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An Inner-loop Controller Guaranteeing Robust Transient Performance for Uncertain MIMO Linear Systems ⋆

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Abstract: An output-feedback controller has been recently proposed that has the following features: (1) it is an inner-loop controller so that it can be added on the existing closed-loop system working in harmony with a pre-designed (possibly non-robust) outer-loop controller, (2) it robustifies the closed-loop system in a way that the uncertain plant under external disturbance becomes a nominal plant without any disturbance, (3) it recovers the trajectory of the nominal closed-loop system in time domain. However, it is restricted to the single-input-single-output systems. In this paper, we extend this result for a class of multi-input-multi-output (MIMO) linear systems having the same number of inputs and outputs. The used tools in this synthesis are the singular perturbation theory and the multi-variable circle criterion.

Keywords: Multi-input-multi-output; Disturbance observer; Performance recovery; Linear systems.

1. INTRODUCTION

Designing efficient controllers to compensate the effect of model uncertainties and external disturbances in control systems has been a central problem in control society, and powerful results are available in the literature. For example, naming only a few, H_2/H_∞ control [Doyle et al., 1989], sliding mode control [Utkin, 1992], adaptive control [Narendra and Annaswamy, 1989, Ioannou and Sun, 1995], backstepping [Krstić et al., 1995], etc. Although these methods proved their efficiency in many applications, they mainly focus on the stability of the uncertain system (robust stability), while little can be addressed on the 'transient' performance. In other words, they don't guarantee that the nominal time-trajectory (the trajectory of the closed-loop system without uncertainties) can be recovered.

Recently, an output-feedback controller has been proposed in [Back and Shim, 2007] which guarantees not only the robust stability, but also the robust transient performance in the sense that the controller recovers the nominal transient trajectory, under the plant uncertainties and the external disturbances. This recovery is approximate in general, but the error in this approximation can be made arbitrarily small. In addition, the controller also guarantees exact tracking when references and disturbances are constant. Moreover, it is an inner-loop controller so that it can be added on the existing closed-loop system working in harmony with a pre-designed (possibly non-robust) outer-loop controller.

There are also some results on the nominal transient recovery in the literature. The work [Freidovich and Khalil, 2006] considers the single-input-single-output (SISO) case with linear nominal model, and the paper [Chakrabortty and Arcak, 2007] studies the multi-input-multi-output (MIMO) case with state feedback. Compared to these results, the work [Back and Shim, 2007] is an output feedback controller and the nominal model can be nonlinear systems, which is quite general.

In this paper, we extend the work [Back and Shim, 2007] for a class of MIMO linear systems having the same number of inputs and outputs. Extension to MIMO case could have been easy if the input gain matrix is nonsingular and has no uncertainty because, if so, the plant could be made just a collection of SISO systems by redefining the inputs. For example, for the system $\dot{x} = Gu$ with $x \in \mathbb{R}^m$, $u \in \mathbb{R}^m$, and nonsingular $G \in \mathbb{R}^{m\times m}$, the input $u = G^{-1}v$ yields $\dot{x} = v$, which is a collection of m independent systems $\dot{x}_i = v_i$. However, if G is not known perfectly, getting G^{-1} becomes impossible, which makes the problem hard. In this paper, we overcome this problem by employing the multi-variable circle criterion so that a MIMO extension of [Back and Shim, 2007] is possible.

The extension proposed in this paper also serves a MIMO extension of a particular control method that is known as 'disturbance observer (DOB)' approach [Ohnishi, 1987], which has been widely applied in the industrial applications (see, for example, Umeno and Hori [1993], Eom et al. [2001], Guvenc and Guvenc [2002], Bohn et al. [2004], Ryoo et al. [2004]), because the controller in [Back and Shim, 2007] is based on the DOB approach, and the MIMO extension of the DOB approach has rarely been discussed in the literature.

Notation: Throughout the paper, 0_k denotes the zero vector in $\mathbb{R}^k$, and I_k denotes the $k \times k$ identity matrix. For two column vectors a and b, we use $[a; b] := [a^T, b^T]^T$ for

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convenience. The notation blockdiag$\{A_1, \ldots, A_k\}$ stands for a block diagonal matrix, whose (i,i)-th block is A_i while the others are zero. When a_i's are scalar, we simply write diag$\{a_1, \ldots, a_k\}$ for the diagonal matrix. Euclidean norm of a vector is denoted by $|\cdot|$. $\mathbb{C}^-$ represents the open left-half complex plane.

2. PROBLEM FORMULATION

We consider a multi-input-multi-output linear system which is put in the following Byrnes-Isidori normal form [Isidori, 1995]:

$$\begin{aligned} \dot{z} &= Sz + Px \\ \dot{x} &= Ax + B(F[z;x] + G(u+d)) \\ y &= Cx, \end{aligned} \tag{1}$$

where $u \in \mathbb{R}^m$ is the control input, $d \in \mathbb{R}^m$ is the unknown disturbance, $y \in \mathbb{R}^m$ is the output, and $x \in \mathbb{R}^r$ and $z \in \mathbb{R}^{n-r}$ are the states of the system such that

$$x = [x_1; \ldots; x_m], \quad x_i := [x_{i1} \ \cdots \ x_{ir_i}]^T \in \mathbb{R}^{r_i}$$

with $r = r_1 + \cdots + r_m$. The matrices $A \in \mathbb{R}^{r\times r}$, $B \in \mathbb{R}^{r\times m}$, and $C \in \mathbb{R}^{m\times r}$ are given by

$$\begin{aligned} A &= \text{blockdiag}\{A_1, \ldots, A_m\}, \quad A_i := \begin{bmatrix} 0_{r_i-1} & I_{r_i-1} \\ 0 & 0_{r_i-1}^T \end{bmatrix}, \\ B &= \text{blockdiag}\{B_1, \ldots, B_m\}, \quad B_i := \begin{bmatrix} 0_{r_i-1} \\ 1 \end{bmatrix}, \\ C &= \text{blockdiag}\{C_1, \ldots, C_m\}, \quad C_i := \begin{bmatrix} 1 & 0_{r_i-1}^T \end{bmatrix}. \end{aligned}$$

In fact, the MIMO linear systems that have the vector relative degree $\{r_1, \ldots, r_m\}$ can always be converted into the above normal form (see Isidori [1995]).

