

## MIMO Tension Modelling and Control for Roll-to-roll Converting Machines

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**Abstract:** Roll-to-roll converting machines have significant interactions in tensions and speeds among web spans. A reasonable MIMO model for web tension plays an important role for high-performance converting machines. In this paper, we derive a nonlinear MIMO web-tension model of a high-speed gravure printing machine considering span length to be time-varying instead of considering it to be fixed. Then a feedback control system is constructed and web-tension control performance is analyzed at transient and steady-state condition via simulation studies using Simulink software.

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### 1. INTRODUCTION

Roll-to-roll converting machines have significant interactions in tensions and speeds among web spans. One main issue in research of roll-to-roll converting machines is to increase web transport velocity as much as possible while controlling tension of the web appropriately in order to obtain high productivity. A reasonable MIMO model for web tension plays an important role for high-performance converting machines.

One important roll-to-roll converting machine is a gravure printing machine used for mass printing products. Control of gravure printing machines is basically composed of tension control and register control. Register control is required for high printing resolution in multi-stage printing systems, in which pre-printed marks and a scanning head are generally used for detecting register errors. However, precise register control is impossible without reasonable web tension control of continuous strip. That is, web tension control is a prerequisite for accurate register control to obtain high printing resolution. If web tension in roll-to-roll printing machine is too high, several problems occur such as large register error, rewinder wrinkling, web tearing, and plastic deformation of the web. On the other hand, if web tension is too small, then problems occur such as web oscillations, loose rewinding and web surface damages.

Usually highly interactive web tension is modeled under the assumption that the span length is fixed (Shin, 2000; Dwivedula et al., 2003; Mathur, 1998; Weiss, 1985), but at unwinder and rewinder units using turrets, the span length changes during operation, especially during roll change and splicing process. Moreover, web length in one span between rollers varies due to dancer motions.

In this paper, we derive a nonlinear MIMO (multi-input multi-output) web-tension model of a high-speed gravure

printing machine considering span length to be time-varying instead of considering it to be time-invariant. Time-varying property of the web span length is due to turret motions of unwinding and rewinding units as well as dancer motions. Furthermore, we analyze web-tension control performance at transient and steady-state operations via computer simulation studies using Simulink. The validity of the web-tension model and control system has been shown via simulation using the model of an actual gravure printing machine.

### 2. GRAVURE PRINTING MACHINE

Fig. 1 shows signal flows of the pilot plant of the gravure printing machine with three color printing function installed at Flexible Display Roll-to-Roll Research Center (FDRC), Konkuk University (manufactured by SAM, Inc.), which is presently used for several research purposes. This system is composed of an unwinder unit including turret and splicing mechanism and a passive dancer, an infeeder unit with a passive dancer, three printing units with color register control devices, an outfeeder unit with a passive dancer, and a rewinder unit including an active dancer. Dancers may be bypassed optionally. Eight loadcells are installed at idle rollers in the middle of continuous process for tension pickup. Web tensions can also be estimated using observers instead of using physical sensors (Lynch et al., 2004).

When a command is given at HMI in the figure, PLC generates appropriate command signals for motion control of each motor, and then the controller of each motor controls tensions and speeds of the web using motion commands and feedback signals for motion and web tension.

Fig. 2 shows a picture of the pilot plant. In this pilot plant, web speed, angular velocity of driving rolls, web tension, angular speed of the turret, radius of the wound roll, arm angles of the dancers can be measured and monitored through control panels and also PCs.

