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New Delay-Dependent Criteria for Robust Stability of Uncertain Singular Systems ⋆

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Abstract: This note concerns the delay-dependent robust stability analysis for uncertain singular time-delay systems. The parameter uncertainty is assumed to be norm-bounded and possibly time-varying, while the time delay considered here is assumed to be constant but unknown. By using a new Lyapunov-krasovskii functional which splits the whole delay interval into two subintervals and defines a different energy function on each subinterval, some delay-dependent conditions are presented for the singular time-delay system to be regular, impulse free and robustly stable. The obtained delay-dependent criteria are effective and less conservative than previous ones, which are illustrated by numerical examples.

1. INTRODUCTION

Singular time-delay systems, which are also referred to as generalized differential-difference equations, descriptor time-delay systems, implicit time-delay systems and semi-state time-delay systems, have been extensively studied in the past years [4], [5]. Singular system model is a natural presentation of dynamic systems and can better describe a large class of systems than regular ones, such as large-scale systems, power systems and constrained control systems [4]. For a long time, the problems of stability analysis for such systems have been the subject of considerable research efforts [1], [2], [3], [13], [17].

Recently, it is known that when the stability problem for singular systems is investigated, the regularity and absence of impulses for continuous systems and causality for discrete systems are required to be considered simultaneously [4], [5], [16], [17]. This make the problems of stability analysis for singular time-delay systems much more complicated than those for state-space ones. For uncertain singular time-delay systems, robust stability problem was discussed in [7] and the H_∞ control as well as filter design problems were also investigated [8], [9]. It should be pointed out that most of the existing results in the literature are delay-independent. When the time-delay is small, these results are often quite conservative. The work of [10], [11], [12], [13], [6] presented some delay-dependent stability condition for singular time-delay systems based on the assumption that the considered system is regular and impulse free. Especially, in [10], a matrix describing the relationship between fast and slow subsystems should be chosen, which is difficult and sometimes impossible. While the stability conditions in [11], [12], [13] need decompose the system matrices, which makes the computation relatively intricate and unreliable. Nevertheless, all these delay-independent or delay-dependent criteria are obtained formulated by linear matrix inequality (LMI) that is based on a fixed Lyapunov-krasovskii functional. In this note, a different idea is that the time delay is divided into even subintervals. Then a new Lyapunov-krasovskii functional which splits the whole delay interval into two even subintervals are used to obtain some new stability conditions based on Lyapunov second method.

The delay-dependent robust stability problem for uncertain singular time-delay systems in this note is investigated. First, we present a new delay-dependent criterion which provides a sufficient condition for a unforced nominal singular time-delay system to be regular, impulse free and stable. Based on this result, the robust stability problem is studied. The obtained results can be regarded as an extension of the results of [3], [15] to their counterparts for uncertain singular time-delay systems.

Throughout this paper, $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, $\mathcal{R}^{m\times n}$ is the set of real matrices with m rows and n columns. I_n is an $n \times n$ identity matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ refer to the maximal and minimal eigenvalues of the matrix P respectively. $\|x\|$ denotes the Euclidean norm of the vector x, that is, $\|x\| = \sqrt{x^T x}$. diag$\{\cdots\}$ denotes a block-diagonal matrix. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that $X-Y$ is positive-definite (respectively, positive-semidefinite).

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2. PROBLEM FORMULATION

Consider an uncertain singular time-delay system described by

$$\begin{aligned} E\dot{x}(t) &= \bar{A}x(t) + \bar{A}_d x(t-d) \\ x(t) &= \phi(t), -d \leq t \leq 0 \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector. d is a constant time delay, $\phi(t)$ is any given initial condition specified on $[-d\ 0]$. The matrix E may be singular and we shall assume that rank $E = r \leq n$. $\bar{A} = (A + \Delta A)$, $\bar{A}_d = (A_d + \Delta A_d)$, A and A_d are constant matrices with appropriate dimensions. ΔA, ΔA_d are unknown matrices representing the admissible uncertainties in the system matrices and can be described as the form of

$$[\Delta A\ \Delta A_d] = MF(t)\,[N_a\ N_d] \tag{2}$$

where M, N_a, N_d are real constant matrices with appropriate dimensions, and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue-measurable elements satisfying

$$F^T(t)F(t) \leq I \tag{3}$$

The nominal unforced counterpart of the system (1) can be written as

$$E\dot{x}(t) = Ax(t) + A_d x(t-d) \tag{4}$$

To fascinate the following discussion, we introduce some definitions and lemmas, which are essential for the development of our main results.

Definition 1. (Dai. [4]).

1. The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero.

2. The pair (E, A) is said to be impulse free if $\deg(\det(sE - A)) = rank\, E$.

Lemma 1. For a given scalar $d^* > 0$, the solution to the system (4) exists and is unique and impulse free on $[0, \infty)$ for any constant time delay d satisfying $0 \leq d \leq d^*$, if the pairs (E, A) and $(E, A + A_d)$ are regular and impulse free.

Proof: If $d > 0$, the desired result follows immediately from the regularity and absence of impulses of the pair (E, A) and by employing the decomposition method [4].

