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Start-up Control of a Hot Strip Mill Tension/Looper System: an Approach based on Model Predictive Control

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Abstract: For the start-up control of tension and looper system, an off-line design method of the control law is considered based on model predictive control approach. By employing a multi-parametric programming for the posed problem, a piecewise affine state feedback control law, which inherits the advantage of the model predictive control, is constructively given. The feature of resulting control system is illustrated with numerical examples.

1. INTRODUCTION

Tension and looper control is the key to successful operations in the hot strip finishing mill and the various control strategies have been applied to improve the performance in the mutual interaction and the transient. For the control of the mutual interaction between the tension and the looper angle, several modern control schemes have been applied and the resulting performance around the equilibrium position has been investigated (Imanari et al. (1997); Seki et al. (1991)). While, a model predictive control (MPC) is newly applied to the start-up control of the tension and looper system (Imura et al. (2004); Asano et al. (2005)) and it is reported that the on-line model predictive control has efficient potential for attenuating the bump against the strip and attaining favorable transient. In the model predictive control, the essential strategy is that the optimal control problem is solved, in the discrete-time setting, on-line over a finite-horizon and the first value of the resulting control sequence is applied. Thus, in the MPC approach to the tension and looper systems, it must be also recognized that the implementation is limited by the complexity of the optimization procedure which is performed in the sample period.

In this paper, we focus on the start-up control of the tension and looper system (Imura et al. (2004); Asano et al. (2005)) and discuss a design method of state feedback control law, which inherits the advantage of the MPC approach. By employing the multi-parametric programming (Bemporad et al. (2002a)) to the start-up control problem, we will show that the MPC law is constructively given by a piecewise affine state feedback control law.

The paper is organized as follows. In Section 2, the tension and looper system is reviewed based on (Imura et al. (2004); Asano et al. (2005)) and a simplified model is newly formulated, which enables us to apply the multi-parametric programming approach in the off-line controller design. In Section 3, a model predictive control problem with LQ cost-functional is posed and the preliminary results are summarized. In Section 4, the calculation method of piecewise affine state feedback control law is proposed by employing multi-parametric quadratic programming (mp-QP). In Section 5, the proposed design method is evaluated with a numerical example. It is illustrated that the control law attenuates the bump against the strip and inherits the advantage of the control scheme for the mutual interaction between the strip and the looper.

2. MODELING

Based on the fundamental results reported by (Imura et al. (2004); Asano et al. (2005)), we first summarize a piecewise affine model for the tension and looper system. The tension and the looper systems are separately modeled and, with account of the interaction which depends on the looper angle, a unified model is obtained.

2.1 Dynamic equations

Based on Fig.1 and Table 1, the dynamic equation of the looper system is described as follows:

$$J\ddot{\theta} = T_{Lref} - \delta\{K_1(\theta)\sigma + K_2(\theta)\} - K_3(\theta) - D\dot{\theta} \quad (1)$$

$$K_1(\theta) = 2Hbr\cos\theta\sin\beta \quad (2)$$

$$K_2(\theta) = 2pHbgr\frac{l}{\cos\beta}\cos\theta \quad (3)$$

$$K_3(\theta) = W_L g r_L \cos(\theta + \theta_G) \quad (4)$$

where K_1, K_2, K_3 denote the looper load torque by the tension, the strip weight and the looper weight, respectively. In the equation (1), δ is a binary variable ($\delta \in \{0,1\}$) and included for describing the behavior both in the contact mode (C-mode) and in the noncontact mode (N-mode). The case $\delta = 1$ corresponds to the C-mode which considers the interaction with the tension system. While the case $\delta = 0$ corresponds to the N-mode as the interaction with the tension system is neglected.

