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Delay-Dependent Robust H_∞ Control of Uncertain Stochastic Delayed Systems ⋆

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Abstract: This paper is concerned with robust H_∞ control for uncertain stochastic time-delay systems with norm-bounded parametric uncertainties. Based on an integral inequality and slack matrix technique, delay-dependent bounded real lemma (BRL) and the condition for the existence of robust H_∞ controller are presented. For all the admissible parametric uncertainties, the designed controller guarantees the resulting closed-loop system is robustly mean-square asymptotically stable with a prescribed H_∞ disturbance attenuation level. The results are formulated in terms of linear matrix inequalities (LMIs). Both model transformation and cross terms bounding techniques are avoided in the derivations. Two numerical examples are provided to show the advantage of the proposed method.

1. INTRODUCTION

Time delays may occur in many practical systems and they may cause instability and poor performance to the systems. Analysis and synthesis of time-delay systems have been received considerable attention in the past decade, see Cao and Xue [2005], Chen et al. [2007, 2008a], de Souza and Li [1999], Fridman and Shaked [2002, 2003], Gao et al. [2007], Gao and Wang [2003], Gu et al. [2003], Han [2005], Han and Yue [2007], He et al. [2007, 2004], Jiang and Han [2007], Lin et al. [2006], Moon et al. [2001], Richard [2003], Suplin et al. [2006], Zhang et al. [2005] and the references therein.

On the other hand, stochastic modeling and control play important roles in many industrial fields. During the past years, increasing efforts have been made on the study of stochastic systems with time delays. LMI techniques have been applied to obtain delay-dependent stability conditions for uncertain stochastic delay systems, see for example Yue and Won [2001], Mao [2002], Yue et al. [2003], Yue and Han [2005], Chen et al. [2005], Xu et al. [2005], Chen et al. [2008a,b,c], and the references therein. Robust H_∞ control problems for uncertain stochastic continuous- and discrete-time systems with delays have been addressed in Xu and Chen [2002, 2004], Xu et al. [2006] and Xu et al. [2004], respectively. These designed controllers guarantee that the closed-loop systems are robustly mean-square asymptotically stable for all admissible uncertainties with specified H_∞ disturbance attenuation degrees. $L_2 - L_\infty$ filtering for such systems has been stated in Gao et al. [2006]. However, the approaches of Xu and Chen [2002, 2004], Xu et al. [2004, 2006], Gao et al. [2006] are all independent on the delays.

Based on Lyapunov-Krasovskii method, Chen et al. [2004] has provided delay-dependent approach to develop controllers by the descriptor transformation together with estimation for cross terms. Most recently, based on free-weighting matrix method, some delay-dependent results have been reported by Xu et al. [2005], Yue and Han [2005], Chen et al. [2008b,c]. In Xia et al. [2007], the authors have discussed delay-dependent $L_2 - L_\infty$ filtering for stochastic delay systems by introducing some slack matrices. Based on input-output method, delay-dependent H_∞ control and filtering for uncertain time-delay systems with state-multiplicative noises have been presented in Gershon et al. [2007]. By this approach, the system is transformed into a deterministic one without delay.

In this paper, Lyapunov-Krasovskii theory is used to deal with delay-dependent robust H_∞ control for a class of stochastic time-delay systems with norm-bounded uncertainties. Based on a stochastic integral inequality (integral inequality method for deterministic delayed systems, please see Han [2005], Zhang et al. [2005], Jiang and Han [2007] etc.) and slack matrix technique, delay-dependent BRL and the condition for the existence of robust H_∞ controller are established in terms of LMIs. The designed controller ensures that the resulting closed-loop systems is robustly mean-square asymptotically stable with a prescribed H_∞ disturbance attenuation level. We avoid the use of any model transformations and bounding techniques for cross terms. The effectiveness of our approaches is verified by two illustrative examples.

