Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Stabilization of 2-D Linear Parameter-Varying Systems using Parameter-Dependent Lyapunov Function: An LMI Approach ⋆

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Abstract: This paper proposes a parameter-dependent state-feedback controller for the 2-D discrete linear parameter-varying (LPV) system with the Fornasini-Machesini (FM) first model. To find the stabilizing conditions of the system, we first transform the closed-loop system to a Roesser-type model, and then derive the conditions to linear matrix inequalities (LMIs) using a parameter-dependent Lyapunov function (PDLF) and a relaxation technique. The simulation results show that the designed controller is valid and the system asymptotically converges to the origin.

Keywords: 2-D system, FM first model, linear matrix inequalities (LMIs), linear parameter-varying (LPV), parameter-dependent Lyapunov function (PDLF),

1. INTRODUCTION

For several decades, there has been much interest in the analysis of two-dimensional (2-D) systems, which has introduced some 2-D dynamic models: for examples, Fornasini-Machesini (FM) first (Fornasini and Marchesini [1976], Kar and Singh [2003], Tong Zhou [2006]) and second model (Fornasini and Marchesini [1978], Hinamoto [1997], Ooba [2000]), Roesser model (Roesser [75]) and so on. These 2-D system theories can be applied not only to the theoretical areas such as iterative learning control (ILC) and signal processing but also to the practical systems such as thermal processes, water steam heating, etc (Bose [1982], Kaczorek [1985], Du and Xie [2002]).

Many of practical systems have included non-linearities such as sector-bounded conditions, input saturations or other non-linear functions. In this case, these systems can be often modeled in the linear parameter-varying (LPV) systems. Thus, in the paper, we consider that the 2-D system has a certain non-linearity, which is described as the LPV system (Becker et al. [1993], Apkarian et al. [1995], Apkarian and Tuan [2000], Park and Choi [2001]). Here, our goal is to propose a method for stabilizing the 2-D LPV system using a parameter-dependent Lyapunov function (PDLF), where the system is regarded as the FM first model and the controller is designed as the state-feedback. To the best of our knowledge, not much academic research has been studied on this area.

The resulting conditions for stabilization can be expressed with linear matrix inequalities (LMIs) using the relaxation technique in Park and Choi [2001], which is solved by the LMI toolbox in MATLAB. In addition, the numerical example shows that the proposed controller is valid.

The paper is organized as follows. Section 2 describes the system handling in the paper and explains the problem. Section 3 explains a new approach and designs the controller for stabilization of 2-D LPV system in detail. Section 4 shows the simulation results. Finally, section 5 concludes the paper with summarization.

2. PROBLEM FORMULATION

Consider the 2-D discrete LPV system with the FM first model:

$$x(t+1,k+1) = A_1(\theta_t)x(t,k+1) + A_2(\theta_t)x(t+1,k) + A_3(\theta_t)x(t,k) + B(\theta_t)u(t,k), \quad (1)$$

where $x(t,k) \in \mathcal{R}^n$ is the state vector, $u(t,k) \in \mathcal{R}^m$ is the control input, and $\theta_t \in \mathcal{R}^r$ denotes a time-varying parameter vector of time-varying parameters $\theta_{t,i}$ with respect to the index t such that, for $i = 1, \cdots, r$,

$$\theta_t = [\,\theta_{t,1}, \cdots, \theta_{t,r}\,]^T, \quad (2)$$

which satisfies the following condition

$$\sum_{i=1}^{r} \theta_{t,i} = 1, \quad 0 \le \theta_{t,i} \le 1, \quad |\theta_{t,i} - \theta_{t-1,i}| \le \delta_i. \quad (3)$$

⋆ This research was supported by the MIC (Ministry of Information and Communication), Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Advancement). (IITA-2008-C1090-0801-0037)
This research was supported by the MIC (Ministry of Information and Communication), Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Advancement). (IITA-2008-C1090-0801-0004)

Then, the system matrices can be described such as

$$A_1(\theta_t) = \sum_{i=1}^{r} \theta_{t,i} A_{1,i}, \quad A_2(\theta_t) = \sum_{i=1}^{r} \theta_{t,i} A_{2,i},$$
$$A_3(\theta_t) = \sum_{i=1}^{r} \theta_{t,i} A_{3,i}, \quad B(\theta_t) = \sum_{i=1}^{r} \theta_{t,i} B_i. \tag{4}$$

The objective of the paper is to design a parameter-dependent state-feedback controller for stabilizing the system (1), which is chosen as

$$u(t,k) = K_1(\theta_t,\theta_{t-1})x(t,k+1) + K_2(\theta_t,\theta_{t-1})x(t+1,k) + K_3(\theta_t,\theta_{t-1})x(t,k), \tag{5}$$

where θ_{t-1} denotes the one-step-past vector of time-varying parameters θ_t.

