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Stability Investigation of a Robotic Swarm with Limited Field of View

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Abstract: This paper presents an analytical study of swarm motion in a quasi static environment, in which, motion of each member is being affected by interactive forces and agents. Interactive effects on each member could be attractive or repulsive due to being far from or close to other members respectively. An agent also can be attractive or repulsive. The method is based on Lyapunov analysis. The aim is to preserve the unity of swarm i.e. not losing any member through motion, while being under influence of an agent. It is also considered that field of view of swarm members is limited; which is the most important characteristic of this work.

1. INTRODUCTION

Schooling and aggregation have been under investigation since 1954. Distributed control structures and artificial swarming have attracted lots of studies in robotics. What ever the mission of an artificial swarm is, one of the most important problems is to preserve unity while trying to coordinate the swarm. Prediction of total swarm size, inter individual distances between members, domain of attraction of attractive objects around the swarm and secure distance from repellent objects in order to save the swarm unity, are considerable factors of many researches.

Observations of natural flocks by Breder (1954), Partridge (1982), and Miller (1996), and computer simulations by Warburton (1990) and Brogan (1997) showed that the assumption of mutual attractive and repulsive forces between members will lead to acceptable social behaviors which are similar to natural flocking behaviors. Artificial potential fields were first introduced for obstacle avoidance by Khatib (1986). Using artificial potential fields and through simulations Reif and Wang (1999) showed that the quasi static modeling also leads to acceptable social behaviors. Gazi and Passino (2002 and 2003 a) considered a scalable swarm with mass less and dimension less members in an n dimensional Euclidian space, and with the assumption of homogeneity and unlimited vision proved that in the absence of any environmental motivation the geometrical center of the swarm will not move. They also showed that the final position of members is unique and is dependent to initial conditions but can not be predicted. Later (2004) they considered the effect of the environment and discussed the cohesion and goal convergence. Gazi (2005) also introduced a sliding mode controller so that the members with considerable inertial effects would follow the high viscosity swarming behaviors in previous studies. Kim, Wang, Ye and Shin (2004) discussed the unreachable goal problem and collision between members and proposed some criteria for design on the basis of artificial potential fields. Baras and Tan (2004) using graph methods modelled the swarm as a Markof random field and tried to control aggregation, segregation and even produce linear configurations via Gibbs sampling methods. Liu, Passino and Polycarpou (2003 a) considered a line of members in one dimensional space with a leader in front. They showed that with the assumption of motion delay, in order not to break this chain velocity of the leader must not exceed a special amount. Later they discussed the problem in n dimensional space (2003 b) with the assumption that the neighborhoods remain unchanged during the motion. A very famous theory in discrete time modeling of swarms is proposed by Vicsek, Czirok, Jacob, Cohen, and Schochet (1995). They showed that using the nearest neighbor rule, a swarm which possesses members with equal velocities and different directions will finally move in a unified direction. They also considered limited sensing extent. Later Jadbabaie, Lin, and Morse (2003), Sarkin (2004), and Gao and Cheng (2005) revised this method and strengthened the proof. Olfati saber and Murray (2004 and 2006) studied the flocking phenomena and combined both graph and energy methods. Martinoli, Easton, and Agassounon (2004) proposed a simulation method and algorithm which decreases the computation time at least by four times. Soysal and Sahin (2005) and Starke, Schanz, and Fukuda (2005) used behavioral control systems to produce cohesive or dispersive behaviors and coordinated a robotic swarm through a field of obstacles. Dougherty (2004) also used behavioral rules and potential fields to control a group of vehicles toward the goal. Gerkey, Thrun and Gordon (2006) developed a technique for coordinating a swarm to execute the searching task in an application domain. Egerested and Hu (2001) and Ogren (2002) used Lyapunov methods to construct a special configuration among agents while trying toward goal and avoiding obstacles. They considered that agents posses a complete dynamic model. Kalantar and Zimmer (2007) studied the under water formation control of vehicles for ocean surface sampling and exploration. Desai, Ostrowski,

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and Kumar (2001 and 2002), tried graph methods to coordinate and reconfigure the agents while moving. They also showed that the method is robust against noise. Fierro et. al. (2002) proposed a framework for deployment of a robotic swarm in an unstructured and unknown environment. Gazi (2003 b) showed that the problem of coordination of some agents along a periodic track and remaining in a predefined configuration will lead to design of a classic servo control system. Howard, Parker and Sukhatme (2006) tried to implement distributed control system on a heterogeneous swarm in order to make them capable of carrying out a specific mission. Bahceci and Sahin (2005) introduced Perceptron (neural network) controllers to achieve acceptable cohesion behavior. Xi, Tan, and Baras (2005) proposed a combination of artificial potential fields and Simulated Annealing technique to control a swarm among environmental potential fields.