The mode transition rule is given as follows:

Table 1. Notation and parameters

Sign	Value	Unit	Description
θ	-	[rad]	Looper angle
σ	-	[MPa]	Interstand tension
V_R	-	[m/s]	Roll velocity
T_{Lref}	-	[rad]	Looper torque
J	2.16×10^3	[Nm^2]	Looper inertia
H	3.1×10^{-3}	[m]	Strip thickness
b	1.0	[m]	Strip width
p	7.85×10^3	[kg/m^3]	Strip density
r	0.6	[m]	Looper arm length
W_L	2.5×10^3	[kg]	Looper weight
l	2.8	[m]	Half of length between stands
r_L	0.125	[m]	Distance between looper axis and center of gravity of looper
D	49.0	[Nms]	Looper damping factor
θ_G	5.0	[deg]	Offset angle between looper angle and center of gravity of looper
β	1.16	[deg]	Strip angle with passline
g	9.8	[m/s^2]	Acceleration of gravity
$f(\sigma)$	0.2154		Forward slip
E	1.96	[GPa]	Young's modulus of strip
L	23.448	[m]	Interstand strip length
T_{ASR}	0.2	[s]	Time constant of mill motor ASR
$\theta_{\min}$	$10\pi/180$	[rad]	Looper angle when the looper is raised to the passline

Table 2. Operating point at C and N modes

Sign	Value	Unit
θ_c	$20\pi/180$	[rad]
σ_c	9.8	[MPa]
T_{Lrefc}	4.2×10^3	[Nm]
V_{Rc}	9.29	[m/s]
θ_n	$10\pi/180$	[rad]

$$\delta = \begin{cases} 0 & \text{if} \quad \theta < \theta_{\min} \\ 1 & \text{if} \quad \theta \geq \theta_{\min} \end{cases} \tag{5}$$

where $\theta_{\min}$ is the looper angle when the looper is raised to the pass line (Table 2).

The tension system is described by

$$\dot{\sigma} = \frac{E}{2l}\left\{-(1+f(\sigma))V_R + \frac{\partial L}{\partial \theta}\dot{\theta}\right\} \tag{6}$$

$$\dot{V}_R = -\frac{1}{T_{ASR}}(V_R - V_{Rref}) \tag{7}$$

and the state transition from the N-mode to the C-mode is assumed by

$$\dot{\theta}(t) = \epsilon_1\dot{\theta}(t_-) \tag{8}$$

$$\sigma(t) = \sigma(t_-) + \epsilon_2\dot{\theta}(t_-) \tag{9}$$

where ϵ_1, ϵ_2 are appropriately estimated constants.

2.2 Piecewise Affine Model

Linearizing the dynamic equations around the operating point (Table 2), a unified model for the tension and looper system is elaborated. In the following, we first linearize the dynamic equation in each mode and derive a unified discrete-time piecewise affine model.

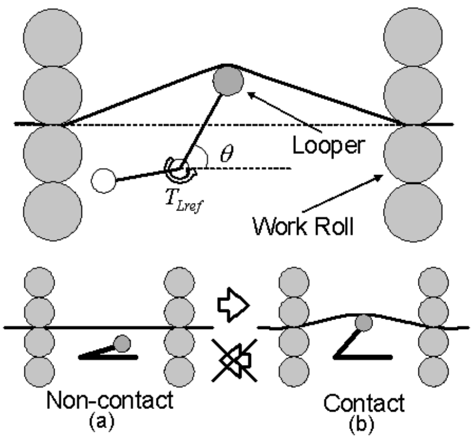

Fig. 1. Tension and looper system (start-up control)

C-mode: Let $(\theta_c, \sigma_c, V_{R_c}, T_{Lrefc}, V_{Rrefc})$ be the operating point such that the equalities

$$T_{Lrefc} = K_1(\theta_c)\sigma_c + K_2(\theta_c) + K_3(\theta_c) \tag{10}$$

$$V_{Rrefc} = V_{Rc} \tag{11}$$

are preserved (Table 2). Denoting the variarion by $(\bar{\theta}, \bar{\sigma}, \bar{V}_R, \bar{T}_{Lref}, \bar{V}_{Rref}) = (\theta - \theta_c, \sigma - \sigma_c, V_R - V_{Rc}, T_{Lref} - T_{Lrefc}, V_{Rref} - V_{Rrefc})$, then linearizing the equations (1)-(4), (6), (7) with $\delta = 1$, the following equation is obtained.