Notations: Throughout this paper, the notations are fairly standard. The superscripts "T" and "-1" stand for the transpose and the inverse of a matrix; $|\cdot|$ denotes the Euclidean norm; $\mathbb{R}^n$ is n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; $\text{diag}\{A_1, A_2, ..., A_n\}$

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represents a diagonal matrix with diagonal elements $A_1, A_2, ..., A_n$; $P > 0$ ($P < 0$) means that the matrix P is positive (negative) definite and symmetric. $\mathcal{E}\{\cdot\}$ denotes the expectation operator. $(\mathbf{\Omega}, \mathcal{F}, \mathcal{P})$ is a probability space, where $\mathbf{\Omega}$ is the sample space, and $\mathcal{F}$ is a σ-algebra of subsets of $\mathbf{\Omega}$. The symmetric term in a symmetric matrix is denoted as $*$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following system

$$\begin{cases} dx(t) = [A(t)x(t) + A_1(t)x(t-h) \\ \quad + B(t)u(t) + B_v v(t)]dt \\ \quad + [H(t)x(t) + H_1(t)x(t-h) \\ \quad + H_v v(t)]dw(t) \\ z(t) = Cx(t) + Du(t) \\ x(\theta) = \psi(\theta), \quad \forall\theta \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $v(t) \in \mathbb{R}^q$ is the disturbance input, $z(t) \in \mathbb{R}^p$ is the controlled output. $h > 0$ is the delay. $\psi(\cdot)$ here is the initial condition assumed to be continuously differentiable on $[-h, 0]$. It is assumed that $w(t)$ is a scalar Wiener process defined on the probability space $(\mathbf{\Omega}, \mathcal{F}, \mathcal{P})$ satisfying $\mathcal{E}\{dw(t)\} = 0, \mathcal{E}\{dw^2(t)\} = dt$. In (1),

$$\begin{aligned} A(t) &= A + \Delta A, \\ A_1(t) &= A_1 + \Delta A_1, \\ B(t) &= B + \Delta B, \\ H(t) &= H + \Delta H, \\ H_1(t) &= H_1 + \Delta H_1, \end{aligned} \tag{2}$$

and A, A_1, B, H, H_1, C, D are known real constant matrices with compatible dimensions, ΔA, ΔA_1, ΔB, ΔH, ΔH_1 are time-varying parametric uncertainties, which can be described by

$$[\,\Delta A \;\, \Delta A_1 \;\, \Delta B \;\, \Delta H \;\, \Delta H_1\,] = LF(t)\,[\,E_1 \;\, E_2 \;\, E_3 \;\, E_4 \;\, E_5\,], \tag{3}$$

where $L, E_1, E_2, E_3, E_4, E_5$ are constant matrices with compatible dimensions, and $F(t)$ is an unknown and time-varying matrix function satisfying $F^T(t)F(t) \le I$.

Definition 1. System (1) with $u(t) = 0$, $v(t) = 0$ is said to be robustly mean-square stable for all admissible uncertainties (3), if for any scalar $\epsilon > 0$ there exists a scalar $\sigma(\epsilon) > 0$ such that

$$\mathcal{E}\{|\,x(t)\,|^2\} < \epsilon, \forall t > 0$$

when

$$\sup_{-h \le s \le 0} \mathcal{E}\{|\psi(s)|^2\} < \sigma(\epsilon).$$

Additionally, system (1) with $u(t) = 0$, $v(t) = 0$ is said to be robustly mean-square asymptotically stable, if

$$\lim_{t\to\infty} \mathcal{E}\{|x(t)|^2\} = 0.$$

holds for any initial conditions.

Definition 2. System (1) with $u(t) = 0$ is said to be robustly mean-square asymptotically stable with a prescribed H_∞ disturbance attenuation level γ for all admissible uncertainties (3), if it is robustly mean-square asymptotically stable in the sense of Definition 1 and $J < 0$ under zero initial conditions, where J is a performance index which is defined as

$$J = \mathcal{E}\{\int_0^\infty [|z(t)|^2 - \gamma^2 |v(t)|^2]dt\}. \tag{4}$$

The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (1) such that the resulting closed-loop system is robustly mean-square asymptotically stable with a prescribed H_∞ disturbance attenuation level γ (i.e. $J < 0$).

To derive our main results, the following two lemmas are necessary.

Lemma 3. (Xie [1996]) Let Φ be a given symmetric matrix, H and G are matrices with approximate dimensions, then for all $F(t)$ satisfying $F^T(t)F(t) \le I$, the following inequality

$$\Phi + HF(t)G + G^T F^T(t) H^T < 0$$

holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Phi + \varepsilon HH^T + \varepsilon^{-1} G^T G < 0.$$

Denoting

$$y(t)dt = dx(t) \tag{5}$$

then by Newton-Leibniz formula, the following holds always

$$\int_{t-h}^{t} y(\alpha)d\alpha = x(t) - x(t-h). \tag{6}$$

Moreover, we can obtain the following lemma.