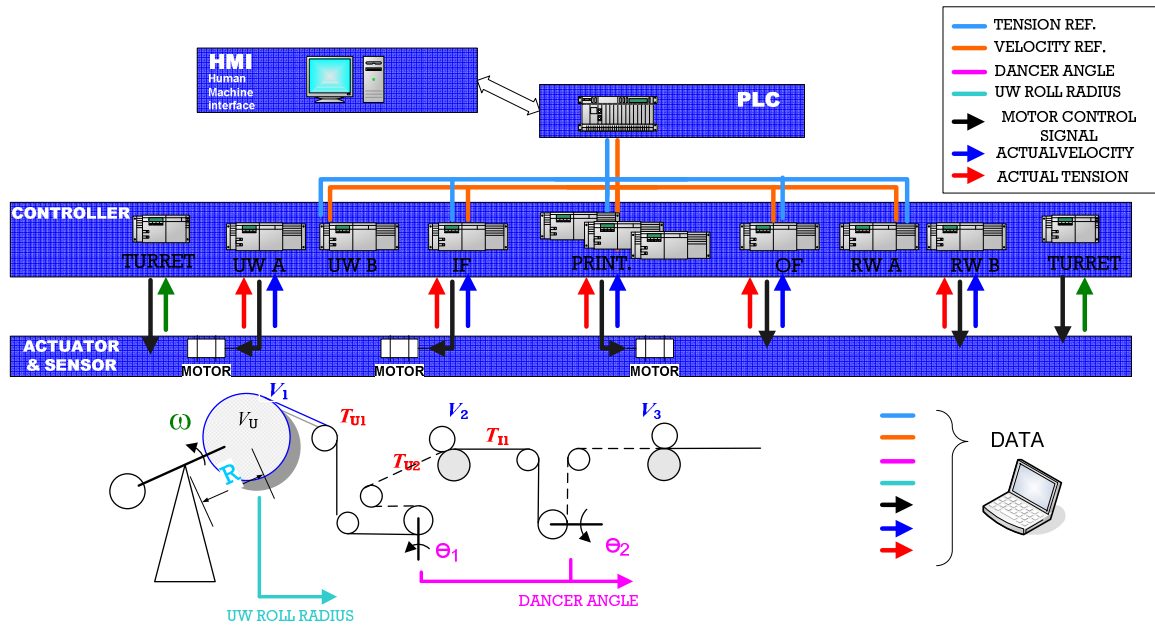


Fig. 1 Signal flow of the pilot plant of the gravure printing machine



Fig. 2 Pilot gravure printing machine installed at FDRC at Konkuk University (Courtesy of SAM, Inc. and FDRC).

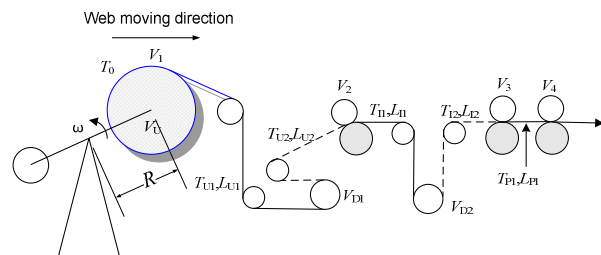
### 3. MIMO NONLINEAR TENSION MODELING

Linear models for web tension of roll-to-roll converting machines have been reported in the literature (Shin, 2000; Dwivedula et al., 2003; Mathur, 1998; Weiss, 1985; Sakamoto et al., 1995; Gravure Education Foundation, 2003). However, linear models have several limitations. For example, they cannot represent appropriately actual dynamics of transient region, e.g., speed increasing or decreasing region, they cannot consider time-varying property of the tension dynamics, and also they cannot predict tension well during turret or splicing process.

In this paper, we derive a nonlinear MIMO model which represents better actual dynamics of transient region and turret operation. A schematic diagram of the pilot gravure printing machine is shown in Fig. 3, in which  $V_i$  represents web speed and  $T_i$  does web tension. In Fig. 3,  $V_{D1}$ ,  $V_{D2}$ ,  $V_{D3}$ ,  $V_{D4}$  represent speeds of dancers. Subscripts U, I, O, R imply unwinder, infeed, outfeed, reworker, respectively.

Customers in developed countries generally require web tension variations to be remained within  $\pm 4\%$  during the whole operation for maintaining high printing quality. However, various uncertainties limit the tension control performance. First of all, linear model is not appropriate for this purpose. In particular, it does not represent well actual web tension of the acceleration and deceleration region of web speed, and it does not represent well tension variation by turret rotation, span length change, dancer motion, and splicing operation.

Within one span, cross sectional area and velocity of the web may vary continuously, but tension and strain are assumed to be constants along web longitudinal direction. All these variables may vary in time.