It is assumed that S, P, F, and G are unknown, which are elements of known matrix sets, $\mathcal{S}$, $\mathcal{P}$, $\mathcal{F}$, and $\mathcal{G}$, respectively.

Assumption 1. The sets $\mathcal{S}$, $\mathcal{P}$, $\mathcal{F}$, and $\mathcal{G}$ are bounded. In particular, for the uncertain input gain matrix G, there exist symmetric positive definite matrices $G^- := \text{diag}\{g_1^-, \ldots, g_m^-\}$ and $G^+ := \text{diag}\{g_1^+, \ldots, g_m^+\}$ such that $0 < G^- < G^+$ and that

$$(G\nu - G^-\nu)^T \Pi^2 (G\nu - G^+\nu) \le 0, \quad \forall \nu \in \mathbb{R}^m,\ \forall G \in \mathcal{G}, \tag{2}$$

where $\Pi = \text{diag}\{\pi_1, \ldots, \pi_m\} := 2(G^+ + G^-)^{-1}$. In addition, the disturbance signal $d(t)$ is at least C^2, and $d(t)$ and $\dot{d}(t)$ are bounded with a known bound l_d such that $|d(t)| \le l_d$. □

Remark 1. Assumption 1 can be extended in such a way that GK replaces the matrix G, where K is any nonsingular matrix. This is possible because the input u and the disturbance d can be considered to be $K^{-1}u^\dagger$ and $K^{-1}d^\dagger$ with new input $u^\dagger$ and disturbance $d^\dagger$, without loss of generality. With the matrix K, the order, the sign, and the magnitude of inputs can also be adjusted to satisfy Assumption 1. □

We now consider a disturbance-free nominal plant of (1):

$$\begin{aligned} \dot{\bar{z}} &= \bar{S}\bar{z} + \bar{P}\bar{x} \\ \dot{\bar{x}} &= A\bar{x} + B(\bar{F}[\bar{z};\bar{x}] + \bar{G}u_r) \\ \bar{y} &= C\bar{x}, \end{aligned} \tag{3}$$

where $\bar{S} \in \mathbb{R}^{(n-r)\times(n-r)}$, $\bar{P} \in \mathbb{R}^{(n-r)\times r}$, $\bar{G} \in \mathbb{R}^{m\times m}$, and $\bar{F} \in \mathbb{R}^{m\times n}$ are the nominal parameters of S, P, G, and F, respectively.

It is assumed that an (dynamic) output feedback outer-loop controller $\mathcal{C}$ is designed *a priori* for the nominal plant (3), which is represented by

$$\begin{aligned} \dot{c} &= \Gamma(c, \bar{y}, y_r), \quad c \in \mathbb{R}^l, \\ u_r &= \gamma(c, \bar{y}, y_r), \quad u_r \in \mathbb{R}^m, \end{aligned} \tag{4}$$

where Γ and γ are C^2 functions, and y_r is a vector of reference command.

Assumption 2. The considered class of reference command $y_r(t)$ is a C^2 function, and $y_r(t)$ and $\dot{y}_r(t)$ are bounded such that $y_r(t) \in S_{y_r}$ where S_{y_r} is a known compact set. For those reference command $y_r(t)$, the nominal closed-loop system (3) and (4) has the following properties:

1. (1) the solution $[\bar{z}(t); \bar{x}(t); c(t)]$ of (3) and (4) evolves in a bounded, connected, and open set $U \subset \mathbb{R}^{n+l}$ if the initial condition $[\bar{z}(0);\ \bar{x}(0);\ c(0)]$ is located in a compact set $S \subset U$,
2. (2) each solution $[\bar{z}(t); \bar{x}(t); c(t)]$ initiated in S is locally asymptotically stable.

□

The item (2) in Assumption 2 is generally met by the usual tracking/regulation controller $\mathcal{C}$. The next one is our last assumption.

Assumption 3. The plant (1) is of minimum phase, that is, S is Hurwitz for all $S \in \mathcal{S}$. □

By Assumption 3, the system $\dot{z} = Sz + Px$ is input-to-state stable. Let U_x and U_z be the projections of the set U to the x and z plane, respectively. Then, define Z as a bounded set where all feasible solutions $z(t)$ reside with $z(0) \in U_z$ for all $x(t) \in U_x$.

In this paper, we design an inner-loop controller of the form

$$\begin{aligned} \dot{\chi} &= \begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \end{bmatrix} = \begin{bmatrix} \Upsilon_1(\chi, y, u_r) \\ \Upsilon_2(\chi, y, u_r) \end{bmatrix} = \Upsilon(\chi, y, u_r) \\ u &= \upsilon(\chi, y, u_r) \end{aligned} \tag{5}$$

where $\chi_1 \in \mathbb{R}^{n-r}$ and $\chi_2 \in \mathbb{R}^{2r}$, such that the real closed-loop system [1]

$$\begin{aligned} \dot{z} &= Sz + Px \\ \dot{x} &= Ax + B(F[z;x] + G(\upsilon(\chi, Cx, \gamma(c, Cx, y_r)) + d)) \\ \dot{c} &= \Gamma(c, Cx, y_r) \\ \dot{\chi} &= \Upsilon(\chi, Cx, \gamma(c, Cx, y_r)) \end{aligned}$$

behaves like the nominal closed-loop system (3) and (4) despite of the model uncertainties and external disturbances. In particular, we are interested in a *transient performance recovery*, that is, the states $[x(t); c(t)]$ of the real solution $[z(t); x(t); c(t); \chi(t)]$ remain close to their nominal counterpart $[x_N(t); c_N(t)]$ where the states $[z_N(t); x_N(t); c_N(t)]$ are the solution of the nominal closed-loop system (3) and (4), with the initial condition $[z_N(0); x_N(0); c_N(0)] = [\chi_1(0); x(0); c(0)]$.

[1] Note that when the outer-loop controller (4) is considered in the real closed-loop system with (5), $\bar{y}$ should be replaced by y, which is evident and will be applied without explicit mentioning throughout the paper.