If $d = 0$, the system (4) reduces to the linear singular system

$$E\dot{x}(t) = (A + A_d)x(t) \tag{5}$$

Noting that the pair $(E, A + A_d)$ is regular and impulse free, we can also obtain the desired result. □

Lemma 2. (Masubuchi [14]). The linear singular system

$$E\dot{x}(t) = Ax(t)$$

is regular, impulse free and stable if and only if there exists a matrix P with appropriate dimensions such that

$$P^T E = E^T P \geq 0,\ P^T A + A^T P < 0$$

Definition 2. For a given scalar $d^* > 0$, the singular time-delay system (4) is said to be regular and impulse free for any constant time delay d satisfying $0 \leq d \leq d^*$, if the pairs (E, A) and $(E, A + A_d)$ are regular and impulse free.

Definition 3. For a given scalar $d^* > 0$, the uncertain singular time-delay system (1) is said to be robustly stable, if this system is regular, impulse free and stable for all admissible uncertainties satisfying (2), (3) and any constant time delay d satisfying $0 \leq d \leq d^*$.

Lemma 3. If there exist matrices Ω_1, Ω_2 and $\Omega_3 \in \mathcal{R}^{n\times n}$ such that the following inequality (6a) holds, then the inequality (6b) is derived from (6a),

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{6a}$$

$$\Omega_1 + \Omega_2 + \Omega_2^T + \Omega_3 < 0. \tag{6b}$$

Proof: Pre-multiplying and post-multiplying (6a) by $[I\ I]$ and $[I\ I]^T$, respectively, we can obtain (6b) immediately. This completes the proof. □

3. MAIN RESULTS

Firstly, the result on stability analysis for the nominal system(4) is summarized in the following theorem. Then a delay-dependent condition for the uncertain system (1) to be robustly stable is presented.

Theorem 1. For a given scalar $d^* > 0$, the singular time-delay system (4) is regular, impulse free and stable for any constant time-delay d satisfying $0 \leq d \leq d^*$, if there exist symmetric positive-definite matrices P, Q_1, Q_2, Z_1, Z_2 and matrices S, X_1, X_2, Y_1, Y_2 such that the linear matrix inequalities (7) are satisfied,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \frac{d^*}{2}A^T(Z_1+Z_2) \\ * & \Xi_{22} & 0 & 0 \\ * & * & -Q_1 & \frac{d^*}{2}A_d^T(Z_1+Z_2) \\ * & * & * & -\frac{d^*}{2}(Z_1+Z_2) \end{bmatrix} < 0 \tag{7a}$$

$$\begin{bmatrix} X_i & Y_i \\ * & Z_i \end{bmatrix} \geq 0,\ \ i = 1, 2 \tag{7b}$$

where $R \in \mathcal{R}^{n\times(n-r)}$ is any column-full-rank matrix satisfying $E^T R = 0$ and

$\Xi_{11} = E^T PA + A^T PE + SR^T A + A^T RS^T + Y_2 E + E^T Y_2^T + \frac{d^*}{2}(X_1 + X_2) + Q_2,\ \Xi_{12} = Y_1 E - Y_2 E,$

$\Xi_{13} = SR^T A_d + E^T PA_d - Y_1 E,\ \Xi_{22} = Q_1 - Q_2$

Proof: By Lemma 3 and Schur complement, it follows from (7) that

$$\begin{aligned} 0 &> \Xi_{11} + \Xi_{12} + \Xi_{12}^T + \Xi_{13} + \Xi_{13}^T + \Xi_{22} - Q_1 \\ &\geq (E^T P + SR^T)(A + A_d) + (A + A_d)^T(PE + RS^T) \end{aligned} \tag{8}$$

By denoting $U = PE + RS^T$ and employing Lemma 2, it can be easily verified that the pair $(E, A + A_d)$ is regular, impulse free and stable. In other words, the system (4) with $d = 0$ is regular, impulse free and stable.

Note rank $E = r \leq n$, there exist two nonsingular matrices $\bar{G}$ and $\bar{H}$ such that

$$\bar{E} = \bar{G}E\bar{H} = \begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix} \tag{9}$$

then, the matrix R can be parameterized as

$$R = \bar{G}^T \begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix} \tag{10}$$

where $\bar{\Phi} \in \mathcal{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

In view of (9), we can define

$$\bar{A} = \bar{G}A\bar{H} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \tag{11a}$$

$$\bar{A}_d = \bar{G}A_d\bar{H} = \begin{bmatrix} \bar{A}_{d11} & \bar{A}_{d12} \\ \bar{A}_{d21} & \bar{A}_{d22} \end{bmatrix} \tag{11b}$$

$$\bar{P} = \bar{G}^{-T}P\bar{G}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix} \tag{11c}$$

$$\bar{Q}_1 = \bar{H}^TQ_1\bar{H} = \begin{bmatrix} \bar{Q}_{111} & \bar{Q}_{112} \\ \bar{Q}_{121} & \bar{Q}_{122} \end{bmatrix} \tag{11d}$$

$$\bar{Q}_2 = \bar{H}^TQ_2\bar{H} = \begin{bmatrix} \bar{Q}_{211} & \bar{Q}_{212} \\ \bar{Q}_{221} & \bar{Q}_{222} \end{bmatrix} \tag{11e}$$

$$\bar{X}_1 = \bar{H}^TX_1\bar{H} = \begin{bmatrix} \bar{X}_{111} & \bar{X}_{112} \\ \bar{X}_{121} & \bar{X}_{122} \end{bmatrix} \tag{11f}$$