Lemma 4. For any constant symmetric positive definite matrix $R \in \mathbb{R}^{n\times n}$, a positive scalar $h > 0$, and the vector function $y(t) \in \mathbb{R}^n$ such that the following integrals are well defined, then there holds

$$-h\int_{t-h}^{t} y^T(s)Ry(s)ds \le \xi^T(t)\Re\xi(t), \tag{7}$$

where $\xi^T(t) = [x^T(t) \quad x^T(t-h)]^T$ and

$$\Re = \begin{bmatrix} -R & R \\ R^T & -R \end{bmatrix}.$$

Proof Similar to Lemma 2 of Han [2005], (7) can be deduced easily by Jensen's inequality (Gu et al. [2003]).

Remark 5. In lemma 4, $y(t)$ is not equivalent to $\dot{x}(t)$ in the deterministic systems (for instances Han [2005], Zhang et al. [2005], Jiang and Han [2007]), due to the existence of the stochastic perturbation $dw(t)$. Lemma 4 reduces to Proposition 3 of Han [2005] if the stochastic perturbation $dw(t) = 0$.

3. MAIN RESULTS

In this section, the delay-dependent robust H_∞ control problem of system (1) will be discussed by means of standard LMI approach. First, we will establish a delay-dependent stochastic BRL for system (1).

3.1 Stochastic BRL

Theorem 6. For all admissible uncertainties (3), system (1) is robustly mean-square asymptotically stable with a prescribed H_∞ disturbance attenuation level γ, if there exist positive definite symmetric matrices P, Q, $R \in \mathbb{R}^{n\times n}$, matrix $S \in \mathbb{R}^{n\times n}$ and positive scalars $\varepsilon_1, \varepsilon_2$ satisfying the following LMI

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & A^TS & PB_v & H^TP & PL & 0 & C^T \\ * & \Omega_{22} & A_1^TS & 0 & H_1^TP & 0 & 0 & 0 \\ * & * & \Omega_{33} & S^TB_v & 0 & S^TL & 0 & 0 \\ * & * & * & -\gamma^2 I & H_v^TP & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & PL & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned} \Omega_{11} =& PA + A^TP + Q - R + \varepsilon_1 E_1^TE_1 + \varepsilon_2 E_4^TE_4, \\ \Omega_{12} =& PA_1 + R + \varepsilon_1 E_1^TE_2 + \varepsilon_2 E_4^TE_5, \\ \Omega_{22} =& -Q - R + \varepsilon_1 E_2^TE_2 + \varepsilon_2 E_5^TE_5, \\ \Omega_{33} =& h^2R - S - S^T. \end{aligned}$$

Proof: Choose the Lyapunov-Krasovskii functional candidate as

$$V(t, x_t) = V_1(t, x_t) + V_2(t, x_t) + V_3(t, x_t), \tag{9}$$

where

$$\begin{aligned} V_1(t, x_t) &= x^T(t)Px(t), \\ V_2(t, x_t) &= \int_{t-h}^{t} x^T(\alpha)Qx(\alpha)\mathrm{d}\alpha, \\ V_3(t, x_t) &= h\int_{-h}^{0}\int_{t+\beta}^{t} y^T(\alpha)Ry(\alpha)\mathrm{d}\alpha d\beta, \end{aligned}$$

with $P > 0,\ Q > 0,\ R > 0$.

Then by Itô differential formula (Mao [1997]), the stochastic differential $dV(t, x_t)$ along the trajectories of system (1) with $u(t) = 0, v(t) = 0$ is

$$dV(t, x_t) = \mathcal{L}V(t, x_t)dt + 2x^T(t)Pg(t)dw(t), \tag{10}$$

where $g(t) = H(t)x(t) + H_1(t)x(t-h)$,

$$\begin{aligned} \mathcal{L}V(t, x_t) =& 2x^T(t)P\,[\,A(t)x(t) + A_1(t)x(t-h)\,] \\ &+ g^T(t)Pg(t) + \mathcal{L}V_2(t, x_t) + \mathcal{L}V_3(t, x_t), \end{aligned}$$

