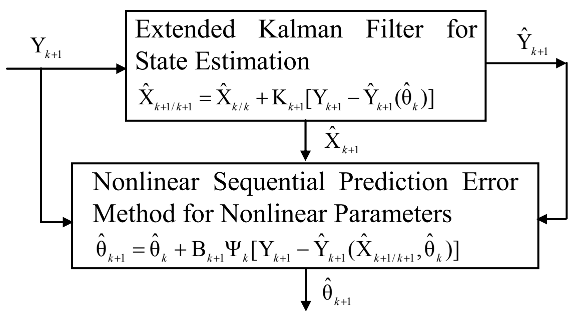


Fig. 4. Sequential Modified Extended Kalman Filter (SMEKF) Algorithm

The extended Kalman filter (EKF) is used for the sequential state estimation based upon the observed data of the response as

- *Prediction*;

$$\hat{X}_{k+1|k} = G(\hat{X}_{k|k}, \bar{f}_k) \tag{11}$$

$$P_{k+1|k} = \Phi_{k+1|k} P_{k|k} \Phi^T_{k+1|k} + Q_k \tag{12}$$

where P_k is the error covariance matrix, and

$$\Phi_{k+1|k} = I + \Delta t \left.\frac{\partial G(X, f; t)}{\partial X}\right|_{X=\hat{X}_{k|k}} \tag{13}$$

- *Filtering*;

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}[Y_{k+1} - H_{k+1}\hat{X}_{k+1|k}] \tag{14}$$

$$\begin{aligned} P_{k+1|k+1} &= [I - K_{k+1}H_{k+1}]P_{k+1|k}[I - K_{k+1}H_{k+1}]^T \\ &\quad + K_{k+1}R_k K_{k+1}{}^T \end{aligned} \tag{15}$$

$$K_{k+1} = P_{k+1|k}H^T_{k+1}[H_{k+1}P_{k+1|k}H^T_{k+1} + R_k]^{-1} \tag{16}$$

where K_k is the Kaman gain matrix, and

$$H_{k+1} = \left[\frac{\partial h_i(X_k; k)}{\partial X_j}\right]_{X_k = \hat{X}_{k+1/k}} \tag{17}$$

The EKF is based on the first order Taylor approximation of the state transition equation on the estimated state trajectory. However, there are limitations if the first order derivatives of the nonlinear terms are not well defined as in the modified Takeda model used in the present study, so that the state transition matrix $\Phi_{k+1|k}$ may not be adequately evaluated. Schei (1997) proposed an algorithm with a finite difference scheme to improve this limitation of the EKF as

$$\Phi_{k+1|k} = \left[\frac{\partial G_i(X)}{\partial X_j}\right] \approx \frac{1}{2e_j}\left[G_i(X + e_j) - G_i(X - e_j)\right] \tag{18}$$

where e_j is small perturbation in the *j*-th component of the state vector X.

In the sequential prediction error method, the adaptation gain matrix B_{k+1} can be obtained as

$$B^{-1}_{k+1} = B^{-1}_k + \Psi_k \Psi^T_k \tag{19}$$

In (19), Ψ_k is the Jacobian matrix of the observation error vector function $e(k+1, \hat{\theta}_k)$ and can be evaluated approximately using a finite difference scheme similarly to the state transition matrix $\Phi_{k+1/k}$ in (18).

5. EXAMPLES FOR IDENTIFICATION OF HYSTERETIC BEHAVIOR OF RC PIERS

5.1 Two-Span Continuous Bridge Model in Longitudinal Direction

This example is a simplified continuous bridge model with a pier in the middle of the deck. It is assumed that a scaled El Centro earthquake (NS, peak ground acceleration (pga) = 0.15g, 1940) is applied in the longitudinal direction, so the effect of the deck may be considered as an additional lumped mass on the top of the pier.

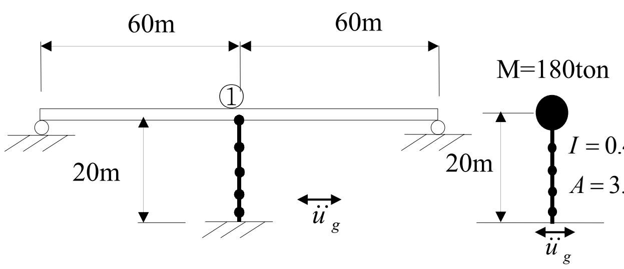
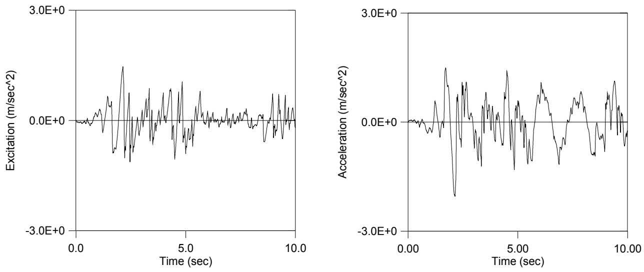


Fig. 5. Two span continuous bridge model subjected earthquake excitation in longitudinal direction

The first natural frequency of this model is obtained as 0.64 Hz, while the damping ratio is assumed as 5% viscous damping for each mode. It is also assumed that 3% in RMS random noises are included in the measured ground excitation and acceleration response. Fig. 6 shows the measured input earthquake acceleration and relative acceleration response at the top of the pier.



