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Observer-Based H_∞ Control for Stochastic Systems with Delays and Nonlinear Perturbations: LMI Approach ⋆

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Abstract: This paper considers the problem of observer-based H_∞ control for a class of Itô-type stochastic delay systems with nonlinear perturbations. An observer-based controller is constructed based on Lyapunov-Krasovskii approach, which guarantees the closed-loop system is robustly stochastically asymptotically stable in the mean square with prescribed H_∞ disturbance attenuation level for all admissible nonlinear perturbations. Sufficient condition for the existence of desired controller is presented in terms of a strict linear matrix inequality (LMI) if the control matrix B is full column rank. A numerical example is provided to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

In many dynamical systems, the states may not be available, thus the existing stabilization methods based on state feedback are not applicable to these systems. In such situations, the controller based on a state observer is very useful to stabilize unstable systems or optimize the performances and dynamical responses of the systems (O'Reilly [1983]). Moreover, the observer has been successfully used to many industrial fields, such as fault detection, temperature control, phase synchronization of chaotic systems, and so on (Tarantino et al. [2000], Mattei [2001], Jana et al. [2006], Meng and Wang [2007]).

Time delays are frequently encountered in many practical systems, the existence of time delays may cause poor performances and instability to the systems. The observer design and observer-based controller design for delay systems have been attracted considerable interests in the past decade. The results of Choi and Chung [1994], Wang et al. [2002] have been obtained by Ricatti-like equations. Based on LMI approach, Lu and Ho [2004], Xu et al. [2004] have established observers for discrete-time uncertain delay systems. Guaranteed cost observer-based control problem for uncertain delay systems with parametric uncertainties has been addressed in Lien [2005a]. The results of Lien [2005a,b] are expressed by means of LMIs with matrix equalities constraints instead of strict LMIs. Although Park [2004] has been formulated by strict LMI, it is conservative since there are some strict requirements on the relationships between the gain matrices and the Lyapunov matrices. By using the singular value decomposition (SVD) technique in Ho and Lu [2003], Chen [2007a,b] have proposed observer-based control laws for uncertain stochastic time-delay systems in terms of LMIs if the measured matrix C if full row rank.

However, the above-mentioned reports are all for deterministic systems, but not for stochastic systems. The study of stochastic systems is very important in both theoretical and practical senses, since many practical systems can be modeled by stochastic differential equations (Mao [1997]).

In this paper, we will deal with observer-based robust H_∞ control for a class of Itô-type stochastic delay systems with nonlinear perturbations. Based on Lyapunov-Krasovskii method and singular value decomposition (SVD) technique, an observer-based controller is designed by means of a strict linear matrix inequality (LMI) if the control matrix B is full column rank. For all admissible nonlinear perturbations, the desired controller ensures that the closed-loop system is robustly stochastically asymptotically stable in the mean square with a prescribed H_∞ disturbance attenuation level $\gamma > 0$. The effectiveness of the method is illustrated by a numerical example.

Notations: Throughout this paper, the notations are standard. $P > 0$ ($P < 0$) means that the matrix P is positive (negative) definite and symmetric; $\|\cdot\|$ refers to the Euclidean norm; $\mathrm{diag}\{A_1, A_2, ..., A_n\}$ denotes a diagonal matrix with diagonal elements $A_1, A_2, ..., A_n$. $\mathcal{E}\{\cdot\}$ represents the expectation operator. $(\Omega, \mathbf{F}, \mathcal{P})$ is a probability space, where Ω is the sample space, $\mathbf{F}$ is the σ-algebra of subsets of Ω, and $\mathcal{P}$ is the probability measure on $\mathbf{F}$. The symmetric term in a symmetric matrix is denoted as $*$.