with

$$\mathcal{L}V_2(t, x_t) = x^T(t)Qx(t) - x^T(t-h)Qx(t-h), \tag{11}$$

and by Lemma 4

$$\begin{aligned} \mathcal{L}V_3(t, x_t) &= h^2 y^T(t)Ry(t) - h\int_{t-h}^{t} y^T(\alpha)Ry(\alpha)\mathrm{d}\alpha \\ &\le h^2 y^T(t)Ry(t) - \xi^T\mathfrak{R}\xi(t). \end{aligned} \tag{12}$$

where $\xi(t)$ and $\mathfrak{R}$ are defined in Lemma 4.

Notice the definition (5) and system (1) with $u(t) = 0, v(t) = 0$, the following is true for any matrix $S \in \mathbb{R}^{n\times n}$

$$\begin{aligned} 0 =& 2y^T(t)S^T\{g(t)dw(t) \\ &+ [A(t)x(t) + A_1(t)x(t-h) - y(t)]dt\}. \end{aligned} \tag{13}$$

From (10) and (13)

$$\begin{aligned} dV(t, x_t) =& \mathcal{L}\tilde{V}(t, x_t)dt \\ &+ 2[x^T(t)P + y^T(t)S^T]g(t)dw(t) \end{aligned} \tag{14}$$

where

$$\begin{aligned} \mathcal{L}\tilde{V}(t, x_t) =& \mathcal{L}V(t, x_t) \\ &+ 2y^T(t)S^T[A(t)x(t) + A_1(t)x(t-h) - y(t)] \\ \le& [\xi^T(t) \quad y^T(t)]\Theta[\xi^T(t) \quad y^T(t)]^T \end{aligned} \tag{15}$$

and

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & A^T(t)S \\ * & \Theta_{22} & A_1^T(t)S \\ * & * & h^2R - S - S^T \end{bmatrix} \tag{16}$$

with

$$\begin{aligned} \Theta_{11} =& PA(t) + A^T(t)P - R + Q + H^T(t)PH(t), \\ \Theta_{12} =& PA_1(t) + R + H^T(t)PH_1(t), \\ \Theta_{22} =& -Q - R + H_1^T(t)PH_1(t). \end{aligned}$$

If $\Theta < 0$, which implies $\mathcal{L}\tilde{V}(t, x_t) < 0$, then the stochastic system (1) is robustly mean-square asymptotically stable by Definition 1 and the stochastic stability theory (Mao [1997]).

Invoking Schur complements (Gu et al. [2003]), $\Theta < 0$ is equivalent to

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & A^T(t)S & H^T(t)P \\ * & \Lambda_{22} & A_1^T(t)S & H_1^T(t)P \\ * & * & h^2R - S - S^T & 0 \\ * & * & * & -P \end{bmatrix} < 0 \tag{17}$$

where

$$\begin{aligned} \Lambda_{11} =& PA(t) + A^T(t)P + Q - R, \\ \Lambda_{12} =& PA_1(t) + R, \\ \Lambda_{22} =& -Q - R. \end{aligned}$$

$\Lambda < 0$ holds if and only if

$$\begin{aligned} \Delta =& \begin{bmatrix} \Delta_{11} & \Delta_{12} & A^TS & H^TP \\ * & \Lambda_{22} & A_1^TS & H_1^TP \\ * & * & h^2R - S - S^T & 0 \\ * & * & * & -P \end{bmatrix} \\ &+ \begin{bmatrix} PL \\ 0 \\ SL \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_1^T \\ E_2^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_1^T \\ E_2^T \\ 0 \\ 0 \end{bmatrix} F^T(t) \begin{bmatrix} PL \\ 0 \\ SL \\ 0 \end{bmatrix}^T \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ PL \end{bmatrix} F(t) \begin{bmatrix} E_4^T \\ E_5^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_4^T \\ E_5^T \\ 0 \\ 0 \end{bmatrix} F^T(t) \begin{bmatrix} 0 \\ 0 \\ 0 \\ PL \end{bmatrix}^T \\ &< 0 \end{aligned} \tag{18}$$

where Λ_{22} is defined in (17), and

$$\begin{aligned} \Delta_{11} =& PA + A^TP + Q - R, \\ \Delta_{12} =& PA_1 + R. \end{aligned}$$

Applying Lemma 3 and Schur complements to $\Delta < 0$ yield

$$\hat{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & A^TS & H^TP & PL & 0 \\ * & \Omega_{22} & A_1^TS & H_1^TP & 0 & 0 \\ * & * & \Omega_{33} & 0 & SL & 0 \\ * & * & * & -P & 0 & PL \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{19}$$

which is implied by $\Omega < 0$.