3. MAIN RESULTS

3.1 Design of the Inner-loop Controller

We present the design procedure of the inner-loop controller (5) in this subsection, with a precise statement of the transient performance recovery achieved by the proposed controller.

First, let $a_i = [a_{i0}\ a_{i1}\ \cdots\ a_{i,r_i-1}]^T$, $i = 1,\dots,m$. For each i, choose $a_{i1},\dots,a_{i,r_i-1}$ such that all the roots of the equation

$$s^{r_i-1} + a_{i,r_i-1}s^{r_i-2} + \cdots + a_{i1} = 0 \tag{6}$$

are in $\mathbb{C}^-$. When $r_i = 1$, there is nothing to choose.

For each i, with $a_{i1},\dots,a_{i,r_i-1}$ fixed, we choose a_{i0} as follows. Define $D(\pi_i g_i^-, \pi_i g_i^+)$ by a closed disk in the complex plane whose diameter is the line segment connecting the points $-1/(\pi_i g_i^-) + j0$ and $-1/(\pi_i g_i^+) + j0$. Let

$$H_i(s) = \frac{1}{s}\frac{a_{i0}}{s^{r_i-1} + a_{r_i-1}s^{r_i-2} + \cdots + a_{i1}}, \tag{7}$$

and find a_{i0} such that the Nyquist plot of $H_i(s)$ is disjoint from the disk $D(\pi_i g_i^-, \pi_i g_i^+)$ and does not encircle the disk. (It is assumed that the Nyquist contour avoids the origin counterclockwise so that the closed Nyquist contour does not contain the pole of $H_i(s)$ at the origin.) Such a_{i0} always exists. Indeed, since $H_i(s)$ has all poles in $\mathbb{C}^-$ except one pole at the origin, the Nyquist plot of $H_i(s)$ is bounded to the left although it is unbounded to the imaginary axis. Hence, by reducing the value of positive a_{i0}, the Nyquist plot shrinks so that the disk D is located outside the closed Nyquist plot. Note that such a_{i0} also guarantees that all the roots of

$$s^{r_i} + a_{i,r_i-1}s^{r_i-1} + \cdots + a_{i1}s + a_{i0} = 0 \tag{8}$$

are in $\mathbb{C}^-$, because the disk D contains the point $-1 + j0$ inside by the fact that $\pi_i = 2/(g_i^+ + g_i^-)$. For later use, define a transfer function matrix

$$H(s) = \mathrm{diag}\{H_1(s),\dots,H_m(s)\}. \tag{9}$$

Now we define saturation functions $\phi : \mathbb{R}^r \to \mathbb{R}^r$ and $\Phi : \mathbb{R}^m \to \mathbb{R}^m$ as globally bounded C^1 functions satisfying

$$\begin{aligned} &\phi(x) = x,\ \forall x \in U_x, \text{ and } \left\|\frac{\partial\phi}{\partial x}(x)\right\| \le \kappa_0,\ \forall x \in \mathbb{R}^r \\ &\Phi(w) = w,\ \forall w \in S_w, \text{ and } \left\|\frac{\partial\Phi}{\partial w}(w)\right\| \le 1,\ \forall w \in \mathbb{R}^m \end{aligned} \tag{10}$$

where κ_0 is a positive constant and

$$\begin{aligned} S_w = \Big\{ &w \in \mathbb{R}^m : w = G^{-1}(\bar{F}[\bar{z}; x] - F[z; x]) \\ &+ (G^{-1} - \Pi)\bar{G}\gamma(c, Cx, y_r) - d, z \in Z, [\bar{z}; x; c] \in U, \\ &y_r \in S_{y_r}, |d| \le l_d, F \in \mathcal{F}, G \in \mathcal{G}\Big\}. \end{aligned}$$

The set S_w indicates the steady-state range of the signal $w(t)$ to be defined in (14e), and will be clarified in Lemma 2. In fact, it is enough to have the saturation levels of ϕ and Φ sufficiently large so that the saturation functions are not active during the steady-state operation.

For $i = 1,\dots,m$, let

$$\Delta_\tau = \mathrm{blockdiag}\{\Delta_{1\tau},\dots,\Delta_{m\tau}\} \tag{11}$$

$$\Delta_{i\tau} = \mathrm{diag}\left\{\frac{1}{\tau^{r_i}}, \frac{1}{\tau^{r_i-1}}, \dots, \frac{1}{\tau}\right\} \tag{12}$$

$$A_{ai\tau} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\frac{a_{i0}}{\tau^{r_i}} & -\frac{a_{i1}}{\tau^{r_i-1}} & \cdots & -\frac{a_{i,r_i-1}}{\tau} \end{bmatrix}. \tag{13}$$

The proposed inner-loop controller (5) is finally given by

$$\dot{\bar{z}} = \bar{S}\bar{z} + \bar{P}\phi(q) \tag{14a}$$

$$\dot{q}_i = A_{ai\tau}q_i + \frac{a_{i0}}{\tau^{r_i}}B_i y_i,\ i = 1,\dots,m \tag{14b}$$

$$\dot{p}_i = A_{ai\tau}p_i + \frac{a_{i0}}{\tau^{r_i}}B_i u_i,\ i = 1,\dots,m \tag{14c}$$

$$u = \Phi(w) + \Pi\bar{G}u_r, \tag{14d}$$

where $q = [q_1;\dots;q_m] \in \mathbb{R}^r$, $q_i = [q_{i1}\ \cdots\ q_{ir_i}]^T \in \mathbb{R}^{r_i}$, $p = [p_1;\dots;p_m] \in \mathbb{R}^r$, $p_i = [p_{i1}\ \cdots\ p_{ir_i}]^T \in \mathbb{R}^{r_i}$, $w = [w_1\ \cdots\ w_m]^T \in \mathbb{R}^m$, and

$$w_i = p_{i1} - \pi_i\dot{q}_{ir_i} + \pi_i\bar{F}_i[\bar{z}; q],\ i = 1,\dots,m. \tag{14e}$$

Here and after, we write F_i, G_i, $\bar{F}_i$, and $\bar{G}_i$ to indicate the i-th row of F, G, $\bar{F}$, and $\bar{G}$, respectively. Note that $\chi_1 = \bar{z}$ and $\chi_2 = [q; p]$.