(a) Ground acceleration (b) Relative acceleration at Node ①

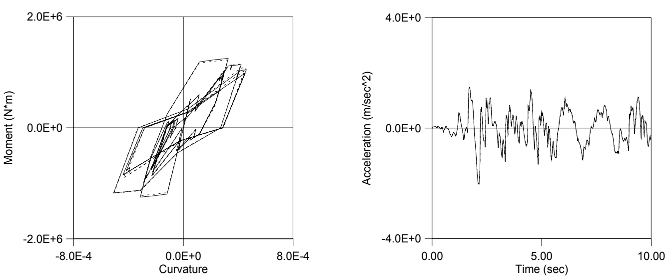
Fig. 6. Input excitation and nonlinear responses

In this example, the performance of two system identification techniques, i.e. the SMEKF and EKF, is compared Table 1 and Figs. 7 and 8 show that the SMEKF has provided excellent estimates for the nonlinear parameters with the acceleration response measurement only at the top of the pier, if the first 3 modes are used in the dynamic analysis.

Table 1. Estimated parameters of bridge pier for earthquake in longitudinal direction

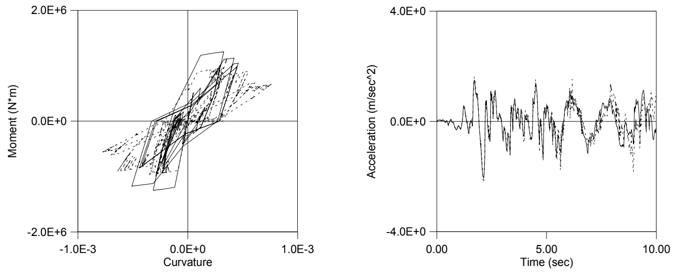
Nonlinear Parameters		M_y (KN·m)	α	β	γ
Exact Values		1200.0	0.0100	1.0000	0.0500
Initial guesses		600.0	0.0050	0.5000	0.0250
w/ 1 mode	SMEKF	849.5	0.0090	1.1373	0.0349
	EKF	901.3	0.0122	0.5649	0.0421
w/ 3 modes	SMEKF	1175.5	0.0093	1.0876	0.0352
	EKF	892.9	0.0122	1.6470	0.0345

The results also show the limitation of the EKF for multi-degrees of freedom with a very limited number of the observation response; i.e. one acceleration measurement at the top of the pier. The recalculated acceleration responses at the top of the pier using the identified nonlinear parameters are compared with the exact value in Figs. 7(b) and 8(b). The response recalculated using the identified nonlinear parameters by the SMEKF have been found to coincide very well with the exact value.



(a) $M-\phi$ relationship (b) Acceleration Response

Fig. 7. Recalculated hystereses and acceleration response using the estimates by the SMEKF and modal sorting (_____ : Exact and - - - - - : Estimated)

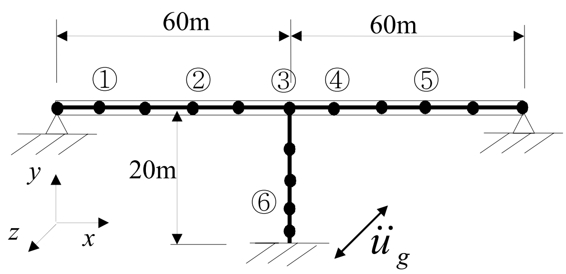


(a) $M-\phi$ relationship (b) Acceleration Response

Fig. 8. Recalculated hystereses and acceleration response using the estimates by the EKF and modal sorting (_____ : Exact and - - - - - : Estimated)

5.2 Two-Span Continuous Bridge Model in Transverse Direction

The second example is a two span continuous bridge model subjected to an earthquake load in the transverse direction. The bottom of the bridge pier is assumed to be damaged by a scaled El Centro earthquake (NS, pga = 0.4g, 1940). The acceleration responses in the transverse direction are assumed to be measured at 5 points on the bridge deck and 1 point near the bottom of the pier. It is assumed that 3% noises in RMS level are also included in the excitation and response measurements.