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2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following system

$$\begin{cases} \mathrm{d}x(t) = [Ax(t) + A_1x(t-d) + f(t,x(t)) \\ \qquad\qquad +g(t,x(t-d)) + Bu(t) + B_v v(t)]\mathrm{d}t \\ \qquad\qquad +[Ex(t) + E_1x(t-d)]\mathrm{d}w(t) \\ y(t) = Cx(t) + C_1u(t) + C_2v(t) \\ z(t) = Dx(t) \\ x(\theta) = \phi(\theta), \quad \forall\theta \in [-d,0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the measurement vector, $z(t) \in \mathbb{R}^q$ is the regulated output, $u(t) \in \mathbb{R}^m$ is the input vector and $v(t) \in \mathbb{R}^l$ is the disturbance input. $d > 0$ is the delay. A, A_1, B, B_v, E, E_1, C, C_1, C_2, D are known real matrices with compatible dimensions. The initial condition function is given by $\phi(\cdot)$, where $\phi(\cdot)$ is a continuously differentiable function on $[-d, 0]$. $w(t)$ is a scalar Wiener process defined on $(\Omega, \mathbf{F}, \mathcal{P})$ satisfying $\mathcal{E}\{dw(t)\} = 0, \mathcal{E}\{dw(t)^2\} = dt$. The functions $f(t, x(t))$, $g(t, x(t-d))$ are unknown nonlinear perturbations satisfying the following Lipschitz condition.

Assumption 1. It is assumed that $f(t, 0) = 0$, $g(t, 0) = 0$ for all $x, y \in \mathbb{R}^n$

$$\begin{aligned} \| f(t,x) - f(t,y) \| &\leq \| F(x-y) \|, \\ \| g(t,x) - g(t,y) \| &\leq \| G(x-y) \|, \end{aligned} \tag{2}$$

where F, $G \in \mathbb{R}^{n\times n}$ are known real constant matrices.

Definition 2. For the admissible nonlinear perturbations (2), system (1) with $u(t) = 0$, $v(t) = 0$ is said to be robustly stochastically stable in the mean square, if for any scalar $\epsilon > 0$ there exists a scalar $\sigma(\epsilon) > 0$ such that

$$\mathcal{E}\{\|x(t)\|^2\} < \epsilon, \forall t > 0$$

when

$$\sup_{-h\leq s\leq 0} \mathcal{E}\{\|\phi(s)\|^2\} < \sigma(\epsilon).$$

Additionally, system (1) with $u(t) = 0$, $v(t) = 0$ is said to be robustly stochastically asymptotically stable in in the mean square, if

$$\lim_{t\to\infty} \mathcal{E}\{\|x(t)\|^2\} = 0.$$

Assumption 3. It is assumed that system (1) is controllable and observable.

Now, we consider a full-order observer-based controller for system (1) as follows

$$\begin{cases} d\hat{x}(t) = [A\hat{x}(t) + A_1\hat{x}(t-d) + f_{\hat{x}} + g_{\hat{x}} \\ \qquad\qquad +Bu(t)]dt + L[y(t) - \hat{y}(t)]dt \\ \qquad\qquad +[E\hat{x}(t) + E_1\hat{x}(t-d)]dw(t) \\ \hat{y}(t) = C\hat{x}(t) + C_1u(t) + C_3v(t) \\ u(t) = -K\hat{x}(t) \end{cases} \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x(t)$, $\hat{y}(t) \in \mathbb{R}^m$ is the observer output, $L \in \mathbb{R}^{n\times p}$ and $K \in \mathbb{R}^{m\times n}$ are the observer and controller gains to be designed, respectively. C_3 is a known real matrix with appropriate dimensions.

For the sake of simplicity, the following denotations are adopted in this paper

$$\begin{aligned} f &= f(t,x(t)), \\ f_{\hat{x}} &= f(t,\hat{x}(t)), \\ g &= g(t,x(t-d)), \\ g_{\hat{x}} &= g(t,\hat{x}(t-d)). \end{aligned} \tag{4}$$

Introduce the estimated error vector $e(t) = x(t) - \hat{x}(t)$, and the augmented vector $\xi(t) = [x^T(t) \quad e^T(t)]^T$, then the corresponding closed-loop augmented system is

$$\begin{cases} d\xi(t) = [\bar{A}\xi(t) + \bar{A}_1\xi(t-d) + \bar{f} + \bar{g} + \bar{B}_v v(t)]dt \\ \qquad\qquad +[\bar{E}\xi(t) + \bar{E}_1\xi(t-d)]dw(t) \\ z(t) = \bar{D}\xi(t) \end{cases} \tag{5}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \\ \bar{A}_1 &= \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix}, \bar{B}_v = \begin{bmatrix} B_v \\ B_v - L(C_2 - C_3) \end{bmatrix}, \\ \bar{f} &= \begin{bmatrix} f \\ f - f_{\hat{x}} \end{bmatrix}, \bar{g} = \begin{bmatrix} g \\ g - g_{\hat{x}} \end{bmatrix}, \\ \bar{E} &= \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \bar{E}_1 = \begin{bmatrix} E_1 & 0 \\ 0 & E_1 \end{bmatrix}, \\ \bar{D} &= [\, D \ 0 \,]. \end{aligned} \tag{6}$$