$$\bar{X}_2 = \bar{H}^TX_2\bar{H} = \begin{bmatrix} \bar{X}_{211} & \bar{X}_{212} \\ \bar{X}_{221} & \bar{X}_{222} \end{bmatrix} \tag{11g}$$

$$\bar{Y}_1 = \bar{H}^TY_1\bar{G}^{-1} = \begin{bmatrix} \bar{Y}_{111} & \bar{Y}_{112} \\ \bar{Y}_{121} & \bar{Y}_{122} \end{bmatrix} \tag{11h}$$

$$\bar{Y}_2 = \bar{H}^TY\bar{G}^{-1} = \begin{bmatrix} \bar{Y}_{211} & \bar{Y}_{212} \\ \bar{Y}_{221} & \bar{Y}_{222} \end{bmatrix} \tag{11i}$$

Left- and right-multiplying Ξ_{11} with $\bar{H}^T$ and $\bar{H}$ yield

$$\bar{\Xi}_{11} = \bar{H}^T\Xi_{11}\bar{H} = \begin{bmatrix} \bar{\Xi}_{11,11} & \bar{\Xi}_{11,12} \\ * & \bar{\Xi}_{11,22} \end{bmatrix} < 0 \tag{12}$$

where $\bar{\Xi}_{11,22} = \bar{S}_2\bar{\Phi}^T\bar{A}_{22} + \bar{A}_{22}^T\bar{\Phi}\bar{S}_2^T + \frac{d^*}{2}(\bar{X}_{122} + \bar{X}_{222}) + \bar{Q}_{222}$, $\bar{S} = \bar{H}^TS = \begin{bmatrix} \bar{S}_1^T & \bar{S}_2^T \end{bmatrix}^T$ and the expressions of $\bar{\Xi}_{11,11}$ and $\bar{\Xi}_{11,12}$ are irrelevant of the following discussion and hence are omitted here. Obviously, (12) implies that $\bar{A}_{22}$ is nonsingular. Suppose, by contradiction, that $\bar{A}_{22}$ is singular, then there exists a vector $\xi \neq 0$ such that $\bar{A}_{22}\xi = 0$ and thus

$$\xi^T\bar{\Xi}_{11,22}\xi \geq \xi^T\bar{Q}_{222}\xi > 0$$

On the other hand, it follows from (12) that $\bar{\Xi}_{11,22} < 0$, which is a contradiction.

We define

$$\tilde{G} = \begin{bmatrix} I & -\bar{A}_{12}\bar{A}_{22}^{-1} \\ 0 & I \end{bmatrix} \bar{G} \tag{13a}$$

$$\tilde{H} = \bar{H} \begin{bmatrix} I & 0 \\ -\bar{A}_{22}^{-1}\bar{A}_{21} & \bar{A}_{22}^{-1} \end{bmatrix} \tag{13b}$$

Then, it is easily shown that

$$\tilde{E} = \tilde{G}E\tilde{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{14a}$$

$$\tilde{A} = \tilde{G}A\tilde{H} = \begin{bmatrix} \tilde{A}_{11} & 0 \\ 0 & I \end{bmatrix} \tag{14b}$$

where $\tilde{A}_{11} = \bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21}$.

Therefore,

$$\begin{aligned} \det(sE - A) &= \det(\tilde{G}^{-1})\det(s\tilde{E} - \tilde{A})\det(\tilde{H}^{-1}) \\ &= \det(\tilde{G}^{-1})(-1)^{(n-r)}\det(sI_r - \tilde{A}_{11})\det(\tilde{H}^{-1}) \end{aligned} \tag{15}$$

which implies that $\det(sE - A)$ is not identically zero and $\deg(\det(sE - A)) = r = \operatorname{rank} E$. Then, the pair (E, A) is regular and impulse free.

In view of (14), we define

$$\tilde{A}_d = \tilde{G}A_d\tilde{H} = \begin{bmatrix} \tilde{A}_{d11} & \tilde{A}_{d12} \\ \tilde{A}_{d21} & \tilde{A}_{d22} \end{bmatrix} \tag{16a}$$

$$\tilde{P} = \tilde{G}^{-T}P\tilde{G}^{-1} = \begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix} \tag{16b}$$

$$\tilde{Q}_1 = \tilde{H}^TQ_1\tilde{H} = \begin{bmatrix} \tilde{Q}_{111} & \tilde{Q}_{112} \\ \tilde{Q}_{121} & \tilde{Q}_{122} \end{bmatrix} \tag{16c}$$

$$\tilde{Q}_2 = \tilde{H}^TQ_2\tilde{H} = \begin{bmatrix} \tilde{Q}_{211} & \tilde{Q}_{212} \\ \tilde{Q}_{221} & \tilde{Q}_{222} \end{bmatrix} \tag{16d}$$

$$\tilde{X}_1 = \tilde{H}^TX_1\tilde{H} = \begin{bmatrix} \tilde{X}_{111} & \tilde{X}_{112} \\ \tilde{X}_{121} & \tilde{X}_{122} \end{bmatrix} \tag{16e}$$

$$\tilde{X}_2 = \tilde{H}^TX_2\tilde{H} = \begin{bmatrix} \tilde{X}_{211} & \tilde{X}_{212} \\ \tilde{X}_{221} & \tilde{X}_{222} \end{bmatrix} \tag{16f}$$