$$\dot{\tilde{x}}(t) = A_c^c\tilde{x}(t) + B_c^c\tilde{u}(t) \tag{12}$$
$$\tilde{x} = [\,\bar{\theta}\ \dot{\bar{\theta}}\ \bar{\sigma}\ \bar{V}_R\,]^T, \quad \tilde{u} = [\,\bar{T}_{Lref}\ \bar{T}_{Rref}\,]^T$$

N-mode: Let (θ_n, T_{Lrefn}) be the operating point which preserves

$$T_{Lrefn} = K_3(\theta_n), \quad V_{Rn} = V_{Rrefn}. \tag{13}$$

Denoting the variation by $(\check{\theta}, \check{T}_{Lref}) = (\theta - \theta_n, T_{Lref} - T_{Lrefn})$, then linearizing the equations (1)-(4) with $\delta = 0$, the equation

$$\dot{\check{x}}(t) = A_n^c\check{x}(t) + B_n^c\check{u}(t) \tag{14}$$
$$\check{x} = [\,\check{\theta}\ \dot{\check{\theta}}\,]^T, \quad \check{u} = \check{T}_{Lref}$$

is obtained. Finally, substituting the coordinate defined for the C-mode: $\check{\theta} = \bar{\theta} + (\theta_c - \theta_n)$, $\check{T}_{Lref} = \bar{T}_{Lref} + (T_{Lrefc} - T_{Lrefn})$, the following affine system is obtained.

$$\dot{x}(t) = A_n^c x(t) + B_n^c u(t) + a_n^c \tag{15}$$
$$x = [\,\bar{\theta}\ \dot{\bar{\theta}}\,]^T, \quad \tilde{u} = \bar{T}_{Lref}$$

Based on the system descriptions (5), (8), (9), (12), (15), a piecewise affine model is derived as follows.

PWA-model: Discretizing the systems (12),(15) with the sample period h, the following model is obtained:

$$\text{N-mode}\ \begin{cases} x_{k+1} = A_n x_k + B_n u_k + a_n \\ I_{k+1} = I_k \\ \text{if}\ \ Cx_k + c < 0,\ \ I_k = 0 \end{cases} \tag{16}$$

$$\text{NC-mode}\ \begin{cases} \tilde{x}_k = E_{nc} x_k + e_{nc} \\ \tilde{x}_{k+1} = A_c \tilde{x}_k + B_c \tilde{u}_k \\ I_{k+1} = 1 \\ \text{if}\ \ Cx_k + c \geq 0,\ \ I_k = 0 \end{cases} \tag{17}$$

$$\text{C-mode}\ \begin{cases} \tilde{x}_{k+1} = A_c \tilde{x}_k + B_c \tilde{u}_k \\ I_{k+1} = I_k \\ \text{if}\ \ \tilde{C}\tilde{x}_k + c \geq 0,\ \ I_k = 1 \end{cases} \tag{18}$$

where

$$x_k = [\,\bar{\theta}(kh)\ \dot{\bar{\theta}}(kh)\,]^T \tag{19}$$

$$\tilde{x}_k = [\,\bar{\theta}(kh)\ \dot{\bar{\theta}}(kh)\ \bar{\sigma}(kh)\ \bar{V}_R(kh)\,]^T \tag{20}$$

$$C = [\,1\ 0\,],\ \ \tilde{C} = [\,1\ 0\ 0\ 0\,],\ \ c = \theta_c - \theta_{\min}, \tag{21}$$

$$A_n = e^{A_n^c h},\ B_n = \int_0^h e^{A_n^c \tau} d\tau B_n^c,\ a_n = \int_0^h e^{A_n^c \tau} d\tau a_n^c,$$

$$A_c = e^{A_c^c h},\quad B_c = \int_0^h e^{A_c^c \tau} d\tau B_c^c. \tag{22}$$

In (16)-(18), it is noted that the NC-mode is newly included in order to describe the state-transition (8), (9) at the first sample time which holds the condition $Cx_k + c \geq 0$. The state x_k and the control u_k in the N-mode are regarded as

$$\tilde{x}_k = [\,x_k^T\ 0\ 0\,]^T,\quad \tilde{u}_k = [\,u_k\ 0\,]^T \tag{23}$$

in the coordinate of the C-mode. Following notations are prepared for describing the state regions for N and NC, C modes.