Theorem 1. Let S_{χ_2} be a compact set for the initial condition $[q(0); p(0)]$, $\bar{S}$ be a compact set slightly smaller than S (i.e., $\bar{S} \subset S$ and their boundaries are disjoint), and $\bar{S}_z$ be the projection of $\bar{S}$ into the z plane. Under Assumptions 1, 2, and 3, for a given $\epsilon > 0$, there exists a $\tau^* > 0$ such that, for each $0 < \tau \le \tau^*$, the solution of the closed-loop system (1), (4), and (5) denoted by $[z(t); x(t); c(t); \chi(t)]$, initiated at $[z(0); x(0); c(0); \chi_1(0); \chi_2(0)] \in \bar{S} \times \bar{S}_z \times S_{\chi_2}$, is bounded for all $t \ge 0$, and satisfies that

$$|[x(t); c(t)] - [\bar{x}_N(t); c_N(t)]| \le \epsilon,\ \forall t \ge 0, \tag{15}$$

where $[\bar{x}_N(t); c_N(t)]$ indicates sub-states of the solution $[\bar{z}_N(t); \bar{x}_N(t); c_N(t)]$ of the nominal closed-loop system (3) and (4), with $[\bar{z}_N(0); \bar{x}_N(0); c_N(0)] = [\chi_1(0); x(0); c(0)]$. □

3.2 Proof of Theorem 1

This subsection provides a proof of Theorem 1. At first it is shown that the closed-loop system can be transformed into the standard singular perturbation form (Lemma 1), and then the stability of the fast subsystem is proved (Lemma 3) with the help of a technical result (Lemma 2). Based on these two results, Theorem 1 is proved by employing the Tikhonov's theorem.

Let $\xi = [\xi_1;\dots;\xi_m]$ with $\xi_i = [\xi_{i1}\ \cdots\ \xi_{ir_1}]^T \in \mathbb{R}^{r_i}$, $i = 1,\dots,m$, and $\eta = [\eta_1;\dots;\eta_m]$ with η_i's and η_{ij}'s being defined like ξ_i's and ξ_{ij}'s. We define $\eta_{[1]} := [\eta_{11}\ \cdots\ \eta_{m1}]^T$ for convenience.

Lemma 1. By the coordinates change

$$\xi_{ij} = \sum_{k=j}^{r_i}\frac{a_{i,k-j}}{a_{i0}}\frac{q_{ik}}{\tau^{r_i-k}} - \frac{x_{ij}}{\tau^{r_i-j}}, \tag{16a}$$

$$\eta_{ij} = \tau^{j-1}\left(p_{ij} - \pi_i q_{ir_i}^{(j)}\right), \tag{16b}$$

where $j = 1,\dots,r_i$, and $i = 1,\dots,m$, the closed-loop system (1), (4), and (14) is written as

$$\begin{aligned}\dot{z} &= Sz + Px \\ \dot{\bar{z}} &= \bar{S}\bar{z} + \bar{P}\phi(\mathcal{T}_\tau[\xi; x]) \\ \dot{x} &= Ax + B\theta_\xi \\ \dot{c} &= \Gamma(c, Cx, y_r)\end{aligned} \tag{17}$$

and, for $i = 1, \ldots, m$,

$$\tau\dot{\xi}_i = A_{i\xi}\xi_i - \tau B_i\theta_{i\xi} \tag{18a}$$

$$\tau\dot{\eta}_i = A_{i\eta}\eta_i + a_{i0}B_i\theta_{i\eta} \tag{18b}$$

where

$$\begin{aligned}\theta_\xi &= [\theta_{1\xi} \ \cdots \ \theta_{m\xi}]^T \in \mathbb{R}^m, \quad \theta_\eta = [\theta_{1\eta} \ \cdots \ \theta_{m\eta}]^T \in \mathbb{R}^m \\ \theta_{i\xi} &= F_i[z; x] + G_i\left(\Phi(w) + \Pi\bar{G}u_r + d\right) \\ \theta_{i\eta} &= -\pi_i\left(F_i[z; x] + G_i(\Phi(w) + \Pi\bar{G}u_r + d)\right) \\ &\qquad + \Phi_i(w) + \pi_i\bar{G}_iu_r \\ u_r &= \gamma(c, Cx, y_r) \\ w_i &= \eta_{i1} + \pi_i\bar{F}_i[\bar{z}; \mathcal{T}_\tau[\xi; x]] \\ A_{i\xi} &= A_i - [a_{i,r_i-1} \ \cdots \ a_{i0}]^T C_i, \quad A_{i\eta} = A_i - B_ia_i^T,\end{aligned}$$

and $\mathcal{T}_\tau$ is defined in Remark 2 below. □

Remark 2. A compact expression for (16a) is $\xi = \tau\Delta_\tau(T_\tau q - x)$ where $T_\tau = \text{blockdiag}\{T_{1\tau}, \ldots, T_{m\tau}\}$ and

$$T_{i\tau} = \frac{1}{a_{i0}}\begin{bmatrix} a_{i0} & a_{i1}\tau & a_{i2}\tau^2 & \cdots & a_{i,r_i-1}\tau^{r_i-1} \\ 0 & a_{i0} & a_{i1}\tau & \ddots & a_{i,r_i-2}\tau^{r_i-2} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & & a_{i0} & a_{i1}\tau \\ 0 & \cdots & \cdots & 0 & a_{i0}\end{bmatrix}.$$

Therefore, $q = \frac{1}{\tau}T_\tau^{-1}\Delta_\tau^{-1}\xi + T_\tau^{-1}x =: \mathcal{T}_\tau[\xi; x]$, in which the matrix $\mathcal{T}_\tau \in \mathbb{R}^{r\times 2r}$ is well-defined for all τ including zero. □

Proof of Lemma 1. We first consider the following transformation:

$$\bar{q}_{ij} = \sum_{k=j}^{r_i}\frac{a_{i,k-j}}{a_{i0}}\tau^{k-j}q_{ik}, \quad 1 \le j \le r_i, \quad 1 \le i \le m.$$