$$\tilde{Y}_1 = \tilde{H}^TY_1\tilde{G}^{-1} = \begin{bmatrix} \tilde{Y}_{111} & \tilde{Y}_{112} \\ \tilde{Y}_{121} & \tilde{Y}_{122} \end{bmatrix} \tag{16g}$$

$$\tilde{Y}_2 = \tilde{H}^TY_2\tilde{G}^{-1} = \begin{bmatrix} \tilde{Y}_{211} & \tilde{Y}_{212} \\ \tilde{Y}_{221} & \tilde{Y}_{222} \end{bmatrix} \tag{16h}$$

$$\tilde{Z}_1 = \tilde{G}^{-T}Z_1\tilde{G}^{-1} = \begin{bmatrix} \tilde{Z}_{111} & \tilde{Z}_{112} \\ \tilde{Z}_{121} & \tilde{Z}_{122} \end{bmatrix} \tag{16i}$$

$$\tilde{Z}_2 = \tilde{G}^{-T}Z_2\tilde{G}^{-1} = \begin{bmatrix} \tilde{Z}_{211} & \tilde{Z}_{212} \\ \tilde{Z}_{221} & \tilde{Z}_{222} \end{bmatrix} \tag{16j}$$

We multiply (7a), on the left and on the right , by $\operatorname{diag}\{\tilde{H}^T, \tilde{H}^T, \tilde{H}^T, \tilde{G}^{-T}\}$ and $\operatorname{diag}\{\tilde{H}, \tilde{H}, \tilde{H}, \tilde{G}^{-1}\}$, respectively, and (7b), by $\operatorname{diag}\{\tilde{H}^T, \tilde{G}^{-T}\}$ and $\operatorname{diag}\{\tilde{H}, \tilde{G}^{-1}\}$, on the left and on the right respectively. Then, one can easily get

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Xi}_{11} & \tilde{\Xi}_{12} & \tilde{\Xi}_{13} & \frac{d^*}{2}\tilde{A}^T(\tilde{Z}_1 + \tilde{Z}_2) \\ * & \tilde{\Xi}_{22} & 0 & 0 \\ * & * & -\tilde{Q}_1 & \frac{d^*}{2}\tilde{A}_d^T(\tilde{Z}_1 + \tilde{Z}_2) \\ * & * & * & -\frac{d^*}{2}(\tilde{Z}_1 + \tilde{Z}_2) \end{bmatrix} < 0 \tag{17a}$$

$$\begin{bmatrix} \tilde{X}_i & \tilde{Y}_i \\ * & \tilde{Z}_i \end{bmatrix} > 0, \ i = 1, \ 2 \tag{17b}$$

where

$\tilde{R} = \tilde{G}^{-T}R = \begin{bmatrix} 0 & \tilde{\Phi}^T \end{bmatrix}^T$, $\tilde{S} = \tilde{H}^TS = \begin{bmatrix} \tilde{S}_1^T & \tilde{S}_2^T \end{bmatrix}^T$,

$\tilde{\Xi}_{11} = \tilde{E}^T\tilde{P}\tilde{A} + \tilde{A}^T\tilde{P}\tilde{E} + \tilde{S}\tilde{R}^T\tilde{A} + \tilde{A}^T\tilde{R}\tilde{S}^T + \tilde{Y}_2\tilde{E} + \tilde{E}^T\tilde{Y}_2^T + \frac{d^*}{2}(\tilde{X}_1 + \tilde{X}_2) + \tilde{Q}_2$, $\tilde{\Xi}_{12} = \tilde{Y}_1\tilde{E} - \tilde{Y}_2\tilde{E}$,

$\tilde{\Xi}_{13} = \tilde{S}\tilde{R}^T\tilde{A}_d + \tilde{E}^T\tilde{P}\tilde{A}_d - \tilde{Y}_1\tilde{E}$, $\tilde{\Xi}_{22} = \tilde{Q}_1 - \tilde{Q}_2$.

Using Schur complement, (17a) implies

$$\begin{bmatrix} \tilde{\Xi}_{11} & \tilde{\Xi}_{13} \\ * & -\tilde{Q}_1 \end{bmatrix} < 0 \tag{18}$$

which can be rewritten as (19) by expanding the block of (18)

$$\begin{bmatrix} \tilde{\Xi}_{11,11} & \tilde{\Xi}_{11,12} & \tilde{\Xi}_{13,11} & \tilde{\Xi}_{13,12} \\ * & \tilde{\Xi}_{11,22} & \tilde{\Xi}_{13,21} & \tilde{S}_2\tilde{\Phi}^T\tilde{A}_{d22} \\ * & * & -\tilde{Q}_{111} & -\tilde{Q}_{112} \\ * & * & * & -\tilde{Q}_{122} \end{bmatrix} < 0 \tag{19}$$

where $\tilde{\Xi}_{11,22} = \tilde{S}_2\tilde{\Phi}^T + \tilde{\Phi}\tilde{S}_2^T + \frac{d^*}{2}(\tilde{X}_{122} + \tilde{X}_{222}) + \tilde{Q}_{222}$,

and other irrelevant variables are omitted here. From (19), it is obvious that

$$\begin{bmatrix} \tilde{S}_2\tilde{\Phi}^T + \tilde{\Phi}\tilde{S}_2^T + \tilde{Q}_{222} & \tilde{S}_2\tilde{\Phi}^T\tilde{A}_{d22} \\ * & -\tilde{Q}_{122} \end{bmatrix} < 0 \tag{20}$$

Left- and right-multiplying (20) by $\zeta = \begin{bmatrix} -\tilde{A}_{d22}^T & I \end{bmatrix}$ and ζ^T, we have

$$\tilde{A}_{d22}^T\tilde{Q}_{222}\tilde{A}_{d22} - \tilde{Q}_{122} < 0 \tag{21}$$

that is $\tilde{A}_{d22}^T\tilde{Q}_{222}\tilde{A}_{d22} - \tilde{Q}_{222} + (\tilde{Q}_{222} - \tilde{Q}_{122}) < 0$.