In this paper a swarm model is proposed and analytically discussed in a continuous time dynamic field. There is no limit on the number of space dimensions and the number of swarm members. In order to be more realistic and similar to a robotic swarm, it is assumed that field of view of every member is not infinity. Stability analysis via Lyapunov method is performed which guarantees the unity preservation while moving toward the goal or running from the repellant zones.

2. MODELING

A swarm as supposed here is homogenous and includes at least two members i.e. $M \geq 2$. Swarm members are considered to be dimensionless, which is a normal assumption in this field of studies. We also consider no time delay in motion of swarm members. Motion and behaviour of swarm members are mostly result of two different phenomena: interactive mutual forces and influence of one or more agents. Interactive mutual forces comprise both attraction and repulsion. Attraction to other members in sight helps to keep the swarm unity, and repulsion from members which are closer than a specified distance avoids collision between swarm members. So there will be a specified equivalent distance named d_{eq} which for distances more than d_{eq} interaction is attractive and for distances less than d_{eq} interaction is repulsive. Contrary to many previous stability investigations on swarms, here, to be more realistic the field of the swarm members' view is not infinity. Hence members out of field of view due to not being clearly sighted, show no mutual attraction. Let us consider inter-individual interaction vector $\mathbf{G_{ij}(.)}$ as follows:

$$\mathbf{G}_{ij} = (x^i - x^j) g_{ij} = \begin{cases} g_r(x^i - x^j) & \|x^i - x^j\| < d_r \\ -\mu(x^i - x^j) & d_r < \|x^i - x^j\| < f_v \\ 0 & \|x^i - x^j\| > f_v \end{cases} \quad (1)$$

where $i,j \in [1, M]$. x^i and x^j represent location of ith and jth members and $\mathbf{G_{ij}}$ corresponds to the effect of the latter on the former. μ is a positive constant coefficient and g_r is a positive definite function. g_r is only defined to avoid collision of members. We can control the amount of d_{eq} by proper choose of the g_r function. We will neglect function g_r in our analysis. Domain of affection of g_r is small. The outer zone around every member is the attraction zone. Intensity of attraction in this zone is controlled with μ. f_v is the extent of inter-individual field of view of swarm members; i.e. if the jth member is farther than f_v from the ith member, then it has no effect on the ith member.

Motion of swarm members is also affected by agents. A swarm member would be influenced by an agent if and only if the agent is not farther than a specified distance. This distance is represented by $\mathbf{f_{av}}$. Therefore regarding an agent, members farther than $\mathbf{f_{av}}$ are not capable to see the agent. Hence there is a circle with center located at the agent's location $\mathbf{C_k}$ with radius equal to $\mathbf{f_{av}}$, in which every swarm member see the agent, and out of the circle no member is capable to see the agent. We name the circle, *vicinity of the agent*. Let us express the agent's effect analytically via the following equation:

$$\sigma_{ik} = \sigma(x^i - C^k) = \begin{cases} g_{ar}(x^i - C^k) & \|x^i - C^k\| < d_{ar} \\ -\beta_k(x^i - C^k) & d_{ar} < \|x^i - C^k\| < f_{av} \\ 0 & \|x^i - C^k\| > f_{av} \end{cases} \quad (2)$$

It is assumed that there may be more than one agent. Index k corresponds to a specific agent where number of all agents is equal to N. Therefore $k \in [1, N]$. g_{ar} is a positive definite function avoiding collision of members and agents. β is a constant coefficient and is positive if the agent attracts swarm members and is negative if the agent repels them. We can control minimum allowed distance between the agent and swarm members by proper choose of the g_{ar} function. We will neglect function g_{ar} in our analysis because domain of affection of g_r is small. The outer zone around the agent is controlled by coefficient β. Greater value of β cause stronger attraction to the agent.