$$\mathcal{N} := \{x_k : Cx_k + c < 0\} \tag{24}$$

$$\mathcal{C} := \{x_k : Cx_k + c \geq 0\} \tag{25}$$

$$\tilde{\mathcal{C}} := \{\tilde{x}_k : \tilde{C}\tilde{x}_k + c \geq 0\} \tag{26}$$

In highlight with the hybrid system model reported by (Imura et al. (2004); Asano et al. (2005)), it is noted that the model adopted here is further simplified in the sense that the time constant of looper motor ACR is neglected. Based on the simplified N-mode model, which feature is characterized on the 2-dimensional state-space, we will discuss a design method of piecewise affine state feedback control law.

3. MODEL PREDICTIVE CONTROL

For the start-up control from the initial position (N-mode) to the operating point (C-mode), we focus on a model predictive control (MPC) problem, which attains the favorable transient over mode-transition. In this section, we formulate the MPC problem for the system (16)-(18) and provide preliminaries for the off-line controller design.

Consider an MPC problem defined as follows.

$$\min_U J,\quad J := \sum_{i=0}^{\infty} \{\tilde{x}_i^T \tilde{Q} \tilde{x}_i + \tilde{u}_i^T \tilde{R} \tilde{u}_i\} \tag{27}$$
$$\text{subj. to}\ \ (16)\text{-}(18)$$
$$U := \{\tilde{u}_0, \tilde{u}_1, \tilde{u}_2, \cdots\},\quad \tilde{Q} > 0,\quad \tilde{R} > 0$$

In the C-mode, the MPC problem (27) coincides with a standard LQ control problem and, if the system response stays in the state region $\tilde{\mathcal{C}}$, the control law is expressed as follows:

$$\tilde{u}_k = K_{\mathrm{LQ}} \tilde{x}_k,\ \ K_{\mathrm{LQ}} = -(\tilde{R} + B_c^T P B_c)^{-1} B_c^T P A_c \tag{28}$$

where $P > 0$ is a stabilizing solution to

$$P = \tilde{Q} + A_c^T P A_c - A_c^T P (\tilde{R} + B_c^T P B_c)^{-1} B_c^T P A_c. \tag{29}$$

In the sequel, we impose the following assumption and clarify the control law which is formulated by the MPC problem (27).

(A) For the resulting system obtained by (27), the state region $\tilde{\mathcal{C}}$ is positively invariant; i.e.

$$(A_c + B_c K_{\mathrm{LQ}})\tilde{x} \in \tilde{\mathcal{C}}\ \text{ if }\ \tilde{x} \in \tilde{\mathcal{C}} \tag{30}$$

holds. □

Under the condition (A), it is noted that the MPC problem (27) is expressed by

$$\min_{U_{N_s}} J_{N_s},\quad J_{N_s} := \sum_{i=0}^{N_s - 1} \{x_i^T Q x_i + u_i^T R u_i\} \tag{31}$$
$$+(E_{nc} x_{N_s} + e_{nc})^T P (E_{nc} x_{N_s} + e_{nc})$$
$$\text{subj. to}\ \ (16)\text{-}(18)$$
$$U := \{\tilde{u}_0, \tilde{u}_1, \tilde{u}_2, \cdots\},$$
$$Q := [\,I_2\ 0_{2\times 2}\,]^T \tilde{Q} [\,I_2\ 0_{2\times 2}\,] > 0$$
$$R := [\,1\ 0\,]^T \tilde{R} [\,1\ 0\,] > 0$$

where N_s is the sample time when the transition to the NC-mode arise (see also Chmielewski et al. (1996); Scokaert et al. (1998)). Furthermore, the solution to (31) is represented by

$$\tilde{u}_k = \begin{cases} [\,u_k\ 0\,]^T,\ k = 0, 1, \cdots, N_s - 1\ \ (\text{N-mode}) \\ K_{\mathrm{LQ}} \tilde{x}_k \quad k = N_s, N_s + 1, \cdots \quad (\text{C-mode}) \end{cases} \tag{32}$$

and the control law is obtained explicitly if the N-mode control sequence $\{u_k : k = 0, 1, \cdots, N_s - 1\}$ is parameterized in terms of the state x_k.