From (17a), $\tilde{\Xi}_{22} < 0$ holds, which implies $(\tilde{Q}_{222} - \tilde{Q}_{122}) > 0$ in (21) and

$$\tilde{A}_{d22}^T\tilde{Q}_{222}\tilde{A}_{d22} - \tilde{Q}_{222} < 0$$

Therefore,

$$\rho(\tilde{A}_{d22}) < 1 \tag{22}$$

Now, we define a transformation of state $x(t)$ as

$$\tilde{x}(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} = \tilde{H}^{-1}x(t) \tag{23}$$

where $\tilde{x}_1(t) \in \mathcal{R}^r$ and $\tilde{x}_2(t) \in \mathcal{R}^{n-r}$. Then, the system (4) can be decomposed as

$$\dot{\tilde{x}}(t) = \tilde{A}_{11}\tilde{x}_1(t) + \tilde{A}_{d11}\tilde{x}_1(t-d) + \tilde{A}_{d12}\tilde{x}_2(t-d) \tag{24a}$$

$$0 = \tilde{x}_2(t) + \tilde{A}_{d21}\tilde{x}_1(t-d) + \tilde{A}_{d22}\tilde{x}_2(t-d) \tag{24b}$$

or equivalently

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{A}_d\tilde{x}(t-d) \tag{25}$$

Considering the system (24), we define the following functional

$$\tilde{V}(\tilde{x}_t) = \tilde{V}_1(\tilde{x}_t) + \tilde{V}_2(\tilde{x}_t) + \tilde{V}_3(\tilde{x}_t) \tag{26}$$

where

$$\tilde{V}_1(\tilde{x}_t) = \tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{E}\tilde{x}(t) = \tilde{x}_1^T(t)\tilde{P}_{11}\tilde{x}_1(t)$$

$$\tilde{V}_2(\tilde{x}_t) = \int_{t-d}^{t-\frac{d}{2}} \tilde{x}^T(\theta)\tilde{Q}_1\tilde{x}(\theta)d\theta + \int_{t-\frac{d}{2}}^{t} \tilde{x}^T(\theta)\tilde{Q}_2\tilde{x}(\theta)d\theta$$

$$\tilde{V}_3(\tilde{x}_t) = \int_{-d}^{-\frac{d}{2}}\int_{t+\theta}^{t} \dot{\tilde{x}}_1^T(s)\tilde{Z}_{111}\dot{\tilde{x}}_1(s)\,dsd\theta + \int_{-\frac{d}{2}}^{0}\int_{t+\theta}^{t} \dot{\tilde{x}}_1^T(s)\tilde{Z}_{211}\dot{\tilde{x}}_1(s)\,dsd\theta$$

From Leibniz-Newton formula, for time $t > d$, we have

$$\int_{t-d}^{t} \dot{\tilde{x}}_1(\theta)\,d\theta = \tilde{x}_1(t) - \tilde{x}_1(t-d) \tag{27}$$

$$\int_{t-d}^{t} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta = \tilde{E}\tilde{x}(t) - \tilde{E}\tilde{x}(t-d) \tag{28}$$

Noting (28) and following the same line as that in [2], [3], we can write the system (24) as

$$\begin{aligned} \tilde{E}\dot{\tilde{x}}(t) = (\tilde{A} + \tilde{A}_d\tilde{E})\tilde{x}(t) - \tilde{A}_d\int_{t-d}^{t} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \\ + \tilde{A}_d(I - \tilde{E})\tilde{x}(t-d) \end{aligned} \tag{29}$$

Then, the derivative of $\tilde{V}_1(\tilde{x}_t)$ along the trajectory of the system (29) with respect to time t satisfies

$$\begin{aligned} \dot{\tilde{V}}_1(\tilde{x}_t) &= 2\tilde{x}^T(t)\tilde{E}^T\tilde{P}(\tilde{A} + \tilde{A}_d\tilde{E})\tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{A}_d(I - \tilde{E})\tilde{x}(t-d) \\ &\quad + 2\tilde{x}^T(t)\tilde{S}\tilde{R}^T\tilde{E}\dot{\tilde{x}}(t) + \eta \\ &= 2\tilde{x}^T(t)\tilde{E}^T\tilde{P}(\tilde{A} + \tilde{A}_d\tilde{E})\tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{A}_d(I - \tilde{E})\tilde{x}(t-d) \\ &\quad + 2\tilde{x}^T(t)\tilde{S}\tilde{R}^T\tilde{A}\tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t)\tilde{S}\tilde{R}^T\tilde{A}_d\tilde{x}(t-d) + \eta \end{aligned} \tag{30}$$

where

$$\begin{aligned} \eta = &-2\tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{A}_d\int_{t-d}^{t-\frac{d}{2}} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \\ &- 2\tilde{x}^T(t)\tilde{E}^T\tilde{P}\tilde{A}_d\int_{t-\frac{d}{2}}^{t} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta. \end{aligned}$$