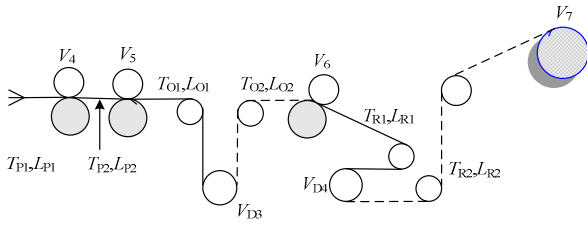


Fig. 3 Schematic diagram of the printing machine

Under these conditions, we obtain the following equations from mass conservation law for the control volume as shown in Fig. 4.

$$\frac{d}{dt} \left( \int_{x_1}^{x_2} \rho(x,t) A(x,t) dx \right) = \rho_1(t) A_1(t) V_1(t) - \rho_2(t) A_2(t) V_2(t) \quad (1)$$

where  $x$  implies web position in longitudinal direction,  $\rho$  implies the density of the web,  $A$  implies cross sectional area of the web, subscripts  $1, 2$  imply the positions of the upstream roller and downstream roller.

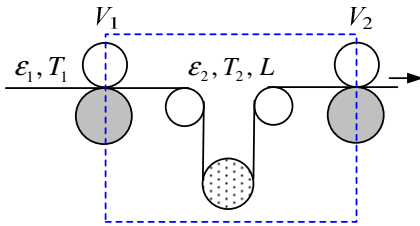


Fig. 4 Control volume for a running web including a dancer roller (dotted roller)

Representing Eq. (1) by means of web strain  $\mathcal{E}$ , we obtain the following equation.

$$\frac{d}{dt} \left[ \frac{L(t)}{1 + \mathcal{E}_2(t)} \right] = \frac{V_1(t)}{1 + \mathcal{E}_1(t)} - \frac{V_2(t)}{1 + \mathcal{E}_2(t)} \quad (2)$$

The strain of the web is very small in general. Thus, assuming  $\mathcal{E}$  is much smaller than 1 in Eq. (2), we can approximately rewrite it as follows.

$$\frac{d}{dt} \left[ (1 - \mathcal{E}_2(t)) L(t) \right] = [1 - \mathcal{E}_1(t)] V_1(t) - [1 - \mathcal{E}_2(t)] V_2(t) \quad (3)$$

From Hooke's law for the web, the following equation can be obtained.

$$T(t) = A(x,t) E(x,t) \mathcal{E}(t) \quad (4)$$

where  $E$  implies Young's modulus that may vary a little due to

the effects of temperature, humidity, etc.

Substituting Eq. (4) for Eq. (3), and using average value  $AE$  instead of  $A(x,t)E(x,t)$ , we obtain a new nonlinear tension model that is able to consider web length variation within a span.

$$L(t) \frac{dT_2(t)}{dt} = V_1(t) T_1(t) - \left[ V_2(t) + \frac{dL(t)}{dt} \right] T_2(t) + AE \left[ V_2(t) - V_1(t) + \frac{dL(t)}{dt} \right] \quad (5)$$

In general, variation of  $A(x,t)E(x,t)$  is very small in a roll-to-roll printing machine. Thus, using the average value  $AE$  instead of  $A(x,t)E(x,t)$  is acceptable for representing dynamic characteristics of the system. Note that neglecting the derivative term of the length in Eq. (5) results in Shin (2000)'s tension model.

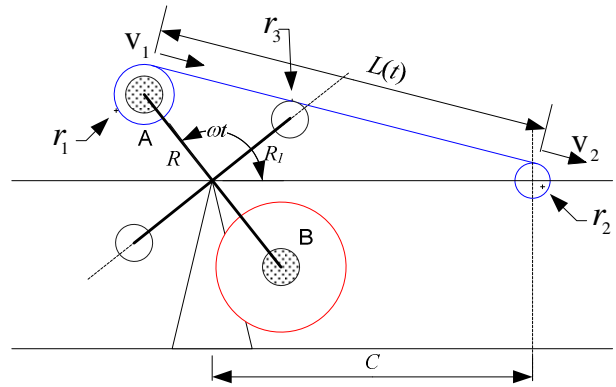


Fig. 5 Configuration of the unwinder turret

Fig. 5 shows a configuration of the unwinder turret, in which A and B wound rolls are alternatively connected to the web and are unwound. The span length  $L(t)$  is calculated from the geometry of Fig. 6 as follows (Lee et al., 2005).

$$L = \sqrt{R^2 + C^2 - 2RC \cos \omega t - (r_1 - r_2)^2} \quad (6)$$

This equation (6) is a closed form and thus easy to calculate online within a prescribed timing limit.