In the next section, we will derive an explicit representation of the control law for the MPC problem (31).

4. OFF-LINE CONTROLLER DESIGN

Employing the multi-parametric programming approach (Bemporad et al. (2002a)), we derive a piecewise affine state feedback control law in the N-mode. The multi-parametric programming approach has been applied to typical control problems with system constraints (e.g. Bemporad et al. (2002a,b); Borrelli (2003); Kojima et al.

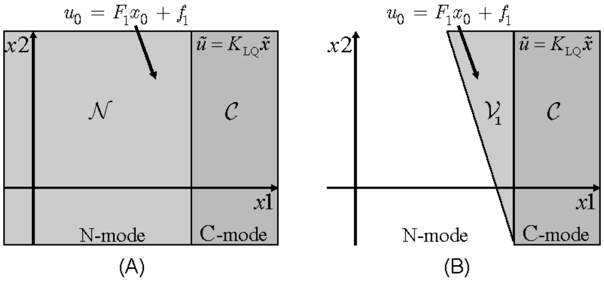

Fig. 2. Partition of region $\mathcal{N}$ ($N_s = 1$)

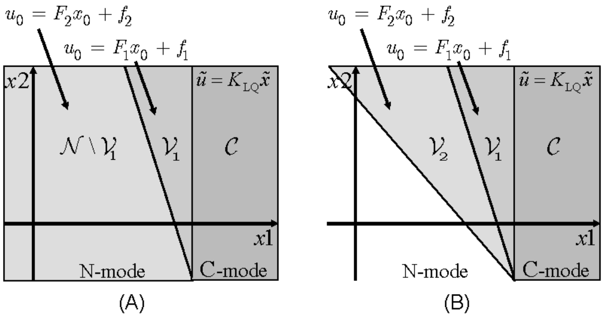

Fig. 3. Partition of region $\mathcal{N}$ ($N_s = 2$)

(2004)), and it enables us to obtain an explicit representation of the control law via off-line optimization.

In this section, we introduce a following result on the multi-parametric quadratic programming (mp-QP) problem and construct a piecewise affine state-feedback control law for the MPC problem (31).

Lemma 1. (Bemporad et al. (2002a)). For a QP-problem

$$\min_{w \in \mathbb{R}^s} \frac{1}{2} w^T \hat{H} w \quad \text{subj.to} \quad \hat{G}w \leq \hat{W} + \hat{S}x_0, \tag{33}$$

let $w = w^*$ be the optimal solution for a given $x_0 = x_0^*$ and $(\tilde{G}, \tilde{W}, \tilde{S})$ be the rows of active constraints such that $\tilde{G}w^* = \tilde{W} + \tilde{S}x_0^*$ holds. Under the assumption such that the rows of $\tilde{G}$ are linearly independent, the optimal solution of (33) is expressed by

$$w = \hat{H}^{-1}\tilde{G}^T(\tilde{G}\hat{H}^{-1}\tilde{G}^T)^{-1}(\tilde{W} + \tilde{S}x_0) \tag{34}$$

in the polyhedral region

$$\hat{G}\hat{H}^{-1}\tilde{G}^T(\tilde{G}\hat{H}^{-1}\tilde{G}^T)^{-1}(\tilde{W} + \tilde{S}x_0) \leq \hat{W} + \hat{S}x_0^*, \tag{35}$$

$$-(\tilde{G}\hat{H}^{-1}\tilde{G}^T)^{-1}(\tilde{W} + \tilde{S}x_0) \leq 0. \tag{36}$$

□

Investigating the relation between the state and the initial value of the control signal, the calculation method of the control law is summarized as follows.