According to Reif and Wang (1999), and biological observations by Breder (1954), Partridge (1982), and Miller et. al. (1996), and also according to computer simulations by Warburton et. al. (1990) and Brogan et. al. (1997), in natural swarms viscosity is considerably high and mass is negligible. Therefore velocity is dependent to external forces, i.e. $c\dot{x}^i = cv^i = u^i$. Here external forces $u^{\mathbf{i}}$ are caused by inter-individual interactions and agents' effects; therefore velocity of each swarm member can be calculated using a formula as:

$$\dot{x}^i = \sum_{k=1}^{N} \sigma_{ik} + \sum_{j=1}^{M} G_{ij} \quad (3)$$

All of the above assumptions are completely in accordance with nature and biological swarms. But our goal is to use this analysis for robotic systems. It may be doubted that the consideration of highly over damped behaviour of a robotic swarm system is justifiable. According to Gazi (2005 And 2007) even if this assumption could not be established, it is possible to design a controller so that every robot follows the mass less viscous dynamic behaviour as a swarm member.

3. DISCUSSION ON SWARM CENTER MOTION

Since here a homogenous swarm is considered, all M members of the swarm are the same, Then the geometrical centre of the swarm would be defined as follows:

$$X = 1/M \sum_{i=1}^{M} x^i \quad (4)$$

Derivation leads to:

$$\dot{X} = +1/M \sum_{i=1}^{M}\sum_{k=1}^{N}\sigma_{ik} + 1/M \sum_{i=1}^{M}\sum_{\substack{j=1 \\ j\neq i}}^{M} G_{ij} \tag{5}$$

First term in the above equation expresses the effect of the agents on the motion of the swarm center. Second term is the effect of interactive forces. If interactive forces are mutual, then $G_{ij} = (x^i - x^j).g_{ij} = -(x^j - x^i).g_{ji} = -G_{ji}$ and this cause the second term to vanish. Hence, providing the mutuality of interactions, swarm center motion only depends on agents. In equilibrium state, every pair of swarm members should feel no desire for motion when they are in a specified distance from each other. Otherwise motion of the center will be observed even in the absence of any external effects.

Suppose that there is just one agent and the first Q members of the swarm are located in the vicinity of an agent where Q<M. Geometrical center of these Q members is named pseudo center of the swarm and is calculated via the following equation:

$$Z = 1/Q \sum_{i=1}^{Q} x^i \tag{6}$$

Due to the limited field of view, the other members are not capable to feel the agents. We need to investigate that if the swarm center converges to the agent's location or not.

Lemma I: Swarm center moves toward an attractive agent's location at most until the whole vicinity of the agent is located inside the swarm.

Proof of Lemma I: Let us define a Lyapunov function V_C as:

$$V_C = \frac{1}{2}\left\| e_C \right\|^2 \tag{7}$$

in which e_C=X-C. It is possible to show that derivative of (7) leads to the following equation:

$$\dot{V}_C = -\frac{\beta Q}{M}\left(e_C^t \, e_C' \right) \tag{8}$$

where e_C'=Z-C. β is defined in equation (2). e_C decreases if (8) is negative. A swarm as we have considered here always tries to follow a minimum energy pattern. Therefore members move so that the boundary of the swarm shows a circular or oval or a convex shape. Simulations also verify this fact. In this condition $e_C.e_C'$ posses a positive value which guarantees negative definiteness of V_C (Fig. 1).

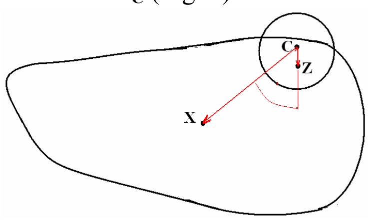

Fig. 1: illustration of vectors e_C'=Z-C and e_C=X-C. Agent is close to the swarm. $e_C.e_C'$ possesses a positive value.

Equation (8) would be equal to zero if e_C' vanishes, which means that the pseudo center of the swarm coincides with the agent's location. Despite of occasional and unstable situations, this coincidence will happen if swarm members are scattered uniformly around the agent. Therefore we can say that the whole agent's vicinity is located inside the swarm. Hence there is no guaranty that swarm center converges to the attractive agent's location unless both their locations coincide at the initial condition or the agent's vicinity is vast enough to accommodate all *M* members of the swarm.

If the agent is repellant then β is negative which causes (8) to be positive under the same circumstances. Therefore if the agent's vicinity is totally located inside the swarm then pseudo center of the swarm coincides with the agent's location and swarm will be trapped. But if the agent's vicinity does not have a complete overlap with swarm then derivative of V_C is positive and swarm center runs away from the agent's location. It is obvious that if some swarm members (or all of them) are outside an agent's vicinity then they feel no desire to move toward or run away from the agent. Hence there would be no reason gathering all swam members uniformly around an attractive agent.