Define the following performance index

$$J = \mathcal{E}\{\int_0^\infty [z^T(t)z(t) - \gamma^2 v^T(t)v(t)]dt\} \tag{7}$$

where $\gamma > 0$ is a given scalar indicating the disturbance attenuation level.

The objective of this paper is to design an observer-based controller with the form of (3) such that system (5) is robustly stochastically asymptotically stable in the mean square with a prescribed level of disturbance attenuation $\gamma > 0$ (i.e. $J < 0$), for all admissible nonlinear perturbations (2).

The following lemma will be very useful to obtain the main result.

Lemma 4. Let $P_1 \in \mathbb{R}^{n\times n}$ be a symmetric matrix, $B \in \mathbb{R}^{n\times m}$ be full column rank (i.e. $\mathrm{rank}(B) = m \leq n$) with the following singular value decomposition (SVD) form

$$B = U \begin{bmatrix} S \\ 0 \end{bmatrix} V^T, \tag{8}$$

then there exists a matrix $\hat{P}_1 \in \mathbb{R}^{m\times m}$ such that $P_1B = B\hat{P}_1$ if and only if

$$P_1 = U \begin{bmatrix} \hat{P}_{11} & 0 \\ 0 & \hat{P}_{22} \end{bmatrix} U^T \tag{9}$$

where $U \in \mathbb{R}^{n\times n}$, $V \in \mathbb{R}^{m\times m}$ are unitary matrices, and $S \in \mathbb{R}^{m\times m}$, $\hat{P}_{11} \in \mathbb{R}^{m\times m}$, $\hat{P}_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$.

Proof: This proof is similar to that of Lemma 3 in Ho and Lu [2003]. If $\mathrm{rank}(B) = m \leq n$, then it has SVD as in the form of (8).

The matrix equality $P_1B = B\hat{P}_1$ can be rewritten as

$$P_1U \begin{bmatrix} S \\ 0 \end{bmatrix} V^T = U \begin{bmatrix} S \\ 0 \end{bmatrix} V^T\hat{P}_1,$$

$$P_1U \begin{bmatrix} S \\ 0 \end{bmatrix} = U \begin{bmatrix} S \\ 0 \end{bmatrix} V^T\hat{P}_1V = U \begin{bmatrix} SV^T\hat{P}_1V \\ 0 \end{bmatrix}.$$

If

$$P_1 = U \begin{bmatrix} \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{12}^T & \hat{P}_{22} \end{bmatrix} U^T,$$

where $\hat{P}_{11} \in \mathbb{R}^{m\times m}, \hat{P}_{12} \in \mathbb{R}^{m\times(n-m)}, \hat{P}_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$, then we have

$$\begin{aligned} P_1 U \begin{bmatrix} S \\ 0 \end{bmatrix} &= U \begin{bmatrix} \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{12}^T & \hat{P}_{22} \end{bmatrix} \begin{bmatrix} S \\ 0 \end{bmatrix} \\ &= U \begin{bmatrix} \hat{P}_{11} S \\ \hat{P}_{12}^T S \end{bmatrix} = U \begin{bmatrix} SV^T \hat{P}_1 V \\ 0 \end{bmatrix}. \end{aligned}$$

Thus, $\hat{P}_{12} = \hat{P}_{12}^T = 0$, and P_1 satisfies (9). Moreover,

$$P_1 B = U \begin{bmatrix} \hat{P}_{11} SV^T \\ 0 \end{bmatrix} = B\hat{P}_1 = U \begin{bmatrix} SV^T \hat{P}_1 \\ 0 \end{bmatrix},$$

$\hat{P}_{11} SV^T = SV^T \hat{P}_1$, and

$$\hat{P}_1 = VS^{-1}\hat{P}_{11}SV^T, \hat{P}_1^{-1} = VS^{-1}\hat{P}_{11}^{-1}SV^T. \quad (10)$$

3. MAIN RESULTS

In this section, we will design observer-based H_∞ controller for the system (1) by using strict LMI approach.