In $\dot{\bar{q}}_{ij}$ coordinates, one has (for simplicity, let $\bar{q}_{i,r_i+1} := 0$)

$$\begin{aligned}\dot{\bar{q}}_{ij} &= \sum_{k=j}^{r_i-1}\frac{a_{i,k-j}}{a_{i0}}\tau^{k-j}q_{i,k+1} + \frac{a_{i,r_i-j}}{a_{i0}}\tau^{r_i-j}\dot{q}_{ir_i} \\ &= \sum_{k=j+1}^{r_i}\frac{a_{i,k-(j+1)}}{a_{i0}}\tau^{k-(j+1)}q_{ik} \\ &\quad + \frac{a_{i,r_i-j}}{a_{i0}}\tau^{r_i-j}\Big(-\sum_{k=1}^{r_i}\frac{a_{i,k-1}}{\tau^{r_i-k+1}}q_{ik} + \frac{a_{i0}}{\tau^{r_i}}x_{i1}\Big) \\ &= \bar{q}_{i,j+1} - \frac{a_{i,r_i-j}}{\tau^j}\Big(\sum_{k=1}^{r_i}\frac{a_{i,k-1}}{a_{i0}}\tau^{k-1}q_{ik} - x_{i1}\Big) \\ &= \bar{q}_{i,j+1} - \frac{a_{i,r_i-j}}{\tau^j}(\bar{q}_{i1} - x_{i1}).\end{aligned} \tag{19}$$

With $\bar{q}_{ij}$'s defined above, ξ_{ij}'s in (16a) are given by $\xi_{ij} = (\bar{q}_{ij} - x_{ij})/\tau^{r_i-j}$. By successively differentiating ξ_{ij}, one obtains that

$$\tau\dot{\xi}_i = A_{i\xi}\xi_i - \tau B_i(F_i[z; x] + G_i(u + d)),$$

which is the equation (18a).

On the other hand, by (16b), it follows that

$$\tau\dot{\eta}_{ij} = \eta_{i,j+1}, \qquad j = 1, \ldots, r_i - 1.$$

For η_{ir_i}, we compute (for simplicity, let $a_{ir_i} := 1$)

$$\begin{aligned}\dot{\eta}_{ir_i} &= -\frac{1}{\tau}\sum_{j=1}^{r_i}a_{i,j-1}\eta_{ij} - \frac{\pi_i}{\tau}\sum_{j=1}^{r_i+1}\tau^{j-1}a_{i,j-1}q_{ir_i}^{(j)} \\ &\quad + \frac{a_{i0}}{\tau}\left(\Phi_i(w) + \pi_i\bar{G}_iu_r\right).\end{aligned}$$

Recalling $\dot{q}_{ir_i} = -\sum_{j=1}^{r_i}\frac{a_{i,j-1}}{\tau^{r_i-j+1}}q_{ij} + \frac{a_{i0}}{\tau^{r_i}}x_{i1}$, one has

$$q_{ir_i}^{(r_i+1)} = -\sum_{j=1}^{r_i}\frac{a_{i,j-1}}{\tau^{r_i-j+1}}q_{ir_i}^{(j)} + \frac{a_{i0}}{\tau^{r_i}}\dot{x}_{ir_i}.$$

Equivalently, $\sum_{j=1}^{r_i+1}\tau^{j-1}a_{i,j-1}q_{ir_i}^{(j)} = a_{i0}\dot{x}_{ir_i}$. Hence,

$$\begin{aligned}\tau\dot{\eta}_{ir_i} &= -a_{i0}\eta_{i1} - a_{i1}\eta_{i2} - \cdots - a_{i,r_i-1}\eta_{ir_i} \\ &\quad - \pi_ia_{i0}\dot{x}_{ir_i} + a_{i0}\left(\Phi_i(w) + \pi_i\bar{G}_iu_r\right) \\ &= -a_{i0}\eta_{i1} - a_{i1}\eta_{i2} - \cdots - a_{i,r_i-1}\eta_{ir_i} \\ &\quad - \pi_ia_{i0}\left(F_i[z; x] + G_i(\Phi(w) + \Pi\bar{G}u_r + d)\right) \\ &\quad + a_{i0}\left(\Phi_i(w) + \pi_i\bar{G}_iu_r\right).\end{aligned}$$

This concludes the proof of Lemma 1. □

Remark 3. From the coordinates change in Lemma 1, the initial conditions are related by

$$\begin{aligned}\xi_i(0) &= \tau\Delta_{i\tau}(T_{i\tau}q_i(0) - x_i(0)) \\ \eta_i(0) &= \tau^{r_i}\Delta_{i\tau}p_i(0) + T_{ix}\Delta_{i\tau}x_i(0) + T_{iq}\Delta_{i\tau}q_i(0),\end{aligned}$$

where $i = 1, \ldots, m$, and T_{ix} and T_{iq} are some constant matrices. It is emphasized that with $x(0)$, $q(0)$, and $p(0)$ in a compact set, the initial condition $\xi(0)$ and $\eta(0)$ may become unbounded for small τ. Nevertheless, it is noted that, for sufficiently small τ,

$$|\xi(0)| \le k^*/\tau^{r^*-1}, \qquad |\eta(0)| \le k^*/\tau^{r^*},$$

where $r^* := \max\{r_1, \ldots, r_m\}$ and k^* is a constant, which implies that the growth is of polynomial order of $(1/\tau)$. □

Note that the closed-loop system (17) and (18) is in the *standard singular perturbation form* with τ being the time separation parameter [Hoppensteadt, 1966, Khalil, 2002]. Now, we present a technical lemma without proof.