Setting $a = \tilde{x}(t)$, $b = \tilde{E}\dot{\tilde{x}}(\theta)$, $N = \tilde{E}^T\tilde{P}\tilde{A}_d$ and employing Lemma 1 in [3], we can show

$$\begin{aligned} \eta \le\ & \frac{d}{2}\tilde{x}^T(t)\tilde{X}_1\tilde{x}(t) + 2\tilde{x}^T(t)(\tilde{Y}_1 - \tilde{E}^T\tilde{P}\tilde{A}_d) \\ &\times \int_{t-d}^{t-\frac{d}{2}} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta + \int_{t-d}^{t-\frac{d}{2}} \dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_1\tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \\ &+ \frac{d}{2}\tilde{x}^T(t)\tilde{X}_2\tilde{x}(t) + 2\tilde{x}^T(t)(\tilde{Y}_2 - \tilde{E}^T\tilde{P}\tilde{A}_d) \\ &\times \int_{t-\frac{d}{2}}^{t} \tilde{E}\dot{\tilde{x}}(\theta)\,d\theta + \int_{t-\frac{d}{2}}^{t} \dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_2\tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \\ =\ & \frac{d}{2}\tilde{x}^T(t)(\tilde{X}_1 + \tilde{X}_2)\tilde{x}(t) + 2\tilde{x}^T(t)(\tilde{Y}_1 - \tilde{E}^T\tilde{P}\tilde{A}_d) \\ &\times (\tilde{E}\tilde{x}(t - \frac{d}{2}) - \tilde{E}\tilde{x}(t-d)) + \int_{t-d}^{t-\frac{d}{2}} \big[\dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_1\tilde{E} \\ &\times \dot{\tilde{x}}(\theta)\,d\theta\big] + 2\tilde{x}^T(t)(\tilde{Y}_2 - \tilde{E}^T\tilde{P}\tilde{A}_d)\big(\tilde{E}\tilde{x}(t) \\ &- \tilde{E}\tilde{x}(t - \frac{d}{2})\big) + \int_{t-\frac{d}{2}}^{t} \dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_2\tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \end{aligned} \tag{31}$$

Additionally, direct computation gives the following expressions for $\dot{\tilde{V}}_2(\tilde{x}_t)$ and $\dot{\tilde{V}}_3(\tilde{x}_t)$,

$$\begin{aligned} \dot{\tilde{V}}_2(\tilde{x}_t) = \tilde{x}^T(t - \frac{d}{2})\tilde{Q}_1\tilde{x}(t - \frac{d}{2}) - \tilde{x}^T(t-d)\tilde{Q}_1\tilde{x}(t-d) \\ + \tilde{x}^T(t)\tilde{Q}_2\tilde{x}(t) - \tilde{x}^T(t - \frac{d}{2})\tilde{Q}_2\tilde{x}(t - \frac{d}{2}) \end{aligned} \tag{32}$$

$$\begin{aligned} \dot{\tilde{V}}_3(\tilde{x}_t) = \frac{d}{2}\dot{\tilde{x}}^T(t)\tilde{E}^T\tilde{Z}_1\tilde{E}\dot{\tilde{x}}(t) - \int_{t-d}^{t-\frac{d}{2}} \dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_1\tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \\ + \frac{d}{2}\dot{\tilde{x}}^T(t)\tilde{E}^T\tilde{Z}_2\tilde{E}\dot{\tilde{x}}(t) - \int_{t-\frac{d}{2}}^{t} \dot{\tilde{x}}^T(\theta)\tilde{E}^T\tilde{Z}_2\tilde{E}\dot{\tilde{x}}(\theta)\,d\theta \end{aligned} \tag{33}$$

Combining manipulations (30)–(33) yield

$$\dot{\tilde{V}}(\tilde{x}_t) \le \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t - \frac{d}{2}) \\ \tilde{x}(t-d) \end{bmatrix}^T \tilde{\Gamma} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t - \frac{d}{2}) \\ \tilde{x}(t-d) \end{bmatrix} \tag{34}$$

where

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} \\ * & \tilde{\Gamma}_{22} & 0 \\ * & * & \tilde{\Gamma}_{33} \end{bmatrix}$$

with

$\tilde{\Gamma}_{11} = \tilde{E}^T\tilde{P}\tilde{A} + \tilde{A}^T\tilde{P}\tilde{E} + \tilde{S}\tilde{R}^T\tilde{A} + \tilde{A}^T\tilde{R}\tilde{S}^T + \tilde{Y}_2\tilde{E} + \tilde{E}^T\tilde{Y}_2^T + \frac{d}{2}(\tilde{X}_1 + \tilde{X}_2) + \tilde{Q}_2 + \frac{d}{2}\tilde{A}^T(\tilde{Z}_1 + \tilde{Z}_2)\tilde{A}$,

$\tilde{\Gamma}_{12} = -\tilde{Y}_2\tilde{E} + \tilde{Y}_1\tilde{E}$,

$\tilde{\Gamma}_{13} = \tilde{S}\tilde{R}^T\tilde{A}_d + \tilde{E}^T\tilde{P}\tilde{A}_d + \frac{d}{2}\tilde{A}^T(\tilde{Z}_1 + \tilde{Z}_2)\tilde{A}_d - \tilde{Y}_1\tilde{E}$,

$\tilde{\Gamma}_{22} = \tilde{Q}_1 - \tilde{Q}_2, \ \tilde{\Gamma}_{33} = -\tilde{Q}_1 + \frac{d}{2}\tilde{A}_d^T(\tilde{Z}_1 + \tilde{Z}_2)\tilde{A}_d$.