Theorem 5. Consider system (1) with $\mathrm{rank}(B) = m \le n$ and its SVD as in the form of (8). For a given scalar $\gamma > 0$, if there exist positive scalars ϵ_1, ϵ_2, matrices $Y \in \mathbb{R}^{m\times n}, Z \in \mathbb{R}^{n\times p}$ and positive definite symmetric matrices $\hat{P}_{11} \in \mathbb{R}^{m\times m}$, $\hat{P}_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$, $P_2, Q_1, Q_2, R_1, R_2 \in \mathbb{R}^{n\times n}$ satisfying $P_1 = U \begin{pmatrix} \hat{P}_{11} & 0 \\ 0 & \hat{P}_{22} \end{pmatrix} U^T > 0$ and

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & \Gamma_3 \end{bmatrix} < 0 \quad (11)$$

where

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} \mathcal{M}_1 & -BY & P_1 A_1 & 0 \\ * & \mathcal{M}_2 & 0 & P_2 A_1 \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} \\ \Gamma_2 &= \begin{bmatrix} E^T P_1 & 0 & P_1 & 0 & P_1 & 0 & P_1 B_v \\ 0 & E^T P_2 & 0 & P_2 & 0 & P_2 & P_2 B_v - Z(C_2 - C_3) \\ E_1^T P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_1^T P_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \Gamma_3 &= -\mathrm{diag}\{P_1, P_2, \epsilon_1 I, \epsilon_1 I, \epsilon_2 I, \epsilon_2 I, \gamma^2 I\} \\ \mathcal{M}_1 &= P_1 A + A^T P_1 + BY + Y^T B^T + D^T D \\ &\quad + Q_1 + \epsilon_1 F^T F + \epsilon_2 G^T G, \\ \mathcal{M}_2 &= P_2 A + A^T P_2 - ZC - C^T Z^T \\ &\quad + Q_2 + \epsilon_1 F^T F + \epsilon_2 G^T G. \end{aligned} \quad (12)$$

then (3) is an observer-based H_∞ controller of system (1), the corresponding controller and observer gain matrices are given by

$$\begin{aligned} K &= VS^{-1}\hat{P}_{11}^{-1}SV^T Y, \\ L &= P_2^{-1} Z. \end{aligned} \quad (13)$$

Proof: Choose the Lyapunov-Krasovskii functional candidate as

$$V(t, \xi_t) = V_1(t, \xi_t) + V_2(t, \xi_t) \quad (14)$$

where

$$\begin{aligned} V_1(t, \xi_t) &= \xi^T(t) P \xi(t), \\ V_2(t, \xi_t) &= \int_{t-d}^{t} \xi^T(s) Q \xi(s) \mathrm{d}s, \end{aligned} \quad (15)$$

with

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0, Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0. \quad (16)$$

In view of Itô differential rule (Mao [1997]), the stochastic differential of $V(t, \xi_t)$ with respect to t along system (5) (with $v(t) = 0$) gives

$$\begin{aligned} dV(t, \xi_t) &= \mathcal{L}V(t, \xi(t))dt \\ &\quad + 2\xi^T(t) P[\bar{E}\xi(t) + \bar{E}_1 \xi(t-d)]dw(t), \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathcal{L}V(t, \xi_t) \; = \; & 2\xi^T(t) P[\bar{A}\xi(t) + \bar{A}_1 \xi(t-d) + \bar{f} + \bar{g}] \\ & + [\bar{E}\xi(t) + \bar{E}_1 \xi(t-d)]^T \\ & \times P[\bar{E}\xi(t) + \bar{E}_1 \xi(t-d)] \\ & + \xi^T(t) Q \xi(t) - \xi^T(t-d) Q \xi(t-d). \end{aligned}$$

It follows from Assumption 1 that

$$\begin{aligned} \| f \| &\le \| Fx(t) \|, \\ \| f - f_{\hat{x}} \| &\le \| Fe(t) \|, \\ \| g \| &\le \| Gx(t) \|, \\ \| g - g_{\hat{x}} \| &\le \| Ge(t) \| \end{aligned} \quad (18)$$

and

$$\begin{aligned} \bar{f}^T \bar{f} &\le \xi^T(t) \bar{F} \xi(t), \\ \bar{g}^T \bar{g} &\le \xi^T(t) \bar{G} \xi(t), \end{aligned} \quad (19)$$

where

$$\bar{F} = \begin{bmatrix} F^T F & 0 \\ 0 & F^T F \end{bmatrix}, \bar{G} = \begin{bmatrix} G^T G & 0 \\ 0 & G^T G \end{bmatrix}. \quad (20)$$