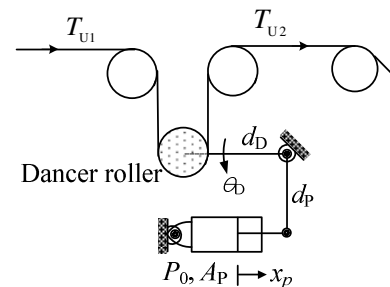


Fig. 6 Dancer mechanism

Fig. 6 shows a dancer mechanism driven by a pneumatic cylinder. In Fig. 6,  $\theta_D$  represents the angle of the dancer arm from a neutral position,  $d_D$  represents the length of the dancer arm from hinge to dancer roller,  $d_p$  represents the length of the dancer from hinge to pneumatic piston, and  $P_0$  and  $A_p$  represent pneumatic pressure and piston area, respectively. The pneumatic cylinder used for dancer system in general is preloaded by a spring within the cylinder as shown in Fig. 7.

From the free body diagram for the dancer roller in Fig. 6, the following equation of motion is obtained (Kang et al., 2006).

$$J_D \dot{V}_D = (T_{U2} - T_{U1})r_D^2 - b_D V_D \quad (7)$$

where  $r_D$  implies the radius of dancer roller,  $b_D$  does the coefficient of viscous friction of dancer roller, and  $V_D$  does web speed at dancer roller. We assume no slip between dancer rollers and web materials.

Then, applying Newton's law of motion to the free body diagram of the dancer arm, we can obtain the following equation.

$$J_{eq} \ddot{\theta}_D = -(m_D d_D \ddot{\theta}_D + T_{U1} + T_{U2})d_D + P_0 A_p - \left[ \frac{F_{\max} - F_{\min}}{x_{p\max}} \left( \frac{x_{p\max}}{2} + d_p \theta_D \right) + F_{\min} \right] d_p - b_{eq} \dot{\theta}_D \quad (8)$$

where  $J_{eq}$  represents the equivalent moment of inertia of the dancer arm with respect to hinge center,  $b_{eq}$  does the equivalent coefficient of viscous friction of the dancer arm, and  $m_D$  does the mass of dancer roller. And  $F_{\max}$ ,  $F_{\min}$  represent maximum and minimum compression force of the spring, and  $x_{p\max}$  does maximum stroke of the piston. In Fig. 7,  $F_s$  represents spring force,  $x_p$  does piston displacement,  $k$  does spring constant, and  $x_p^*$  does operating condition. The right side graph of Fig. 7 shows real values for the printing machine installed in FDRC at Konkuk University.

If we neglect mass, moment of inertia, and viscous friction of dancer roller in Eqs. (7) and (8), then  $T_{U1} = T_{U2} = T$ , and we can obtain the following equation for the roller.

$$J_{eq} \ddot{\theta}_D + b_{eq} \dot{\theta}_D + \frac{F_{\max} - F_{\min}}{x_{p\max}} d_p^2 \theta_D = (P_0 A_p - \frac{F_{\max} + F_{\min}}{2}) d_p - 2T d_D \quad (9)$$

It is known that keeping different web-tension values for different web-spans is better than keeping constant tension values for the whole web (Weiss, 1985). Usually tension zones are divided into four; unwind zone, infeed zone, outfeed zone, and rewind zone. In printing zone, tension increases along web flowing, or is same with infeed zone at steady state.

Optimal tension value for each zone may vary according to web properties and operating conditions. For simulation purpose in this paper, we set tension command of unwind, infeed, outfeed, and rewind zone as 100%, 110%, 120%, 50% of unwind zone tension, respectively.