Algorithm:

[Step 1]

Let $N_s = 1$ and parameterize the optimal solution in terms of the initial state $x_0 \in \mathcal{N}$. Then specify the initial state region such that the resulting control causes the transition to NC-mode at $N_s = 1$.

For $N_s = 1$, the MPC problem (31) is rewritten by

$$\begin{aligned} \min_{u_0} J_1, \quad J_1 = x_0^T Q x_0 + u_0^T R u_0 & \\ + (E_{nc}x_1 + e_{nc})^T P (E_{nc}x_1 + e_{nc}) & \quad (37) \\ \text{subj.to} \quad x_1 = A_n x_0 + B_n u_0 + a_n & \end{aligned}$$

and, applying Lemma 1, the optimal control for (37) is expressed as follows.

$$u_0 := F_1 x_0 + f_1, \quad x_0 \in \mathcal{N} \tag{38}$$

Since the control (38) yields the transition

$$x_1 = A_n x_0 + B_n(F_1 x_0 + f_1) + a_n =: G_1 x_0 + g_1 \tag{39}$$

at $N_s = 1$, the state-region such that (37) coincides with the solution of (31) is represented as follows (Fig. 2).

$$\mathcal{V}_1 = \{\, x_0 \;:\; G_1 x_0 + g_1 \in \mathcal{C} \} \tag{40}$$

[Step 2]

Let $N_s = 2$ and parameterize the optimal solution in terms of the initial state $x_0 \in \mathcal{N} \setminus \mathcal{V}_1$. Then specify the subset of state region such that the resulting control causes the transition to $\mathcal{V}_1$ at 1 unit-time later.

For $N_s = 2$, the MPC problem (31) is rewritten by

$$\begin{aligned} \min_{U_2} J_2, \quad J_2 = \sum_{i=0}^{1} x_i^T Q x_i + u_i^T R u_i & \\ + (E_{nc}x_2 + e_{nc})^T P (E_{nc}x_2 + e_{nc}) & \quad (41) \\ \text{subj.to} \quad x_{k+1} = A_n x_k + B_n u_k + a_n, \quad k = 1, 2 & \\ U_2 = \{u_0, u_1\} & \end{aligned}$$

and, applying Lemma 1, the initial value of the optimal control sequence is expressed as follows.

$$u_0 := F_2 x_0 + f_2, \quad x_0 \in \mathcal{N} \setminus \mathcal{V}_1 \tag{42}$$

Since (42) yields the transition

$$x_1 = A_n x_0 + B_n(F_2 x_0 + f_2) + a_n =: G_2 x_0 + g_2 \tag{43}$$

at 1 unit-time later, the state-region such that (41) coincides with the solution of (31) is represented as follows (Fig. 3).

$$\mathcal{V}_2 = \{\, x_0 \;:\; G_2 x_0 + g_2 \in \mathcal{V}_1 \} \tag{44}$$

[Step 3]

Let $N_s = k$ ($k = 2, 3, \cdots$) in the MPC problem (31) and parameterize the optimal control u_0 in terms of the initial state $x_0 \in \mathcal{N} \setminus (\bigcup_{i=1}^{k-1} \mathcal{V}_i)$. Applying Lemma 1, the initial value of the resulting control sequence is parameterized by

$$u_0 := F_k x_0 + f_k, \quad x_0 \in \mathcal{N} \setminus (\bigcup_{i=1}^{k-1} \mathcal{V}_i) \tag{45}$$

and, further, the state region such that the state moves to $\mathcal{V}_{k-1}$ at 1 unit-time later is obtained as follows:

$$\mathcal{V}_k = \{\, x_0 \,:\, G_k x_0 + g_k \in \mathcal{V}_{k-1} \} \tag{46}$$

where $G_k := A_n + B_n F_k$, $g_k := B_n f_k$ holds the equality:

$$x_1 = A_n x_0 + B_n(F_k x_0 + f_k) + a_n =: G_k x_0 + g_k. \tag{47}$$

[Step 4]

Repeat Step 3 until the initial position of the start-up control is covered by the region (46).