Then, $\Omega < 0$ will ensure the robust mean-square asymptotic stability of system (1).

Furthermore, the stochastic differential $dV(t, x_t)$ along the trajectories of system (1) with $u(t) = 0$ is

$$\begin{aligned} dV(t,x_t) =& \mathcal{L}V_v(t,x_t)dt \\ &+ 2x^T(t)P[g(t)+H_v v(t)]dw(t), \end{aligned} \quad (20)$$

where

$$\begin{aligned} \mathcal{L}V_v(t,x_t) =& 2x^T(t)P\,[\,A(t)x(t)+A_1(t)x(t-h)+B_v v(t)\,] \\ &+ [g(t)+H_v v(t)]^T P[g(t)+H_v v(t)] \\ &+ \mathcal{L}V_2(t,x_t)+\mathcal{L}V_3(t,x_t), \end{aligned}$$

with $\mathcal{L}V_2(t,x_t), \mathcal{L}V_3(t,x_t)$ defined in (11) and (12).

By (1) and (5), the following holds for any matrix $S \in \mathbb{R}^{n\times n}$

$$\begin{aligned} 0 =& 2y^T(t)S^T\{g(t)dw(t)+[A(t)x(t) \\ &+ A_1(t)x(t-h)+B_v v(t)-y(t)]dt\}. \end{aligned} \quad (21)$$

It follows by (20) and (21)

$$\begin{aligned} dV(t,x_t) =& \mathcal{L}\tilde{V}_v(t,x_t)dt + 2[x^T(t)P+y^T(t)S^T] \\ &\times [g(t)+H_v v(t)]dw(t) \end{aligned} \quad (22)$$

where

$$\mathcal{L}\tilde{V}_v(t,x_t) \le \zeta^T(t)\hat{\Theta}\zeta(t) \quad (23)$$

and $\zeta^T(t) = [\xi^T(t) \quad y^T(t) \quad v^T(t)]$

$$\hat{\Theta} = \begin{bmatrix} \Theta_{11} & \Theta_{12} & A^T(t)S & PB_v + H^T(t)PH_v(t) \\ * & \Theta_{22} & A_1^T(t)S & H_1^T(t)PH_v(t) \\ * & * & \Omega_{33} & S^TB_v \\ * & * & * & H_v^T(t)PH_v(t) \end{bmatrix}. \quad (24)$$

Under zero initial condition and the mean-square asymptotic stability, we have by Itô formula,

$$\begin{aligned} J =& \mathcal{E}\{\int_0^\infty [|z(t)|^2 - \gamma^2|v(t)|^2]dt\} \\ =& \mathcal{E}\{\int_0^\infty [|z(t)|^2 - \gamma^2|v(t)|^2 + \mathcal{L}\tilde{V}_v(t)]dt\} \\ \le& \mathcal{E}\{\int_0^\infty \zeta^T(t)\Theta_v\zeta(t)dt\} \end{aligned} \quad (25)$$

where

$$\Theta_v = \begin{bmatrix} \Theta_{11}+C^TC & \Theta_{12} & A^T(t)S & PB_v + H^T(t)PH_v(t) \\ * & \Theta_{22} & A_1^T(t)S & H_1^T(t)PH_v(t) \\ * & * & \Omega_{33} & S^TB_v \\ * & * & * & -\gamma^2 I + H_v^T(t)PH_v(t) \end{bmatrix}. \quad (26)$$

Following the similar lines as in the previous part of this proof, it can be verified that $\Theta_v < 0$ is equivalent to $\Omega < 0$.