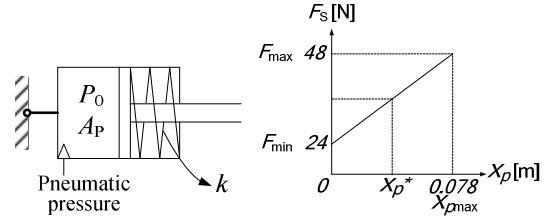


Fig. 7 Preloaded pneumatic cylinder

Applying web-tension model, Eq. (5), and dancer model, Eq. (9) for the 3 color printing machine results in the following nonlinear MIMO plant model, in which  $\dot{l}_U, \dot{l}_R$  are constant linear speeds by unwind and rewind turrets.

$$\begin{aligned} L_U \dot{T}_U &= V_U T_{U0} - (V_I + \dot{l}_U) T_U + (V_I - V_U + \dot{l}_U) A E \\ \dot{l}_U &= \dot{l}_U + 2d_D \dot{\theta}_{D1}, \quad \dot{l}_U = v_U \text{ (given)} \\ L_I \dot{T}_I &= V_I T_U - (V_{P1} + \dot{l}_I) T_I + (V_{P1} - V_I + \dot{l}_I) A E \\ \dot{l}_I &= 2d_D \dot{\theta}_{D2} \\ J_{eq} \ddot{\theta}_{D2} + b_{eq} \dot{\theta}_{D2} + \frac{F_{\max} - F_{\min}}{x_{p\max}} d_p^2 \theta_{D2} &= (P_{I0} A_p - \frac{F_{\max} + F_{\min}}{2}) d_p - 2T_I d_D \\ L_{P1} \dot{T}_{P1} &= V_{P1} T_I - V_{P2} T_{P1} + (V_{P2} - V_{P1}) A E \\ L_{P2} \dot{T}_{P2} &= V_{P2} T_{P1} - V_{P3} T_{P2} + (V_{P3} - V_{P2}) A E \\ L_O \dot{T}_O &= V_{P3} T_{P2} - (V_O + \dot{l}_O) T_O + (V_O - V_{P3} + \dot{l}_O) A E \\ \dot{l}_O &= 2d_D \dot{\theta}_{D3} \\ J_{eq} \ddot{\theta}_{D3} + b_{eq} \dot{\theta}_{D3} + \frac{F_{\max} - F_{\min}}{x_{p\max}} d_p^2 \theta_{D3} &= (P_{O0} A_p - \frac{F_{\max} + F_{\min}}{2}) d_p - 2T_O d_D \\ L_R \dot{T}_R &= V_O T_O - (V_R + \dot{l}_R) T_R + (V_R - V_O + \dot{l}_R) A E \\ \dot{l}_R &= \dot{l}_R + 2d_D \dot{\theta}_{D4}, \quad \dot{l}_R = v_R \text{ (given)} \\ J_{eq} \ddot{\theta}_{D4} + b_{eq} \dot{\theta}_{D4} + \frac{F_{\max} - F_{\min}}{x_{p\max}} d_p^2 \theta_{D4} &= (P_{R0} A_p - \frac{F_{\max} + F_{\min}}{2}) d_p - 2T_R d_D \end{aligned} \quad (10)$$

#### 4. TENSION CONTROL SCHEME

A schematic configuration of the tension control system is shown in Fig. 8. In the present gravure printing machine, a master velocity command is given to the motor of the first printing unit, and for the other driving motor, adjusted

velocity commands are inputted. For each motor, a motor driver and tension controller is attached as shown in Fig. 8. Tension controllers use PI control types.

Parameters for web materials and the gravure printing machine are summarized in Table 1 and Table 2.

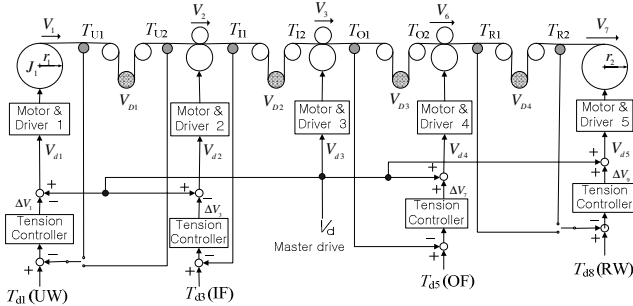


Fig. 8 Schematic configuration of web tension control

Table 1 Parameters for web materials

Web material	PET	OPP	AL	Paper
Thickness ( $\mu\text{m}$ )	12	20	6, 9	40 (120gsm)
Width (m)	1	1	1	1
Web velocity (mpm)	200, 350, 500			
Young's modulus (Gpa)	3.6	2.1	68	
Required tension ( $\text{kg}_f/\text{m}$ )	5, 10, 15, 20			10, 20, 30, 40
Acceleration time (sec)	60	60	60	60