Lemma 2. Assuming that $[z(t); \bar{z}(t); x(t); c(t); y_r(t)] \in Z \times U \times S_{y_r}$ and $|d(t)| \le l_d$, define the vector functions $\eta_{[1]}^* : \mathbb{R} \to \mathbb{R}^m$ and $\theta_\eta^*, \psi : \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^m$ as

$$\begin{aligned}\eta_{[1]}^*(t) &:= (G^{-1} - \Pi)(\bar{F}[\bar{z}(t); x(t)] + \bar{G}u_r(t)) \\ &\qquad - G^{-1}(F[z(t); x(t)] + Gd(t)) \\ \theta_\eta^*(\nu, t) &:= (I_m - \Pi G)(\Phi(\nu + \Pi\bar{F}[\bar{z}(t); x(t)]) + \Pi\bar{G}u_r(t)) \\ &\qquad - \Pi(F[z(t); x(t)] + Gd(t)), \\ \psi(\nu, t) &:= \nu + \eta_{[1]}^*(t) - \theta_\eta^*(\nu + \eta_{[1]}^*(t), t)\end{aligned}$$

with $u_r(t) = \gamma(c(t), Cx(t), y_r(t))$. Then, $\psi(0, t) = 0$ and $\psi(\nu, t)$ belongs to the sector $[\Pi G^-, \Pi G^+]$, that is,

$$(\psi(\nu, t) - \Pi G^-\nu)^T(\psi(\nu, t) - \Pi G^+\nu) \le 0, \ \forall\nu \in \mathbb{R}^m, \ \forall t \in \mathbb{R}. \tag{20}$$

□

Remark 4. Assuming that the slow variables are fixed parameters such that $[z; \bar{z}; x; c; y_r] \in Z \times U \times S_{y_r}$ and $|d| \le l_d$, the equilibrium point (ξ^*, η^*) of the fast subsystem (18) when $\tau = 0$ can be computed. From (18a), we have $\xi^* = 0$. Note that this results in $q = x$ since $q = \frac{1}{\tau}T_\tau^{-1}\Delta_\tau^{-1}\xi + T_\tau^{-1}x$, $T_\tau = I_r$ if $\tau = 0$, and $\frac{1}{\tau}\Delta_\tau^{-1}$ is well-defined for $\tau = 0$. On the other hand, from (18b), we have $\eta_{i2}^* = \cdots = \eta_{ir_i}^* = 0$, $i = 1, \ldots, m$, and

$$\eta_{i1}^* + \pi_i \left(F_i[z;x] + G_i(\Phi(w^*) + \Pi \bar{G} u_r + d) \right) \\ - \pi_i \bar{G}_i u_r - \Phi_i(w^*) = 0, \quad i = 1, \ldots, m, \tag{21}$$

where $w^* = [w_1^* \ \cdots \ w_m^*]^T$ and $w_i^* = \eta_{i1}^* + \pi_i \bar{F}_i[\bar{z};x]$. Interestingly, this equation is the same as $\psi(0,t) = 0$ when t is fixed and $\eta_{[1]}^*$ is defined as $[\eta_{11}^* \ \cdots \ \eta_{m1}^*]^T$. Therefore, $\eta_{[1]}^*$, defined in Lemma 2, is the unique solution of (21) for $1 \leq i \leq m$. (Uniqueness follows from the sector condition of $\psi(\nu, t)$.) □

Now we consider the behavior of the fast subsystem (18). Let $\sigma = t/\tau$. Then, in new time scale σ, the system (18) becomes

$$\begin{aligned} \frac{d\xi_i}{d\sigma} &= A_{i\xi}\xi_i - \tau B_i \theta_{i\xi}(\eta_{[1]}, \xi, \tau, \tau\sigma) \\ \frac{d\eta_i}{d\sigma} &= A_{i\eta 0}\eta_i + B_i a_{i0} \left(-\eta_{i1} + \theta_{i\eta}(\eta_{[1]}, \xi, \tau, \tau\sigma) \right) \end{aligned} \tag{22}$$

where $A_{i\eta 0}$ is the same as $A_{i\eta}$ except the fact that the $(r_i, 1)$-element of $A_{i\eta 0}$ is zero. Here, by abuse of notation, the arguments of the functions $\theta_{i\xi}$ and $\theta_{i\eta}$ are simplified to $(\eta_{[1]}, \xi, \tau, \tau\sigma)$ in order to avoid unwanted notational burden. In fact, the slow variables $z(t)$, $\bar{z}(t)$, $x(t)$, $c(t)$, $d(t)$, and $y_r(t)$, depending on $t = \tau\sigma$, are replaced by the argument $\tau\sigma$ in the functions taking these slow variables as their arguments. Note that $\theta_{i\xi}$ and $\theta_{i\eta}$ are globally bounded with respect to $\eta_{[1]}$ and ξ thanks to the saturation function Φ.

Define $\tilde{\xi}(t) := \xi(t) - \xi^*(t) = \xi(t)$ and $\tilde{\eta}(t) := \eta(t) - \eta^*(t)$, in which $\xi^*(t) = 0$ and $\eta_{i2}^*(t) = \cdots = \eta_{ir_i}^*(t) = 0$, $i = 1, \ldots, m$ by Remark 4, and $\eta_{i1}^*(t)$, $i = 1, \ldots, m$, are given in Lemma 2 as the i-th element of $\eta_{[1]}^*(t)$. For convenience, define $\tilde{\eta}_{[1]} = \eta_{[1]} - \eta_{[1]}^*$.

Lemma 3. Let $T > 0$. Suppose that $z(t) \in Z$ and $[\bar{z}(t); x(t); c(t)] \in U$ for all $0 \leq t \leq T$. Then, there exists $\tau_1 > 0$ such that, for each $0 < \tau \leq \tau_1$, the solution of (22), initiated from any $\xi(0)$ and $\eta(0)$, satisfies

$$\left| [\tilde{\xi}(t); \tilde{\eta}(t)] \right| \leq k e^{-\lambda \frac{t}{\tau}} \left| [\tilde{\xi}(0); \tilde{\eta}(0)] \right| + \delta(\tau), \ \forall t \in [0, T] \tag{23}$$

with some positive constants k and λ that are independent of τ, and a class-$\mathcal{K}$ function δ. □