Substituting (5) into (1), we have the closed-loop system as follows:

$$x(t+1,k+1) = \bar{A}_1(\theta_t,\theta_{t-1})x(t,k+1) + \bar{A}_2(\theta_t,\theta_{t-1})x(t+1,k) + \bar{A}_3(\theta_t,\theta_{t-1})x(t,k), \tag{6}$$

where

$$\bar{A}_1(\theta_t,\theta_{t-1}) = A_1(\theta_t) + B(\theta_t)K_1(\theta_t,\theta_{t-1}),$$
$$\bar{A}_2(\theta_t,\theta_{t-1}) = A_2(\theta_t) + B(\theta_t)K_2(\theta_t,\theta_{t-1}),$$
$$\bar{A}_3(\theta_t,\theta_{t-1}) = A_3(\theta_t) + B(\theta_t)K_3(\theta_t,\theta_{t-1}).$$

Let

$$z(t,k+1) \triangleq x(t+1,k+1) - \bar{A}_1(\theta_t,\theta_{t-1})x(t,k+1),$$

then (6) can be described as a Roesser-type model:

$$E(\theta_t,\theta_{t-1}) \begin{bmatrix} x(t+1,k) \\ z(t,k+1) \end{bmatrix} = F(\theta_t,\theta_{t-1}) \begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}, \tag{7}$$

where

$$E(\theta_t,\theta_{t-1}) = \begin{bmatrix} I & 0 \\ -\bar{A}_2(\theta_t,\theta_{t-1}) & I \end{bmatrix},$$
$$F(\theta_t,\theta_{t-1}) = \begin{bmatrix} \bar{A}_1(\theta_t,\theta_{t-1}) & I \\ \bar{A}_3(\theta_t,\theta_{t-1}) & 0 \end{bmatrix}.$$

3. STABILIZING THE 2-D LPV SYSTEM USING THE PARAMETER-DEPENDENT LAYPUNONV FUNCTION

Let us consider a candidate PDLF for the 2-D discrete system such as

$$V\left(\begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}\right) = \begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}^T P(\theta_{t-1}) \begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}, \tag{8}$$

whose difference is given by

$$\Delta V\left(\begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}\right) = \begin{bmatrix} x(t+1,k) \\ z(t,k+1) \end{bmatrix}^T P(\theta_t) \begin{bmatrix} x(t+1,k) \\ z(t,k+1) \end{bmatrix} - \begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix}^T P(\theta_{t-1}) \begin{bmatrix} x(t,k) \\ z(t,k) \end{bmatrix} < 0, \tag{9}$$

where

$$P(\theta_{t-1}) \triangleq P_1(\theta_{t-1}) \oplus P_2(\theta_{t-1}),$$
$$P(\theta_t) \triangleq P_1(\theta_t) \oplus P_2(\theta_t),$$

and $\oplus$ means the direct sum.

Using (9), in the following theorem, we provide a PDLF-based stabilizer in terms of parameterized linear matrix inequalities (PLMIs).

Theorem 1. The closed-loop system (7) is globally asymptotically stable, if there exist matrices $\bar{P}_1(\theta_t)$, $\bar{P}_2(\theta_t)$, $\bar{P}_1(\theta_{t-1})$, $\bar{P}_2(\theta_{t-1})$, $W_1(\theta_t)$, $W_2(\theta_t)$, $W_3(\theta_t)$, $\bar{K}_1(\theta_t,\theta_{t-1})$, $\bar{K}_2(\theta_t,\theta_{t-1})$ and $\bar{K}_3(\theta_t,\theta_{t-1})$ such that

$$\begin{bmatrix} \bar{P}(\theta_{t-1}) & (*) & (*) \\ F(\theta_t,\theta_{t-1})\bar{P}(\theta_{t-1}) & \begin{matrix} E(\theta_t,\theta_{t-1})W^T(\theta_t) \\ +(*) \end{matrix} & (*) \\ 0 & W^T(\theta_t) & \bar{P}(\theta_t) \end{bmatrix} > 0,$$
$$\bar{P}(\theta_{t-1}) > 0, \quad \bar{P}(\theta_t) > 0, \tag{10}$$

where

$$\bar{P}(\theta_{t-1}) = \bar{P}_1(\theta_{t-1}) \oplus \bar{P}_2(\theta_{t-1}),$$
$$\bar{P}(\theta_t) = \bar{P}_1(\theta_t) \oplus \bar{P}_2(\theta_t),$$
$$W^T(\theta_t) = \begin{bmatrix} W_1^T(\theta_t) & 0 \\ W_2^T(\theta_t) & W_3^T(\theta_t) \end{bmatrix}.$$