4. SWARM UNITY

It is necessary to establish and discuss conditions so that the unity of the swarm is preserved near an agent. We are going to use agents to coordinate swarm and unity preservation here means not to make any member to exit from the range of view of other members. Attractive and repellent agents have different influences on swarm unity. Attractive agent near swarm would weaken the unity because it may attract and separate one or more members from other swarm members. Therefore contrary to the stabilizing character of an attractive agent, it may induce instability in swarm. The opposite phenomena would be observed for a repellent agent. It means that a repellent agent near swarm would force them together and strengthen the unity.

To investigate proper conditions for preservation of the swarm unity let us define Unity function as follows:

$$V = \sum_{i=1}^{M}\sum_{\substack{j=1 \\ j\neq i}}^{M} \frac{1}{4M}\left\| x^i - x^j \right\|^2 \tag{9}$$

We need to establish conditions for descending behavior of unity function *V*. So we shall discuss its derivative:

$$\dot{V} = -M\dot{V}_C - \beta\sum_{i=1}^{Q}\left\| x^i - C \right\|^2 - \frac{1}{2}\sum_{i=1}^{M}\sum_{\substack{j=1 \\ j\neq i}}^{M} g_{ij}\left\| x^i - x^j \right\|^2 \tag{10}$$

Lemma II: Swarm unity will be preserved beside an attractive agent if the following inequality is satisfied:

$$\frac{\beta}{\mu}\frac{Q}{M} < \frac{f_v^{\,2}}{2f_{av}^{\,2}} \tag{11}$$

Proof of Lemma II: If the agent is not located inside the swarm at initial conditions, then manipulation of (10) leads to the following inequality. Due to limitation of pages we have ignored the procedure:

$$\dot{V} < \sum_{i=1}^{Q}\sum_{\substack{j=1 \\ j\neq i}}^{M}\left\{ \frac{\beta Q}{M}\left\| x^i - C \right\|^2 - \frac{1}{2}\mu\left\| x^i - x^j \right\|^2 \right\} \tag{12}$$

If (12) is going to be negative definite then (11) must be satisfied. If vicinity of the attractive agent is vast enough to accommodate all *M* members then (11) will reduce to (13).

Lemma III: Swarm unity will be preserved beside a repellant agent if the following inequality is satisfied:

$$\frac{\beta}{\mu} < \frac{f_v^{\,2}}{2f_{av}^{\,2}} \tag{13}$$

Proof of Lemma III: If the agent is not located inside the swarm at initial conditions, then manipulation of (10) leads to the following inequality:

$$\dot{V} < \sum_{i=1}^{Q}\left\{\beta\left\|x^i - C\right\|^2 - \frac{1}{2}\sum_{\substack{j=1 \\ j\neq i}}^{M}\mu\left\|x^i - x^j\right\|^2\right\} \tag{14}$$

If (14) is going to be negative definite then (13) must be satisfied. Note that all presented proofs are based on the assumption that every swarm member has at least one member in sight.

5. VALIDATION VIA SIMULATION

In order to verify results a few simulations is performed. The aim is to preserve swarm unity both near attractive and repellent agents. Values of parameters used in simulation are reported in table 1. Both g_r and g_{ar} are selected with too much gradient therefore all inter-individual distances d_{eq} are forced to be approximately equal to 1m. Values of β and μ satisfy (11) and (13), Hence swarm unity is expected to be preserved. The swarm possesses 32 members.

Table 1. Values of Parameters used in Simulation

μ	0.4	β	0.2
F_v	1.5m	f_{av}	2m
d_r	1m	d_{ar}	0.3m

5.1 Swarm Near a Repellant Agent

Fig. 2 illustrates behaviour of the swarm near a repellant agent. In Fig. 2A swarm runs away from the agent's location and distance between swarm center and the agent's location grows. This behaviour continues till all members exit from the agent vicinity. After this time swarm center shows no movement as illustrated in Fig. 2 A4. In Fig. 1B pseudo center of the swarm and agent's location coincide at the initial condition and swarm is trapped around the repellant agent. As Fig. 2B shows little movement can be observed during simulation which means more arrangement around the repellant agent. In both cases unity is preserved.