It is clear that for any scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$

$$\begin{aligned} \mathcal{L}V(t, \xi_t) &\le \mathcal{L}V(t, \xi_t) + \epsilon_1(\xi^T(t)\bar{F}\xi(t) - \bar{f}^T \bar{f}) \\ &\quad + \epsilon_2(\xi^T(t)\bar{G}\xi(t) - \bar{g}^T \bar{g}) \\ &\le \theta^T(t) \Phi \theta(t) \end{aligned} \quad (21)$$

where $\theta^T(t) = [\xi^T(t) \quad \xi^T(t-d) \quad \bar{f}^T \quad \bar{g}^T]$ and

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P & P \\ * & \Phi_{22} & 0 & 0 \\ * & * & -\epsilon_1 I & 0 \\ * & * & * & -\epsilon_2 I \end{bmatrix} \quad (22)$$

with

$$\begin{aligned} \Phi_{11} &= \bar{A}^T P + P\bar{A} + Q + \bar{E}^T P \bar{E} + \epsilon_1 \bar{F} + \epsilon_2 \bar{G}, \\ \Phi_{12} &= P\bar{A}_1 + \bar{E}^T P \bar{E}_1, \\ \Phi_{22} &= -Q + \bar{E}_1^T P \bar{E}_1. \end{aligned}$$

By stochastic stability theory (Mao [1997]), if $\Phi < 0$ then $\mathcal{L}V(t, \xi_t) < 0$, which inplies that system (5) is robustly stochastically asymptotically stable in the mean square.

Employing Schur complement lemma (Boyd et al. [1994]), $\Phi < 0$ is equivalent to

$$\Phi_0 = \begin{bmatrix} \mathcal{M}_0 & P\bar{A}_1 & \bar{E}^T P & P & P \\ * & -Q & \bar{E}_1^T P & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 \\ * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0 \quad (23)$$

where $\mathcal{M}_0 = \bar{A}^T P + P\bar{A} + Q + \epsilon_1 \bar{F} + \epsilon_2 \bar{G}$.

According to Lemma 4, for any matrix $\mathrm{rank}(B) = m$, U, S and V can be easily obtained via SVD (8). For any symmetric matrix $P_1 \in \mathbb{R}^{n\times n}$, there exists a matrix $\hat{P}_1 \in \mathbb{R}^{m\times m}$ such that $P_1 B = B\hat{P}_1$ with $\hat{P}_1 = VS^{-1}\hat{P}_{11}SV^T$, which implies $P_1BK = B\hat{P}_1K$.

Substituting (6) and (20) into (23) and setting

$$\begin{aligned}\hat{P}_1K &= Y,\\ P_2L &= Z,\end{aligned} \tag{24}$$

result in

$$\Delta = \begin{bmatrix} \Delta_1 & \Delta_2 \\ * & \Delta_3 \end{bmatrix} < 0 \tag{25}$$

where

$$\begin{aligned}
\Delta_1 &= \begin{bmatrix} \mathcal{M}_3 & -BY & P_1A_1 & 0 \\ * & \mathcal{M}_4 & 0 & P_2A_1 \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix}\\
\Delta_2 &= \begin{bmatrix} E^TP_1 & 0 & P_1 & 0 & P_1 & 0 \\ 0 & E^TP_2 & 0 & P_2 & 0 & P_2 \\ E_1^TP_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_1^TP_2 & 0 & 0 & 0 & 0 \end{bmatrix}\\
\Delta_3 &= -\operatorname{diag}\{P_1, P_2, \epsilon_1 I, \epsilon_1 I, \epsilon_2 I, \epsilon_2 I\}\\
\mathcal{M}_3 &= P_1A + A^TP_1 + BY + Y^TB^T\\
&\quad + Q_1 + \epsilon_1 F^TF + \epsilon_2 G^TG,\\
\mathcal{M}_4 &= \mathcal{M}_2.
\end{aligned} \tag{26}$$