In this case, the state-feedback controller (5) is given by

$$K_1(\theta_t,\theta_{t-1}) = \bar{K}_1(\theta_t,\theta_{t-1})\bar{P}_1^{-1}(\theta_{t-1}),$$
$$K_2(\theta_t,\theta_{t-1}) = \bar{K}_2(\theta_t,\theta_{t-1})W_1^{-T}(\theta_t),$$
$$K_3(\theta_t,\theta_{t-1}) = \bar{K}_3(\theta_t,\theta_{t-1})\bar{P}_1^{-1}(\theta_{t-1}).$$

Proof. From the Lyapunov difference (9), we can derive the following PLMI: for $\bar{P}(\theta_{t-1}) \triangleq P^{-1}(\theta_{t-1})$, $\bar{P}(\theta_t) \triangleq P^{-1}(\theta_t)$,

$$\begin{bmatrix} \bar{P}(\theta_{t-1}) & (*) \\ F(\theta_t,\theta_{t-1})\bar{P}(\theta_{t-1}) & (2,2) \end{bmatrix} > 0, \tag{11}$$

where

$$(2,2) = E(\theta_t,\theta_{t-1})\bar{P}(\theta_t)E^T(\theta_t,\theta_{t-1}).$$

Using the following relation for (2,2) of (11),

$$0 \le (E - W\bar{P}^{-1})\bar{P}(E^T - \bar{P}^{-1}W^T),$$
$$E\bar{P}E^T \ge EW^T + WE^T - W\bar{P}^{-1}W^T, \tag{12}$$

and using the Schur-complement with respect to $W\bar{P}^{-1}W^T$, then we have the resultant PLMIs (10). ∎

Unfortunately, it is extremely difficult to directly solve the parameter-dependent conditions of the Theorem 1 because it is required to solve an infinite number of LMIs. Therefore, it is important to reduce the solvability of PLMI to the solvability of a finite number of LMIs. To do this, we use the convex relaxation technique in Park and Choi [2001] with the following assumptions:

$$\bar{P}_1(\theta_t) = \sum_{i=1}^{r} \theta_{t,i}\bar{P}_{1,i}, \quad \bar{P}_1(\theta_{t-1}) = \sum_{i=1}^{r} \theta_{t-1,i}\bar{P}_{1,i},$$

$$\bar{P}_2(\theta_t) = \sum_{i=1}^{r} \theta_{t,i}\bar{P}_{2,i} + \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_{t,i}\theta_{t,j}\bar{P}_{2,i,j},$$

$$\bar{P}_2(\theta_{t-1}) = \sum_{i=1}^{r} \theta_{t-1,i}\bar{P}_{2,i} + \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_{t-1,i}\theta_{t-1,j}\bar{P}_{2,i,j},$$

$$\bar{K}_1(\theta_t, \theta_{t-1}) = \sum_{i=1}^{r} \theta_{t,i}\bar{K}_{1,t,i} + \sum_{i=1}^{r} \theta_{t-1,i}\bar{K}_{1,t-1,i},$$

$$\bar{K}_2(\theta_t, \theta_{t-1}) = \sum_{i=1}^{r} \theta_{t,i}\bar{K}_{2,t,i} + \sum_{i=1}^{r} \theta_{t-1,i}\bar{K}_{2,t-1,i},$$

$$\bar{K}_3(\theta_t, \theta_{t-1}) = \sum_{i=1}^{r} \theta_{t,i}\bar{K}_{3,t,i} + \sum_{i=1}^{r} \theta_{t-1,i}\bar{K}_{3,t-1,i},$$

$$W_1(\theta_t) = \sum_{i=1}^{r} \theta_{t,i}W_{1,i}, \quad W_2(\theta_t) = \sum_{i=1}^{r} \theta_{t,i}W_{2,i},$$