5.2 Swarm Beside an Attractive Agent

Fig.3 shows the effect of an attractive agent near swarm. At initial conditions two members are inside the agent vicinity. Due to the distance between pseudo center Z and the agent position C, pseudo center and real center X start moving toward the agent. This motion continues till swarm members cover the whole agent vicinity. At the time Z has come very close to C and this cause pseudo center Z and real center X both to stop moving. Also none of the inter-member relations is broken due to the agent effect which means that satisfaction of equation (11) and lemma II has preserved the swarm unity near the attractive agent.

In both simulations we can see that in final conditions members do not seem to be gathered around a point and boundary of the swarm resembles more to a convex polygonal. This polygonal would be smoothened by increasing field of view of members. This will also decrease the final distance between swarm center and for example an agent's location, and will bring members more uniformly around it.

It may be questioned that how much it is possible to influence a swarm by agents. The answer comes back to the constraints like extent of view and attraction-repulsion functions defined by the designer.

6. CONCLUSIONS

Stability analysis and preservation of swarm unity is discussed analytically in this paper. Contrary to previous stability analysis in continuous time dynamic modelling of a swarm, limited field of view is considered here which leads to more realistic swarming behaviour. In a recent previous work by the author (Etemadi, et. al. 2007), fading field of view is considered and the assumption of limited field of view shows more reasonable results, especially about the convergence of the swarm center to the agent's location and behaviour of the swarm near an agent. Conditions for preservation of swarm unity near attractive or repellent agents are discussed via Unity Function.

The method introduces very secure conditions of stability. Although Lyapunov method naturally is a conservative method but we believe that it is possible to improve the stability conditions by extending secure domains via definition of new Lyapunov functions which is under study yet. It should be noted that the results are going to be used for coordination of robotic swarms which is still under study. The problem of multi agent and a swarm also would be very similar to the presented procedure especially when there are agents with non-overlapping vicinities, but still needs a little more concentration.

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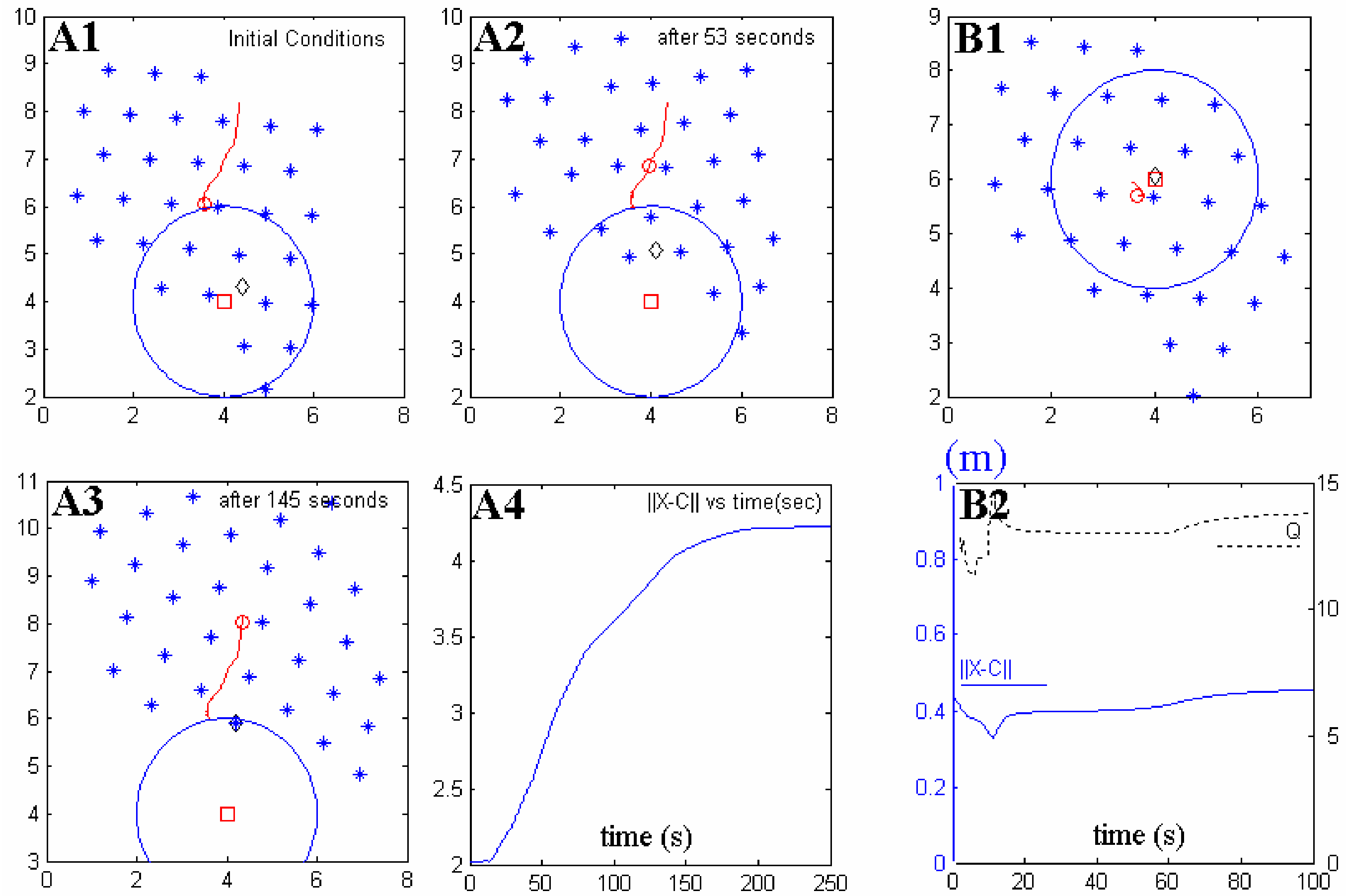