It can be seen that $\Delta < 0$ is implied by $\Gamma < 0$. Therefore, system (5) is robustly stochastically asymptotically stable in the mean square for all admissible nonlinear perturbations if $\Gamma < 0$. The controller and observer gain matrices can be determined as, from (24)

$$\begin{aligned}K &= \hat{P}_1^{-1}Y = VS^{-1}\hat{P}_{11}^{-1}SV^TY,\\ L &= P_2^{-1}Z.\end{aligned} \tag{27}$$

On the other hand, for any nonzero $v(t)$, the stochastic differential of the (14) along system (5) is

$$\begin{aligned}dV(t,\xi_t) &= \mathcal{L}V_v(t,\xi_t)dt\\ &\quad + 2\xi^T(t)P[\bar{E}\xi(t) + \bar{E}_1\xi(t-d)]dw(t)\end{aligned} \tag{28}$$

where

$$\begin{aligned}\mathcal{L}V_v(t,\xi_t) &= 2\xi^T(t)P[\bar{A}\xi(t) + \bar{A}_1\xi(t-d)\\ &\quad + \bar{f} + \bar{g} + \bar{B}_v v(t)]\\ &\quad + [\bar{E}\xi(t) + \bar{E}_1\xi(t-d)]^T\\ &\quad \times P[\bar{E}\xi(t) + \bar{E}_1\xi(t-d)]\\ &\quad + \xi^T(t)Q\xi(t) - \xi^T(t-d)Q\xi(t-d).\end{aligned}$$

It can be followed that for any scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$

$$\begin{aligned}\mathcal{L}V_v(t,\xi_t) &\le \mathcal{L}V_v(t,\xi_t) + \epsilon_1(\xi^T(t)\bar{F}\xi(t) - \bar{f}^T\bar{f})\\ &\quad + \epsilon_2(\xi^T(t)\bar{G}\xi(t) - \bar{g}^T\bar{g})\\ &\le \theta_v^T(t)\Phi\theta_v(t)\end{aligned} \tag{29}$$

where $\theta_v^T(t) = [\xi^T(t) \quad \xi^T(t-d) \quad \bar{f}^T \quad \bar{g}^T \quad v^T(t)]$ and

$$\Phi_0 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P & P & P\bar{B}_v \\ * & \Phi_{22} & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & 0 \end{bmatrix} \tag{30}$$

with $\Phi_{11}, \Phi_{12}, \Phi_{22}$ defined in (22).

Noting the zero initial condition and the asymptotic mean-square stability, for any nonzero $v(t)$, we have

$$\mathcal{E}\{\int_0^\infty \mathcal{L}V_v(t,\xi_t)dt\} = \mathcal{E}\{\int_0^\infty dV(t,\xi_t)\} = 0,$$

and

$$\begin{aligned}J &= \mathcal{E}\{\int_0^\infty [z^T(t)z(t) - \gamma^2 v^T(t)v(t)]dt\}\\ &= \mathcal{E}\{\int_0^\infty [z^T(t)z(t) - \gamma^2 v^T(t)v(t) + \mathcal{L}V_v(t,\xi_t)]dt\}\\ &\le \mathcal{E}\{\int_0^\infty \theta_v^T(t)\Psi\theta_v(t)dt\}\end{aligned} \tag{31}$$

where

$$\Psi = \Phi_0 + \operatorname{diag}\{\bar{D}^T\bar{D}, 0, 0, 0, 0, 0, -\gamma^2 I\}. \tag{32}$$

Applying Lemma 4 and Schur complement lemma again, it can be deduced that $\Psi < 0$ is equivalent to $\Gamma < 0$.

Thus, if $\Gamma < 0$, then $\Psi < 0$ and $J < 0$, which guarantees system (5) robustly stochastically asymptotically stable in the mean square with a specified disturbance attenuation level γ. This means that the observer-based controller (3) is a robust H_∞ controller of the system (1). The controller and observer gain matrices are constructed by (13). The proof is completed. ■

Remark 6. If the control matrix B is full column rank, i.e. $\operatorname{rank}(B) = m \le n$, then we can develop a robust H_∞ controller by means of strict LMI for system (1) shown as Theorem 5. The condition that B is full column rank can be satisfied in many practical situations.