Based on the algorithm (Step 1)-(Step 4), the control law in the N-mode of (32) is explicitly given in the following form.

$$u_k = \begin{cases} F_1 x_k + f_1 & x_k \in \mathcal{V}_1 \\ F_2 x_k + f_2 & x_k \in \mathcal{V}_2 \\ F_3 x_k + f_3 & x_k \in \mathcal{V}_3 \\ \vdots & \\ F_\ell x_k + f_\ell & x_k \in \mathcal{V}_\ell \end{cases} \tag{48}$$

Thus, a piecewise affine state feedback control law is constructively given based on (32) and (48),

5. SIMULATION

The feature of the resulting control system is discussed based on a discrete-time system (16)-(18) which is obtained with the sampling time $h = 0.02$[s]. For the MPC problem (31) with

$$\tilde{Q} = I_4, \quad \tilde{R} = I_2, \tag{49}$$

the LQ control in the C-mode is obtained by

$$\tilde{u}_k = \begin{bmatrix} 3110 & 3602 & -0.003 & 0.0002 \\ -4.511 & 1.885 & 0.966 & 0.422 \end{bmatrix} \tilde{x}_k, \quad x_k \in \tilde{\mathcal{C}} \tag{50}$$

and, further, it is verified that the condition (A) holds in this case.

Applying the calculation procedure summarized in Section 4, the control law is obtained as follows.

$$u_k = \begin{cases} [\,-340.90 \;\; -46.234\,]\, x_k - 0.5467 & x_k \in \mathcal{V}_1 \\ [\,-395.19 \;\; -61.415\,]\, x_k - 1.3667 & x_k \in \mathcal{V}_2 \\ [\,-449.60 \;\; -78.740\,]\, x_k - 2.5010 & x_k \in \mathcal{V}_3 \\ \qquad \vdots & \\ [\,-2142.7 \;\; -1703.3\,]\, x_k - 458.75 & x_k \in \mathcal{V}_{38} \end{cases} \tag{51}$$

For the state $x_k = (\bar{\theta}(kh), \dot{\bar{\theta}}(kh))$ $(k = 0, 1, \cdots)$, the relation between the state regions and the control laws in (51) are summarized by Fig. 4 [1]. The state-trajectory is depicted by Fig. 5 and it is observed that the control is switched to the standard LQ control (50) at 38 unit-time. While the control input and the state responses are summarized by Fig. 6. In this case, the solution of the MPC problem yields a control strategy such that the magnitude of control input is attenuated before the mode transition to C-mode.

[1] In Fig. 4, original coordinate of the looper system (1) is adopted: $(\theta, \dot{\theta}) = (\bar{\theta} + \theta_c, \dot{\bar{\theta}})$.

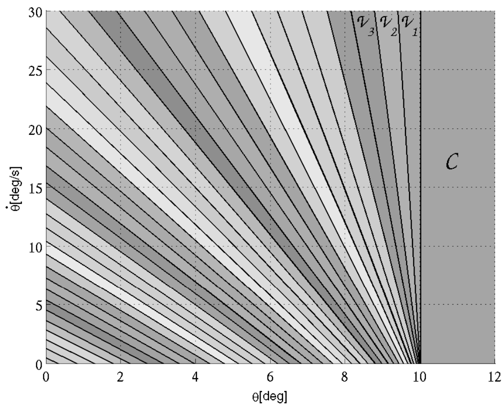

Fig. 4. Feedback control law: $\tilde{Q} = I_4$, $\tilde{R} = I_2$

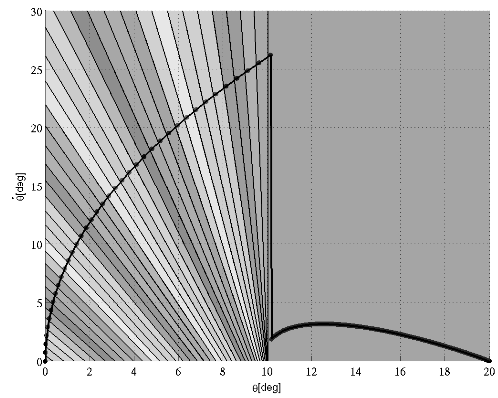

Fig. 5. State trajectory: $\theta = 0$ [deg], $\dot{\theta} = 0$ [deg/s]