Table 2 Parameters for the gravure printing machine

PET material	Required tension ( $\text{kg}_f/\text{m}$ )	Span length (m)	Moment of inertia ( $\text{kg}\cdot\text{m}^2$ )	Total weight ( $\text{kg}_f$ )	Motor power (kW)
UW	10	4.4	92	731	11
IF	11	5.4	4	113	10
OF	12	5.4	4	113	10
RW	5	4.4	92	731	11

### 5. SIMULATIONS USING SIMULINK

Using the derived plant model, Eq. (10), a Simulink model is constructed as shown in Fig. 9 and Fig. 10. In general, the side view of the unwinder roll is not an exact circle but an oval shape by distortion, and this distorted unwinder roll causes a disturbance with constant frequency. If this frequency is near natural frequency of the converting machine, then tension control performance becomes bad. In this paper, we let  $T_{U0} = 10 \sin \omega t + 100$  (N/m) to simulate this kind of disturbance. Fig. 11 shows the time schedule for the system simulation and Fig. 12 shows simulation results of the tension

control system. At about 4 m/s web speed in this parameter setting, the control system has resonance, and thus the interval around 4 m/s web speed should be passed rapidly by when a starting or ending procedure of the system operation is conducted.

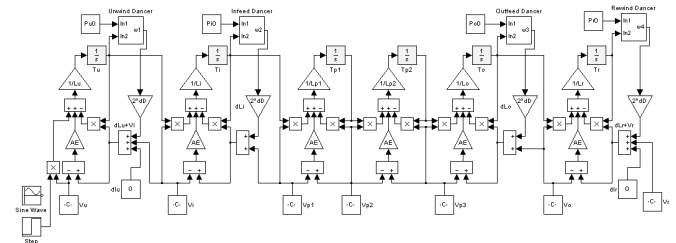


Fig. 9 Nonlinear tension model neglecting dancer inertias

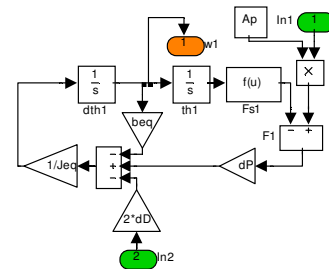


Fig. 10 Dancer subsystem neglecting dancer roller inertias

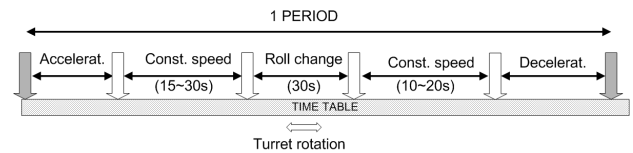


Fig. 11 Time schedule for system simulations

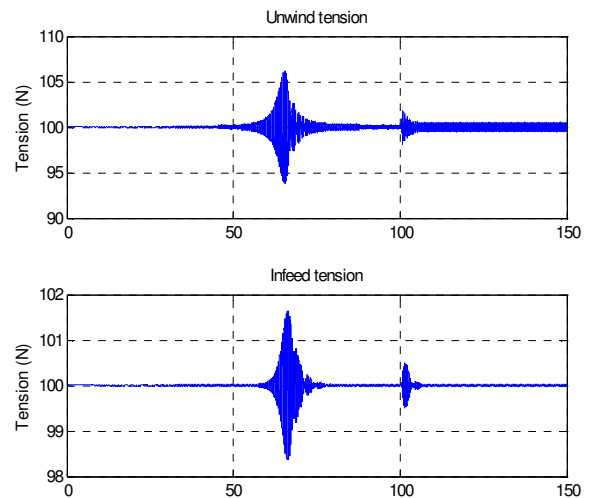


Fig. 12 Simulation results for a disturbance due to unwind roll distortion

## 6. CONCLUSIONS

In this paper, a nonlinear MIMO web-tension model of a roll-to-roll printing machine is derived that considers span length to be time-varying instead of being fixed. With this nonlinear model and control logic, web tension control of the gravure printing machine is simulated by Simulink software. At about 4 m/s web speed in the present parameter setting, the control system has resonance, and thus the interval around 4 m/s web speed should be passed rapidly by when the starting or ending procedure of the system operation is conducted.

Further experimental study is necessary to verify above-simulated results.

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