$$W_3(\theta_t) = \sum_{i=1}^{r} \theta_{t,i}W_{3,i}.$$

Corollary 2. (Simplified stabilizing condition via LMI) The closed-loop system (7) is globally asymptotically stable, if there exist matrices $\bar{P}_{1,i}$, $\bar{P}_{2,i}$, $\bar{P}_{2,i,j}$, $\bar{K}_{1,t,i}$, $\bar{K}_{1,t-1,i}$, $\bar{K}_{2,t,i}$, $\bar{K}_{2,t-1,i}$, $\bar{K}_{3,t,i}$, $\bar{K}_{3,t-1,i}$, $W_{1,i}$, $W_{2,i}$, $W_{3,i}$, Λ, Λ_i, Σ, Σ_i and Ξ_i such that, for all $i = 1, \cdots, r$, $j = 1, \cdots, r$,

$$\begin{bmatrix} \Upsilon & (*) & (*) \\ [\,\Gamma_{ij}\,]_{r\times 1} & [\,\Delta_{ij}\,]_{r\times r} & (*) \\ [\,\Omega_{ij}\,]_{r\times 1} & [\,\Pi_{ij}\,]_{r\times r} & [\,\Psi_{ij}\,]_{r\times r} \end{bmatrix} > 0, \tag{13}$$

$$\bar{P}_{1,i} > 0, \quad \bar{P}_{2,i} > 0, \quad \bar{P}_{2,i,j} > 0, \tag{14}$$

$$\Lambda + \Lambda^T \geq 0, \quad \Lambda_i + \Lambda_i^T \geq 0, \tag{15}$$

$$\Sigma + \Sigma^T \geq 0, \quad \Sigma_i + \Sigma_i^T \geq 0, \tag{16}$$

$$\Xi_i + \Xi_i^T \geq 0, \tag{17}$$

where

$$\Upsilon = \mathbf{N}_0,$$

$$\Gamma_{i1} = \mathbf{M}_{1-i} + \mathbf{N}_{1-i},$$

$$\Omega_{i1} = \mathbf{M}_{2-i} + \mathbf{N}_{2-i},$$

$$\Delta_{ij} = \begin{cases} \mathbf{M}_{3-ii} + \mathbf{N}_{3-ii} & i = j, \\ \bar{\mathbf{M}}_{3-ij} + \mathbf{N}_{3-ij} & i \neq j, \end{cases}$$

$$\Pi_{ij} = \begin{cases} \mathbf{M}_{4-ii} + \mathbf{N}_{4-ii} & i = j, \\ \mathbf{M}_{4-ij} & i \neq j, \end{cases}$$

$$\Psi_{ij} = \mathbf{M}_{5-ij} + \mathbf{N}_{5-ij},$$

$$\mathbf{M}_{1-i} = \begin{bmatrix} \Theta & \Theta & \Theta \\ \Theta & \begin{bmatrix} W_{1,i} & W_{2,i} \\ 0 & W_{3,i} \end{bmatrix} & \begin{bmatrix} W_{1,i} & W_{2,i} \\ 0 & W_{3,i} \end{bmatrix} \\ \Theta & \Theta & \begin{bmatrix} \frac{1}{2}\bar{P}_{1,i} & 0 \\ 0 & \frac{1}{2}\bar{P}_{2,i} \end{bmatrix} \end{bmatrix},$$

$$\mathbf{M}_{2-i} = \begin{bmatrix} \begin{bmatrix} \frac{1}{2}\bar{P}_{1,i} & 0 \\ 0 & \frac{1}{2}\bar{P}_{2,i} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ \bar{P}_{2,i} & 0 \end{bmatrix} & \Theta \\ \Theta & \Theta & \Theta \\ \Theta & \Theta & \Theta \end{bmatrix},$$

$$\mathbf{M}_{3-ii} = \begin{bmatrix} \Theta & (*) & (*) \\ \begin{bmatrix} B_i\bar{K}_{1,t,i} & 0 \\ B_i\bar{K}_{3,t,i} & 0 \end{bmatrix} & \begin{bmatrix} 0 & (*) \\ -A_{2,i}W_{1,i}^T & \\ -B_i\bar{K}_{2,t,i} & 0 \end{bmatrix} & (*) \\ \Theta & \Theta & \begin{bmatrix} 0 & 0 \\ 0 & \bar{P}_{2,i,i} \end{bmatrix} \end{bmatrix},$$