Proof of Lemma 3. Assume, without loss of generality, $\tau < 1$. The dynamics of $\tilde{\xi}$ and $\tilde{\eta}$ in σ time scale can be derived as, with $'$ denoting $d/d\sigma$,

$$\tilde{\xi}' = A_\xi \tilde{\xi} - \tau B \theta_\xi(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tilde{\xi}, \tau, \tau\sigma) \tag{24}$$

$$\tilde{\eta}' = A_{\eta 0}\tilde{\eta} + B a_{[0]} \big(- \tilde{\eta}_{[1]} - \eta_{[1]}^* \\ + \theta_\eta(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tilde{\xi}, \tau, \tau\sigma) \big) - \bar{B}(\eta_{[1]}^*)' \tag{25}$$

where

$$\begin{aligned} \bar{B} &:= \mathrm{blockdiag}\{\bar{B}_1, \ldots, \bar{B}_m\}, \quad \bar{B}_i := [1; 0_{r_i - 1}], \\ a_{[0]} &:= \mathrm{diag}\{a_{10}, \ldots, a_{m0}\}, \\ A_\xi &:= \mathrm{blockdiag}\{A_{1\xi}, \ldots, A_{m\xi}\}, \\ A_{\eta 0} &:= \mathrm{blockdiag}\{A_{1\eta 0}, \ldots, A_{m\eta 0}\}. \end{aligned}$$

For now, let us consider the following system

$$\tilde{\xi}' = A_\xi \tilde{\xi} \tag{26}$$

$$\tilde{\eta}' = A_{\eta 0}\tilde{\eta} + B a_{[0]} \big(- \tilde{\eta}_{[1]} - \eta_{[1]}^* + \theta_\eta^*(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tau\sigma) \big), \tag{27}$$

where θ_η^* is defined in Lemma 2. Clearly, the system (24)–(25) is a perturbed system of (26)–(27). Hence, we first prove the exponential stability of the system (26)–(27) and then derive (23).

The system (26) is exponentially stable since A_ξ is Hurwitz. Hence, there exists a positive definite symmetric matrix P such that

$$P_\xi A_\xi + A_\xi^T P_\xi = -I_r. \tag{28}$$

On the other hand, the system (27) can be viewed as a feedback system

$$\tilde{\eta}' = A_{\eta 0}\tilde{\eta} + B a_{[0]} u^\dagger, \quad y^\dagger = \tilde{\eta}_{[1]} = C\tilde{\eta} \tag{29a}$$

$$u^\dagger = -\psi(y^\dagger, \tau\sigma). \tag{29b}$$

because $\psi(\tilde{\eta}_{[1]}, \tau\sigma) = \tilde{\eta}_{[1]} + \eta_{[1]}^*(\tau\sigma) - \theta_\eta^*(\tilde{\eta}_{[1]} + \eta_{[1]}^*(\tau\sigma), \tau\sigma)$ from Lemma 2. Noting that the transfer function matrix from $u^\dagger$ to $y^\dagger$ is $H(s)$ of (9), and that $H(s)[I + G^- H(s)]^{-1}$ is diagonal and Hurwitz, and $[I + G^+ H(s)][I + G^- H(s)]^{-1}$ is diagonal and strictly positive real [Khalil, 2002], by construction, it follows from the circle criterion [Khalil, 2002, Sec. 7.1.1] that the system (27) is exponentially stable and admits a quadratic Lyapunov function $W = \tilde{\eta}^T P_{\tilde{\eta}} \tilde{\eta}$, where $P_{\tilde{\eta}}$ is a symmetric positive definite matrix, such that $W' \leq -\kappa |\tilde{\eta}|^2$ with $\kappa > 0$.

Note that the perturbation terms of (25) with respect to the system (27) are $B a_{[0]}(\theta_\eta(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tilde{\xi}, \tau, \tau\sigma) - \theta_\eta^*(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tau\sigma))$ and $\bar{B}(\eta_{[1]}^*)'$. From the relation

$$\begin{aligned} &|\theta_\eta(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tilde{\xi}, \tau, \tau\sigma) - \theta_\eta^*(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tau\sigma)| \\ &\quad = |(I_m - \Pi G)(\Phi(\tilde{\eta}_{[1]} + \eta_{[1]}^* + \Pi \bar{F}[\bar{z}; \mathcal{T}_\tau[\xi; x]]) \\ &\qquad\qquad - \Phi(\tilde{\eta}_{[1]} + \eta_{[1]}^* + \Pi \bar{F}[\bar{z}; x]))|, \end{aligned}$$

it follows that there exist $k_1 > 0$ and $k_2 > 0$ such that

$$\begin{aligned} &|\theta_\eta(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tilde{\xi}, \tau, \tau\sigma) - \theta_\eta^*(\tilde{\eta}_{[1]} + \eta_{[1]}^*, \tau\sigma)| \\ &\quad \leq \|(I_m - \Pi G)\| \|\Pi \bar{F}\| \left| [\bar{z}; \tfrac{1}{\tau} T_\tau^{-1} \Delta_\tau^{-1} \xi + T_\tau^{-1} x] - [\bar{z}; x] \right| \\ &\quad \leq \|(I_m - \Pi G)\| \|\Pi \bar{F}\| \left(\left| \tfrac{1}{\tau} T_\tau^{-1} \Delta_\tau^{-1} \xi \right| + \left| (T_\tau^{-1} - I) x] \right| \right) \\ &\quad \leq k_1 |\xi| + k_2 \tau, \end{aligned}$$

where the mean value theorem and the fact $\|(\partial \Phi)/(\partial w)\| \leq 1$, $\forall w$ are used in the first inequality, and the last inequality follows from the boundedness of $\left\| \frac{1}{\tau} T_\tau^{-1} \Delta_\tau^{-1} \right\|$ and $\left\| \frac{1}{\tau}(T_\tau^{-1} - I) \right\|$ for $\tau < 1$. Regarding the second perturbation, it can be shown that there exists a constant k_3 such that $\left| (\eta_{[1]}^*)' \right| \leq \tau k_3$.