Fig. 2: **A1,2,3)** Repellant Agent pushes the swarm away till all swarm members exit from the agent vicinity. **A4)** Changes of the distance between swarm center and agent position vs. time.
B) Entrapment of swarm around the repellant agent. **B1)** Positions of swarm members. **B2)** Distance between swarm center and the agent position (solid graph) show little change. Q (dashed graph) also follows the same pattern.
(Stars *: swarm members, square□ : agent's location, circle ○: swarm center, and diamond ◊: pseudo center of the swarm, continues line: swarm center motion path.)

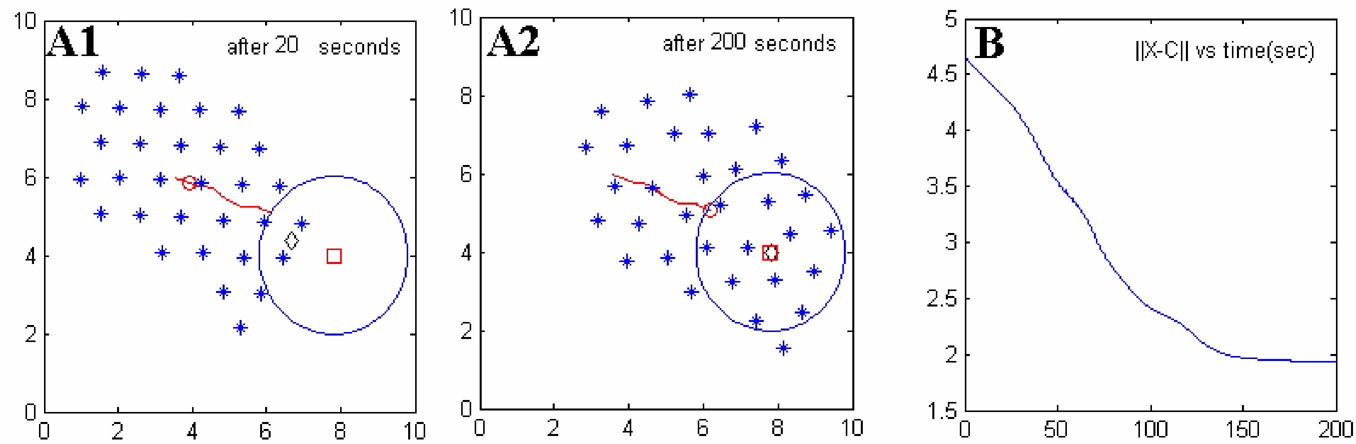

Fig. 3: Attractive agent pulls the swarm and swarm center till the whole agent vicinity is filled with swarm members
A) Initial and final positions of swarm member **B)** change of the distance between swarm center and the agent vs. time.
(Stars *: swarm members, square□ : agent's location, circle ○: swarm center, and diamond ◊: pseudo center of the swarm, continues line: swarm center motion path.)