Therefore, we can conclude that if LMI (8) holds, then system (1) will be robustly mean-square asymptotically stable with a specified H_∞ disturbance attenuation level γ. This completes the proof. ■

Remark 7. The existence of the positive definite symmetric term $hy^T(t)Ry(t)$, which is shown as $h\dot{x}^T(t)R\dot{x}(t)$ in the deterministic delay systems (Han [2005], Zhang et al. [2005], Jiang and Han [2007]), drives us to introduce the free matrix S in (16). If $S = 0$, (16) becomes to $\Theta = \left(\begin{smallmatrix} \Theta_{11} & \Theta_{12} & 0 \\ * & \Theta_{22} & 0 \\ 0 & 0 & hR \end{smallmatrix}\right)$, and $\Theta < 0$ is infeasible. Therefore, the slack matrix S is introduced, which can make it to be solvable.

Remark 8. Neither model transformation nor cross term estimating technique is employed in Theorem 6. Therefore, Theorem 6 is expected to be less conservative.

3.2 Robust H_∞ Control

We now investigate the problem of H_∞ controller design for system (1) based on Theorem 6 as follows.

Theorem 9. Given scalars λ, $h > 0, \gamma > 0$, for all admissible uncertainties (3), if there exist positive scalars δ_1, δ_2, positive definite symmetric matrices X, $\bar{Q}$, $\bar{R} \in \mathbb{R}^{n\times n}$ and matrix $Y \in \mathbb{R}^{m\times n}$ satisfying (27) (shown at the top of the next page), where

$$\begin{aligned} \Upsilon_{11} =& AX + XA^T + BY + Y^TB^T + \bar{Q} - \bar{R} + \delta_1 LL^T, \\ \Upsilon_{12} =& A_1X + \bar{R}, \\ \Upsilon_{22} =& -\bar{Q} - \bar{R}, \\ \Upsilon_{13} =& \lambda(XA^T + Y^TB^T) + \delta_1\lambda LL^T, \\ \Upsilon_{33} =& h^2\bar{R} - 2\lambda X + \delta_1\lambda^2 LL^T, \\ \Upsilon_{55} =& -X + \delta_2 LL^T, \\ \Upsilon_{16} =& XC^T + Y^TD^T, \\ \Upsilon_{17} =& XE_1^T + Y^TE_3^T, \end{aligned}$$

then $u(t) = Kx(t)$ is a robust H_∞ controller of system (1). Moreover, the controller gain is constructed by $K = YX^{-1}$.

Proof: Let $S = S^T = \lambda P$, where λ is a tuning parameter. Replacing A, E_1 and C by $A_K = A+BK$, $E_K = E_1+E_3K$ and $C_K = C+DK$ in (8), respectively, we have

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \lambda A_K^TP & PB_v & H^TP & PL & 0 & C_K^T \\ * & \Omega_{22} & \lambda A_1^TP & 0 & H_1^TP & 0 & 0 & 0 \\ * & * & \Xi_{33} & \lambda PB_v & 0 & \lambda PL & 0 & 0 \\ * & * & * & -\gamma^2 I & H_v^TP & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & PL & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (28)$$

where

$$\begin{aligned} \Xi_{11} =& PA_K + A_K^TP + Q - R \\ &+ \varepsilon_1 E_K^TE_K + \varepsilon_2 E_4^TE_4, \\ \Xi_{12} =& PA_1 + R + \varepsilon_1 E_K^TE_2 + \varepsilon_2 E_4^TE_5, \\ \Xi_{33} =& h^2R - 2\lambda P. \end{aligned}$$

Taking a congruent transformation to the above inequality (28) by diag$\{P^{-1}, P^{-1}, P^{-1}, I, P^{-1}, I, I, I\}$, and setting

$$\begin{aligned} X = P^{-1},\ \bar{Q} = XQX, \\ \bar{R} = XRX,\ Y = KX, \end{aligned} \quad (29)$$

result in (30) (shown at the top of the next page), where

$$\begin{aligned} \Pi_{11} =& AX + XA^T + BY + Y^TB^T + \bar{Q} - \bar{R} \\ &+ \varepsilon_1(E_1X+E_3Y)^T(E_1X+E_3Y) + \varepsilon_2 XE_4^TE_4X, \\ \Pi_{12} =& A_1X + \bar{R} \\ &+ \varepsilon_1(E_1X+E_3Y)^TE_2X + \varepsilon_2 XE_4^TE_5X, \\ \Pi_{22} =& -\bar{Q} - \bar{R} + \varepsilon_1 XE_2^TE_2X + \varepsilon_2 XE_5^TE_5X, \\ \Pi_{33} =& h^2\bar{R} - 2\lambda X. \end{aligned}$$