By Schur complement, it is easy to show that (17) guarantees $\dot{\tilde{V}}(\tilde{x}_t) < 0$ and therefore

$$\begin{aligned} \lambda_1\|\tilde{x}_1(t)\|^2 - \tilde{V}(\tilde{x}_0) &\le \tilde{x}_1^T(t)\tilde{P}_{11}\tilde{x}_1(t) - \tilde{V}(\tilde{x}_0) \\ &\le \tilde{V}(\tilde{x}_t) - \tilde{V}(\tilde{x}_0) = \int_0^t \dot{\tilde{V}}(\tilde{x}_\theta)d\theta \\ &\le -\lambda_2\int_0^t \|\tilde{x}(\theta)\|^2 d\theta \le -\lambda_2\int_0^t \|\tilde{x}_1(\theta)\|^2 d\theta \end{aligned} \quad (35)$$

where

$\lambda_1 = \lambda_{\min}(\tilde{P}_{11}) > 0, \ \lambda_2 = -\lambda_{\max}(\tilde{\Gamma}) > 0$.

Then, it is obvious that

$$\lambda_1\|\tilde{x}_1(t)\|^2 + \lambda_2\int_0^t \|\tilde{x}_1(\theta)\|^2 d\theta \le \tilde{V}(\tilde{x}_0) \quad (36)$$

Noticing (22), we have

$$\lim_{t\to\infty} \tilde{x}(t) = 0 \quad (37)$$

From the congruence transformation $x(t) = \tilde{H}\tilde{x}(t)$, we have $\lim_{t\to\infty} x(t) = 0$. This completes the proof. □

Remark 1. Theorem 1 presents a new delay-dependent stability criterion for singular time-delay system (4) by employing the new Lyapunov-Krasovskii functional. When $E = I$, the singular time-delay system (4) reduces to a state-space time-delay system and it can be easily shown that Theorem 1 coincides with Theorem 1 in [3] provided $Z_1 = Z_2$ and $Q_1 = Q_2$. Therefore, Theorem 1 can be regarded as a generalization of the reported results on state-space time-delay systems to singular time-delay systems.

Though the result of Theorem 1 is delay-dependent, the delay-independent condition can be obtained as a particular case for certain values of the tuning variables. Setting $Y_1 = Y_2 = 0$, $Q_1 = Q_2 = Q$, $X_1 = X_2 = Z_1 = Z_2 = \frac{\varepsilon I}{d^*}$ in (7) and letting $\varepsilon \to 0^+$ result in the following corollary.

Corollary 1. The singular time-delay system (4) is regular, impulse free and stable, if there exist symmetric positive-definite matrices P, Q and a matrix S such that the linear matrix inequality (38) is satisfied,

$$\begin{bmatrix} \Theta_{11} & E^TPA_d + SR^TA_d \\ * & -Q \end{bmatrix} < 0 \quad (38)$$

where $R \in \mathcal{R}^{n\times(n-r)}$ is any full-column-rank matrix satisfying $E^TR = 0$ and

$\Theta_{11} = E^TPA + A^TPE + SR^TA + A^TRS^T + Q$

As mentioned in [11], [12], the method of [2], [3] are much conservative since the transformed system is not equivalent to the original one and neither is the result of Theorem 1. Following the similar philosophy as that in [13], we represent the system (4) to the following equivalent form

$$\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_d & 0 \end{bmatrix}\begin{bmatrix} x(t-d) \\ y(t-d) \end{bmatrix} \quad (39)$$

where $y(t) = E\dot{x}(t)$. Then, by Theorem 1, it is easy to see that the system (39) is regular, impulse free and stable for $0 \le d \le d^*$, if there exist symmetric positive-definite matrices P, Q_i, Z_i and matrices X_i, Y_i satisfying (7) where E is replaced by $\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}$, A by $\begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}$ and A_d by $\begin{bmatrix} 0 & 0 \\ A_d & 0 \end{bmatrix}$, $i = 1, 2$. As a particular case, we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & \varepsilon I \end{bmatrix}, R = \begin{bmatrix} R_1 & 0 \\ 0 & P_3 \end{bmatrix}, S = \begin{bmatrix} S_1 & S_2 \\ 0 & I \end{bmatrix},$$

$$Q_i = \begin{bmatrix} Q_{i1} & 0 \\ 0 & \varepsilon I \end{bmatrix}, X_i = \begin{bmatrix} X_{i1} & X_{i2} \\ * & X_{i3} \end{bmatrix},$$

$$Y_i = \begin{bmatrix} Y_{i1} & 0 \\ Y_{i2} & 0 \end{bmatrix}, Z_i = \begin{bmatrix} Z_{i1} & 0 \\ 0 & \varepsilon I \end{bmatrix}, \ i = 1, 2.$$

where P_1, $P_3 \in \mathcal{R}^{n\times n}$ are nonsingular matrices with P_1 symmetric and positive-definite, $R_1 \in \mathcal{R}^{n\times(n-r)}$ satisfies $E^TR_1 = 0$ and rank $R_1 = n - r$, $S_1 \in \mathcal{R}^{n\times(n-r)}$, $S_2 \in \mathcal{R}^{n\times n}$, $\varepsilon > 0$. It is obvious that $\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}^T R = 0$ and $R \in \mathcal{R}^{2n\times(2n-r)}$ is with full column rank. By denoting $P_2 = P_3S_2^T$ and letting $\varepsilon \to 0^+$, the following theorem can be obtained by employing Schur complement.