Deck : Total mass = 180ton
$A_d = 6\text{m}^2$, $I_z = 5\text{m}^4$
$I_y = 90\text{m}^4$, $J_x = 95\text{m}^4$
Pier : $A_p = 3.1\text{m}^2$, $I_x = 0.4\text{m}^4$

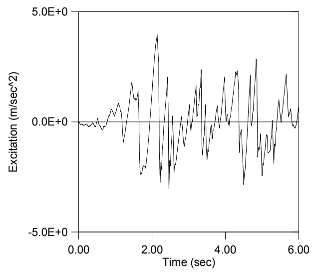


Fig. 9. Two span continuous bridge model and input ground motion in the transverse direction

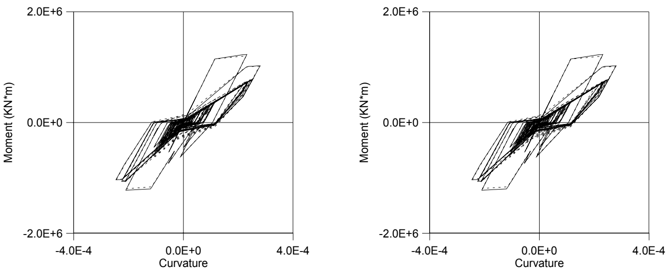
The fundamental natural frequency of this bridge model is obtained as 9.3 Hz in the transverse direction (the 3rd mode). The viscous damping ratio is assumed as 5% for each mode. In this example, the mode superposition with modal sorting is utilized to reduce the problem size for the system identification. The modal contribution values in (6) were estimated at Node ⑥ near the bottom of the bridge pier in Fig. 9, in which local damage was expected during the earthquake.

Table 2 shows two estimated nonlinear parameters using various numbers of the sorted modes and SMEKF. The accuracy of the estimate for β is not so good as shown in this table. However, very good estimate has been obtained for the hysteresis as shown in Fig. 11, which indicates the insensitivity of the parameter for the overall hysteretic behavior in the present example case.

Table 2. Estimated parameters of bridge pier pier for earthquake in transverse direction

Nonlinear Parameters	M_y (KN·m)	α	β	γ
Exact Values	1200.0	0.0100	1.0000	0.2000
Initial guesses	600.0	0.0050	0.5000	0.1000
w/ 2-modes	1156.8	0.0096	0.4956	0.1674
w/ 5-modes	1174.2	0.0096	0.4894	0.1789
w/ 12-modes	1167.8	0.0096	0.5509	0.1789

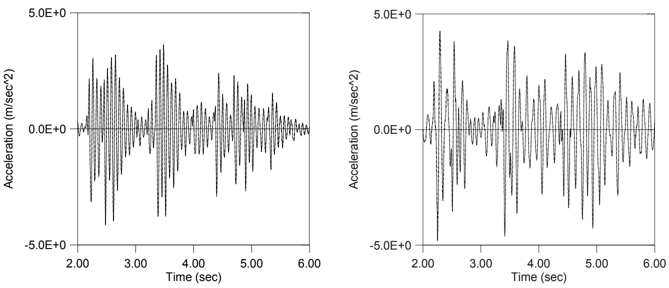
The results indicate that the accuracy of the estimated parameters got improved with the increasing number of the modes included. Reasonable estimates of the parameters were obtained with the SMEKF, which can reproduce excellent hystereses as shown in Fig. 10.



(a) with 5-modes (b) with 12-modes

Fig. 10. Estimated $M-\phi$ relationships by the SMEKF and modal sorting (_____ : Exact and - - - - - : Estimated)

In Fig. 11, the recalculated acceleration responses with the identified parameters using 12 sorted modes are compared with the exact responses. It can be found that very accurate responses have been estimated.



(a) Accel. at Node ③ (b) Accel. at Node ⑥

Fig. 11. Recalculated acceleration responses by the SMEKF with 12 sorted modes (_____ : Exact and - - - - - : Estimated)

6. CONCLUSIONS

The hysteretic behavior of a RC bridge pier is modeled using the modified Takeda model. As the modified Takeda model defined the hysteresis with various rules of loading and reloading, a finite difference scheme is employed in the conventional extended Kalman filter (EKF) for calculating the state transition matrix. A mode superposition method with modal sorting is proposed to reduce the problem size for the nonlinear system identification using Kalman filtering techniques. The sequential modified extended Kalman filter (SMEKF) is also proposed to improve the convergence and to prevent the erroneous estimation results in practical structural dynamic system with a large number of DOFs, in which the EKF is used for the state estimation and the nonlinear sequential prediction error method is for the parameter identification.

From the various example analyses of bridge structures, it has been found that both of the SMEKF and a mode superposition with modal sorting technique are very effective to identify the nonlinear hysteretic behavior and parameters involved in a locally damaged bridge pier with a limited measurement data for the acceleration responses of the bridge structure. The system identification for nonlinear structural dynamic systems was customarily carried out with displacement or velocity responses. However, in this study a nonlinear parameter identification method has been developed using the acceleration measurements only, which are much easier to measure in the practical bridge structures, such as long-span bridges.

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