Taking $\delta_i = \varepsilon_i (i = 1, 2)$, then it can be proved that $\Pi < 0$ is equivalent to the LMI (26) by some simple manipulations. Thus, the controller gain can be determined as $K = YX^{-1}$ by (28). The proof is completed. ■

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & B_v & XH^T & \Upsilon_{16} & \Upsilon_{17} & XE_4^T \\ * & \Upsilon_{22} & \lambda XA_1^T & 0 & XH_1^T & 0 & XE_2^T & XE_5^T \\ * & * & \Upsilon_{33} & \lambda B_v & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & H_v^T & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\delta_1 I & 0 \\ * & * & * & * & * & * & * & -\delta_2 I \end{bmatrix} < 0 \quad (27)$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \lambda XA_K^T & B_v & XH^T & L & 0 & XC_K^T \\ * & \Pi_{22} & \lambda XA_1^T & 0 & XH_1^T & 0 & 0 & 0 \\ * & * & \Pi_{33} & \lambda B_v & 0 & \lambda L & 0 & 0 \\ * & * & * & -\gamma^2 I & H_v^T & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & L & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (30)$$

4. ILLUSTRATIVE EXAMPLES

In this section, the effectiveness of our method will be demonstrated by two numerical examples.

Example 10. Consider the following system (Chen et al. [2004])

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$B_v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, H_v = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, C = [\,0\ \ 1\,], D = 0.1, \quad (31)$$
$$L = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, E_1 = E_2 = E_4 = E_5 = 0.2I.$$

The delay-independent method of Xu and Chen [2002] is not applicable to this example. If $h = 0.3$, then the minimum disturbance attenuation level by Chen et al. [2004] is $\gamma_{\min} = 1.65$ (with $\delta = 0.8$). However, applying Theorem 9 we can obtain $\gamma_{\min} = 1.1339$ when $\lambda = 1$, and the following solutions

$$X = \begin{bmatrix} 3.3150 & 0.7277 \\ 0.7277 & 1.4629 \end{bmatrix},$$
$$Y = [\,-2.4027\ \ -2.5734\,],$$
$$\bar{Q} = \begin{bmatrix} 1.3552 & 1.0794 \\ 1.0794 & 0.8890 \end{bmatrix},$$
$$\bar{R} = \begin{bmatrix} 3.0601 & 0.2355 \\ 0.2355 & 1.2327 \end{bmatrix},$$
$$\delta_1 = 20.1076, \delta_2 = 27.1985.$$

Then, the corresponding controller is given by

$$u(t) = Kx(t) = [\,-0.3802\ \ -1.5700\,]\,x(t).$$

When $\lambda = 0.1$, the minimum disturbance attenuation level is $\gamma_{\min} = 0.0001$.

Example 11. Consider the stochastic time-delay system (1) with (Gershon et al. [2007])

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -0.4 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0.1 \\ -0.1 & -0.04 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_v = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
$$H = \begin{bmatrix} 0 & 0.3 \\ -0.2 & -0.04 \end{bmatrix}, H_1 = \begin{bmatrix} 0 & 0.18 \\ -0.09 & -0.15 \end{bmatrix}, \quad (32)$$
$$C = \begin{bmatrix} -0.5 & 4 \\ 0 & 0 \end{bmatrix}.$$

It can be seen that minimum disturbance attenuation level by Xu and Chen [2002] and Gershon et al. [2007] are $\gamma_{\min} = 4.855$ and $\gamma_{\min} = 4.5822$, respectively. Compared with these results, we can obtain $\gamma_{\min} = 3.5155$ ($\lambda = 0.1, h = 0.5$) and the following gain matrix

$$K = [\,4.0321\ \ -19.4478\,].$$

It is shown by Examples 10 and 11, that the presented method of this paper is much less conservative than the existing ones in the literature.

5. CONCLUSIONS

A stochastic bounded real lemma (BRL) and a robust H_∞ controller in terms of linear matrix inequalities (LMIs) for uncertain stochastic time-delay systems have been presented in this paper. The delay-dependent results are obtained based on an integral inequality and slack matrix technique. Neither model transformation and cross terms bounding techniques is involved. Our method is less conservative than the exiting ones in the literature, which has been shown by two numerical examples.

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