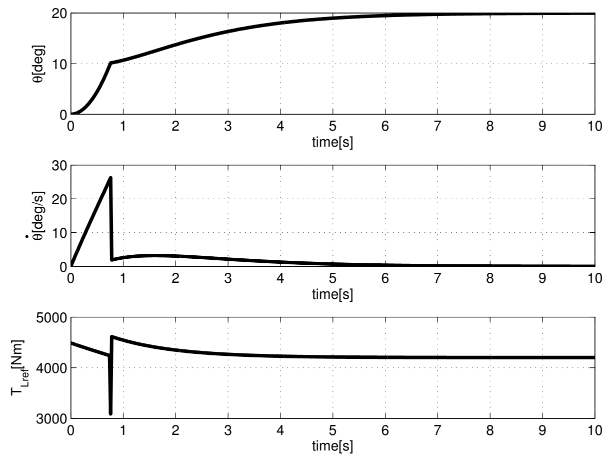

Fig. 6. Time response: $\theta = 0$ [deg], $\dot{\theta} = 0$ [deg/s]

Fig. 7, 8 are the system responses calculated with a fictitious state perturbation $\theta = 9$ [deg], $\dot{\theta} = -25$ [deg/s]. In this design example, the response is recovered from the perturbation and the appropriate mode transition is attained by the proposed state feedback control.

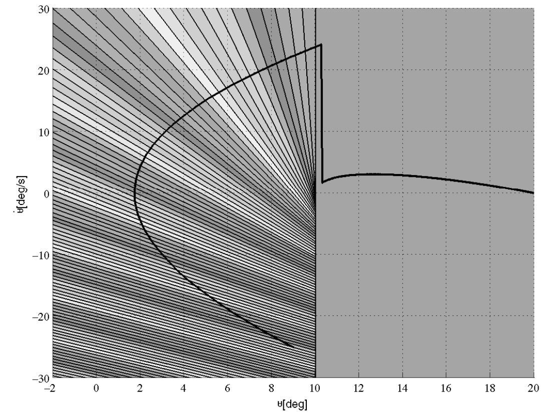

Fig. 7. State trajectory: $\theta = 9$ [deg], $\dot{\theta} = -25$ [deg/s]

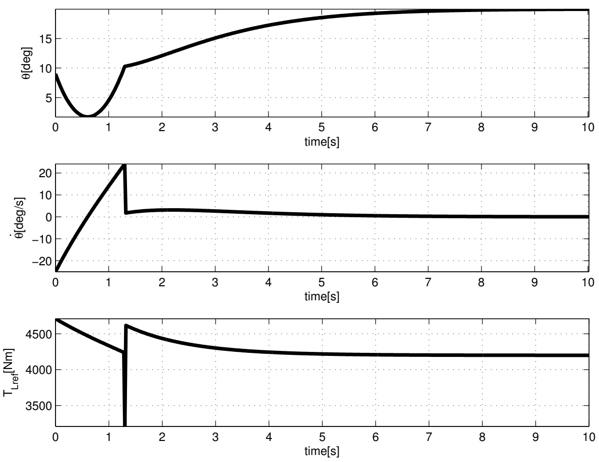

Fig. 8. Time response: $\theta = 9$ [deg], $\dot{\theta} = -25$ [deg/s]

6. CONCLUSION

For the start-up control of tension and looper system, an off-line design method of the control law is discussed based on a linear-quadratic model predictive control. By employing a multi-parametric programming, a piecewise affine state feedback control law is obtained and the feature of the resulting system is evaluated with numerical examples.

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