Remark 7. It should be pointed out that the results of Lien [2005a,b] are expressed by means of LMIs with the matrix equality $P_1B = BP_1$. The LMI with some matrix equality, which can be solved by free software SCILAB, can not be solved directly by Matlab LMI Toolbox.

Remark 8. If the augmented vector is chosen as $\eta(t) = [\hat{x}^T(t) \quad e^T(t)]^T$, we have the following augmented system

$$\begin{cases} d\eta(t) = [\tilde{A}\eta(t) + \tilde{A}_1\eta(t-d)\\ \qquad\quad + \tilde{f} + \tilde{g} + \tilde{B}v(t)]dt\\ \qquad\quad + [\tilde{E}\eta(t) + \tilde{E}_1\eta(t-d)]dw(t)\\ z(t) = \tilde{D}\eta(t)\end{cases} \tag{33}$$

where

$$\begin{aligned}\tilde{A} &= \begin{bmatrix} A+BK & LC \\ 0 & A-LC \end{bmatrix}, \tilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix},\\ \tilde{B}_v &= \begin{bmatrix} 0 \\ B_v \end{bmatrix}, \tilde{f} = \begin{bmatrix} f_{\hat{x}} \\ f - f_{\hat{x}} \end{bmatrix},\\ \tilde{g} &= \begin{bmatrix} g_{\hat{x}} \\ g - g_{\hat{x}} \end{bmatrix}, \tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix},\\ \tilde{E}_1 &= \begin{bmatrix} E_1 & 0 \\ 0 & E_1 \end{bmatrix}, \tilde{D} = [\, D \;\; 0 \,].\end{aligned} \tag{34}$$

Then we can obtain the corresponding result, which is formulated in terms of an LMI if the matrix C is full row rank. The details are omitted here.

Remark 9. It should be noted that the LMI (11) is independent on the matrix C_1, because it is eliminated by the form of the observer (3).

Remark 10. Theorem 5 is delay-independent. By adopting the method provided in Chen et al. [2004], Xia et al. [2007], Gershon et al. [2007], Chen and Xue [2008], Chen et al. [2008], we can obtain delay-dependent result that may be less conservative than Theorem 5.

4. NUMERICAL EXAMPLE

In this section, a numerical example will be provided to demonstrate the effectiveness of the presented approach in Section 3.

Example 11. Consider system (1) with the following parameters

$$
\begin{aligned}
A &= \begin{bmatrix} 1.5 & 1 & -2 \\ 2 & -2 & 1.5 \\ 1 & 0.6 & -3 \end{bmatrix}, A_1 = \begin{bmatrix} 1.6 & 1.2 & -1 \\ 1.1 & 1.8 & 2 \\ -3 & 1.5 & 1 \end{bmatrix}, \\
B &= \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \\ -0.5 & 1 \end{bmatrix}, B_v = \begin{bmatrix} 0.2 & -0.8 \\ 0.6 & 0.5 \\ -0.2 & 0.3 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & 1.2 & 0.5 \\ 2 & 1 & 0.5 \end{bmatrix}, \\
E &= E_1 = I, F = G = 0.1I.
\end{aligned} \tag{35}
$$

It is obvious that the control matrix B is full column rank. The minimal disturbance attenuation level for this example is $\gamma_{\min} = 0.0002$, by solving the LMI (11). When $\gamma = 0.5$, the controller and observer gains are given by

$$
\begin{aligned}
K &= \begin{bmatrix} -12.5853 & 1.1061 & 5.0746 \\ 2.4897 & -6.0522 & -6.3558 \end{bmatrix}, \\
L &= \begin{bmatrix} -247.6303 & 126.3515 \\ -143.6455 & 165.8887 \\ 45.5314 & 14.3938 \end{bmatrix}.
\end{aligned} \tag{36}
$$

5. CONCLUSIONS

Observer-based H_∞ control for a class of stochastic delay systems with nonlinear perturbations has been addressed in this paper. Base on Lyapunov-Krasovskii approach and SVD technique, an observer-based H_∞ controller has been designed. The designed controller ensures the closed-loop augmented system is robustly stochastically asymptotically stable in the mean square with a prescribed disturbance attenuation level γ for all admissible nonlinear perturbations. If B is full column rank, then the existence of such controller can be presented in terms of a strict LMI. The effectiveness of the result has been shown by an illustrative example.

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