Finally, let $V(\tilde{\xi}, \tilde{\eta}) = \alpha \tilde{\xi}^T P_\xi \tilde{\xi} + \tilde{\eta}^T P_{\tilde{\eta}} \tilde{\eta}$ with α to be determined. Then, for (24)–(25),

$$\begin{aligned} V' &\leq -\alpha |\tilde{\xi}|^2 - 2\tau\alpha \tilde{\xi}^T P_\xi B \theta_\xi - \kappa |\tilde{\eta}|^2 \\ &\quad + 2\tilde{\eta}^T P_{\tilde{\eta}} \left(B a_{[0]}(\theta_\eta - \theta_\eta^*) - \bar{B}(\theta_{[1]}^*)' \right) \\ &\leq -\alpha |\tilde{\xi}|^2 + \tau\alpha \varrho_1 |\tilde{\xi}| - \kappa |\tilde{\eta}|^2 \\ &\quad + |\tilde{\eta}| (\tau(\varrho_2 k_2 + \varrho_3 k_3) + \varrho_2 k_1 |\tilde{\xi}|) \end{aligned}$$

where $\varrho_1 = 2|P_\xi B| \max\{\theta_\xi\}$, $\varrho_2 = 2|P_{\tilde{\eta}} B a_{[0]}|$, and $\varrho_3 = 2|P_{\tilde{\eta}} \bar{B}|$. (Note that θ_ξ is bounded on $Z \times U$ with bounded d and y_r, which can be seen from the definition of θ_ξ in Lemma 1.) With $\alpha = (k_1 \varrho_2)^2/\kappa$, we have

$$V' \leq -\frac{\alpha}{2} |\tilde{\xi}|^2 - \frac{\kappa}{2} |\tilde{\eta}|^2 + \tau\alpha \varrho_1 |\tilde{\xi}| + \tau(\varrho_2 k_2 + \varrho_3 k_3) |\tilde{\eta}|.$$

Hence, it follows that $V' \leq -\bar{\alpha}_1 V + \tau \bar{\alpha}_2 \sqrt{V}$ with some $\bar{\alpha}_1 > 0$ and $\bar{\alpha}_2 > 0$. By the comparison lemma [Khalil, 2002] and the quadratic property of V, the claim follows. □

Remark 5. By repeating the proof of Lemma 3, one can easily prove that if $\tau = 0$, the fast subsystem (22) (i.e., the boundary-layer system of (17) and (18)) is globally exponentially stable at the equilibrium $[\xi^*; \eta^*]$. □

Assuming that $z(t) \in Z$ and $[\bar{z}(t); x(t); c(t)] \in U$, the stability of the boundary-layer system enables us to derive the *quasi-steady-state system* as follows:

$$\begin{aligned} \dot{z} &= Sz + Px \\ \dot{\bar{z}} &= \bar{S}\bar{z} + \bar{P}x \\ \dot{x} &= Ax + B\left(\bar{F}[\bar{z}; x] + \bar{G}\gamma(c, Cx, y_r)\right) \\ \dot{c} &= \Gamma(c, Cx, y_r) \end{aligned} \tag{30}$$

which is obtained by putting ξ^* and η^* into the slow system (17) and using the fact

$$\begin{aligned} \theta_\xi|_{\tau=0,\xi=\xi^*,\eta=\eta^*} &= F[z; x] + G(w^* + \Pi\bar{G}u_r + d) \\ &= \bar{F}[\bar{z}; x] + \bar{G}u_r \end{aligned}$$

where $w^* = \eta^*_{[1]} + \Pi\bar{F}[\bar{z}; x] = G^{-1}(\bar{F}[\bar{z}; x] - F[z; x]) + (G^{-1} - \Pi)\bar{G}u_r - d$ and $u_r = \gamma(c, Cx, y_r)$. It is interesting to observe that the quasi-steady-state system (30) is nothing but the nominal closed-loop system augmented by the z-dynamics.

With the arguments so far, we are ready to prove the statements of Theorem 1. The system (17) and (18), that is in the standard singular perturbation form, is equivalent to the closed-loop system (1), (4), and (14) with $\bar{y}$ being replaced by y in (4). Then, there exists $T_1 > 0$, which is independent of τ, such that the state $[x(t); c(t)]$ of the solution $[z(t); \bar{z}(t); x(t); c(t)]$ of (17) and (18) remains in $U_z \times U$ for $0 \le t \le T_1$, with any initial condition $[z(0); \bar{z}(0); x(0); c(0); \xi(0); \eta(0)] \in \bar{S}_z \times \bar{S} \times \mathbb{R}^r \times \mathbb{R}^r$ and with any $0 < \tau \le 1$, because the compact set $\bar{S}_z \times \bar{S}$ is contained in the open set $U_z \times U$, and the vector field of (17) is bounded, independently of τ, by the saturation functions. On the other hand, there exists $T_2 > 0$, independent of τ, such that $|[x(t); c(t)] - [x_N(t); c_N(t)]| \le \epsilon/2$ for $0 \le t \le T_2$, because $[x(0); c(0)] = [x_N(0); c_N(0)]$ and, again, the vector field of (17) is bounded. Finally, let $T = \min\{T_1, T_2\}$. Then, Lemma 3 is applicable, which yields that

$$|[\xi(T) - \xi^*; \eta(T) - \eta^*(T)]| \le ke^{-\lambda\frac{T}{\tau}}\left(\frac{k^*}{\tau^{r^*}} + \hat{k}\right) + \delta(\tau)$$

where we used that $|[\tilde{\xi}(0); \tilde{\eta}(0)]| \le (k^*/\tau^{r^*} + \hat{k})$ from Remark 3, and $\hat{k}$ is a bound for $\theta^*_{[1]}$. Therefore, by taking a sufficiently small τ, we have the property that $\xi(T) \to \xi^*$ and $\eta(T) \to \eta^*(T)$ as $\tau \to 0$. Then, the Tikhonov's theorem can now be applied for the time interval $[T, \infty)$. Indeed, the boundary-layer system (i.e., $\tau = 0$) is uniformly globally exponentially stable by Remark 5, and the solution $[\bar{z}_N(t); \bar{x}_N(t); c_N(t)]$ (i.e., the nominal trajectory) of the quasi-steady-state system is uniformly locally asymptotically stable by Assumption 2. With these, all the assumptions in [Hoppensteadt, 1966] hold. Then, by the Tikhonov's theorem, Theorem 1 is proved.

4. CONCLUSION

We have proposed an inner-loop controller for a class of MIMO linear systems, which guarantees robust transient performance as well as robust steady-state performance, assuming that an outer-loop controller shows a satisfactory performance for the nominal plant without any disturbances. This has been achieved by employing some saturations functions on top of the disturbance observer structure discussed in [Shim and Joo, 2007]. Nonlinear extension of the proposed result can be one of further studies.

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