Theorem 2. For a given scalar $d^* > 0$, the singular time-delay system (4) is regular, impulse free and stable for any constant time delay d satisfying $0 \le d \le d^*$, if there exist symmetric positive-definite matrices P_1, Q_{i1}, Z_{i1} and matrices P_2, P_3, S_1, X_{i1}, X_{i2}, X_{i3}, Y_{i1}, Y_{i2} $(i = 1, 2)$ such that the linear matrix inequalities (40) hold,

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & 0 \\ * & * & * & -Q_{11} \end{bmatrix} < 0 \quad (40a)$$

$$\begin{bmatrix} X_{i1} & X_{i2} & Y_{i1} \\ * & X_{i3} & Y_{i2} \\ * & * & Z_{i1} \end{bmatrix} \ge 0, \ i = 1, 2 \quad (40b)$$

where $R_1 \in \mathcal{R}^{n\times(n-r)}$ is any full-column-rank matrix satisfying $E^TR_1 = 0$ and

$$\begin{aligned} \Xi_{11} &= P_2^TA + A^TP_2 + Y_{21}E + E^TY_{21}^T + Q_{21} \\ &\quad + \frac{d^*}{2}(X_{11} + X_{21}) \\ \Xi_{12} &= E^TP_1 + S_1R_1^T - P_2^T + A^TP_3 + E^TY_{22}^T \\ &\quad + \frac{d^*}{2}(X_{12} + X_{22}) \end{aligned}$$

$\Xi_{22} = -P_3 - P_3^T + \frac{d^*}{2}(X_{13} + X_{23}) + \frac{d^*}{2}(Z_{11} + Z_{21})$,

$\Xi_{13} = Y_{11}E - Y_{21}E, \ \Xi_{23} = Y_{12}E - Y_{22}E, \ \Xi_{33} = Q_{11} - Q_{21}$,

$\Xi_{14} = P_2^TA_d - Y_{11}E, \ \Xi_{24} = P_3^TA_d - Y_{12}E$.

Based on the result of Theorem 2, we can easily obtain the following robust stability result.

Theorem 3. For a given scalar $d^* > 0$, the uncertain singular time-delay system (1) is robustly stable for any constant time delay d satisfying $0 \leq d \leq d^*$, if there exist symmetric positive-definite matrices P_1, Q_{i1}, Z_{i1} and matrices P_2, P_3, S_1, X_{i1}, X_{i2}, X_{i3}, Y_{i1}, Y_{i2} $(i = 1, 2)$ such that the linear matrix inequalities (41) and (40b) hold,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & P_2^T M & N_a^T \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & P_3^T M & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -Q_{11} & 0 & N_d^T \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (41)$$

where R_1, Ξ_{11}, Ξ_{12} and Ξ_{22} follow the same definitions as those in (40).

4. NUMERICAL EXAMPLES

Example 1. Consider the following singular time-delay system studied in [12], [13],

$$\begin{bmatrix} \dot{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 1 \\ 0 & 0.5 \end{bmatrix} x(t-d)$$

This system is stable for $0 \leq d \leq 0.5$ in [12]. Applying Theorem 1 in this note gives $d^* = 0.5773$. Table 1 gives the maximum upper bound of d^* obtained by different methods. Note that the result of [10] fails to deal with this system. It is obvious to see that the condition of Theorem 2 is much simpler than that of [12], [13] with fewer variables.

Table 1. Comparison of delay-dependent stability condition of example 1

Methods	d^*	Number of variables
Fridman [12]	0.5000	19
Theorem 1	0.5773	10
Fridman [13]	1.1500	24
Boukas [10]	—	—
Theorem 2	1.1547	18

Example 2. The next example concerns system (1) the following parameters,

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} -0.5 & 0 \\ 0 & 1 \end{bmatrix}, \; A_d = \begin{bmatrix} -1 & 0.1 \\ 1 & 0 \end{bmatrix},$$
$$\|\Delta A\| \leq 0.2, \; \|\Delta A_d\| \leq 0.2,$$

By choosing $R_1 = [0 \;\; 1]^T$ and applying Theorem 3, it can be confirmed that system (1) is regular, impulse free and robustly asymptotically stable for any constant delay $d^* = 1.3891$.

5. CONCLUSION

The problem of robust stability for uncertain singular time-delay systems was investigated. Some new delay-dependent robust stability criteria were obtained based on a new Lyapunov-Krasovskii functional. The obtained condition can be checked by using the standard interior-point algorithm. Numerical examples were also provided to demonstrate the feasibility and superiority of the proposed approach.

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