$$\bar{\mathbf{M}}_{3-ij}^T = \begin{bmatrix} \Theta & \Theta & \Theta \\ \begin{bmatrix} B_i\bar{K}_{1,t,j} & 0 \\ B_i\bar{K}_{3,t,j} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ -A_{2,i}W_{1,j}^T & \\ -B_i\bar{K}_{2,t,j} & 0 \end{bmatrix} & \Theta \\ \Theta & \Theta & \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2}\bar{P}_{2,i,j} \end{bmatrix} \end{bmatrix},$$

$$\mathbf{M}_{4-ij}^T = \begin{bmatrix} \Theta & \Theta & \Theta \\ \begin{bmatrix} A_{1,i}\bar{P}_{1,j} + & \\ B_i\bar{K}_{1,t-1,j} & 0 \\ A_{3,i}\bar{P}_{1,j} + & \\ B_i\bar{K}_{3,t-1,j} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ -B_i\bar{K}_{2,t-1,j} & 0 \end{bmatrix} & \Theta \\ \Theta & \Theta & \Theta \end{bmatrix},$$

$$\mathbf{M}_{5-ij} = \begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \bar{P}_{2,i,j} \end{bmatrix} & (*) & (*) \\ \begin{bmatrix} 0 & \bar{P}_{2,i,j} \\ 0 & 0 \end{bmatrix} & \Theta & (*) \\ \Theta & \Theta & \Theta \end{bmatrix},$$

$$\Theta = [\,0\,]_{2n\times 2n},$$

$$\mathbf{N}_0 = (\Lambda + \Lambda^T) + (\Sigma + \Sigma^T) - \sum_{i=1}^{r} \delta_i^2(\Xi_i + \Xi_i^T),$$

$$\mathbf{N}_{1-i} = (-\Lambda_i) + (-2\Lambda),$$

$$\mathbf{N}_{2-i} = (-\Sigma_i) + (-2\Sigma),$$

$$\mathbf{N}_{3-ij} = \begin{cases} (\Lambda + \Lambda^T) + (\Lambda_i + \Lambda_i^T) + (\Xi_i + \Xi_i^T) & i = j, \\ \Lambda & i \neq j, \end{cases}$$

$$\mathbf{N}_{4-ii} = (-2\Xi_i),$$

$$\mathbf{N}_{5-ij} = \begin{cases} (\Sigma + \Sigma^T) + (\Sigma_i + \Sigma_i^T) + (\Xi_i + \Xi_i^T) & i = j, \\ (\Sigma + \Sigma^T) & i \neq j. \end{cases}$$

Proof. With the following vector:

$$[\,I,\ \theta_{t,1}I,\ \cdots,\ \theta_{t,r}I,\ \theta_{t-1,1}I,\ \cdots,\ \theta_{t-1,r}I\,]^T,$$

PLMIs (10) can be relaxed into the LMIs conditions (13)–(17), which are the discrete-time version of Park and Choi [2001]. For more details of relaxation, refer to Park and Choi [2001]. ∎

4. SIMULATION RESULTS

We consider the 2-D LPV discrete-time FM first model (1) under the conditions:

$$r=2,\quad \delta_1=\delta_2=1,\quad \theta_{t,1}=sin^2 t,\quad \theta_{t,2}=cos^2 t,$$
$$A_{1,1}=\begin{bmatrix}0.1 & 1.0\\ 0.1 & 1.5\end{bmatrix},\quad A_{1,2}=\begin{bmatrix}0.2 & 1.0\\ 0.5 & 1.5\end{bmatrix},$$
$$A_{2,1}=\begin{bmatrix}0.1 & 1.0\\ 0.1 & -1.5\end{bmatrix},\quad A_{2,2}=\begin{bmatrix}0.0 & 1.0\\ 0.2 & -2.0\end{bmatrix},$$
$$A_{3,1}=\begin{bmatrix}0.0 & 1.0\\ 0.0 & 1.3\end{bmatrix},\quad A_{3,2}=\begin{bmatrix}-0.1 & 1.0\\ 0.0 & 2.0\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.0\\ 1.0\end{bmatrix},\qquad B_2=\begin{bmatrix}0.5\\ 1.6\end{bmatrix}.$$

Assume that the boundary conditions $\{x(1,k), x(t,1)|t \geq 1, k \geq 1\}$ of the state are set to uniformly distributed random numbers between 0 and 1 except the initial conditions:

$$x(1,1)=\begin{bmatrix}50\\ 100\end{bmatrix},\quad x(2,1)=\begin{bmatrix}25\\ 150\end{bmatrix},\quad x(1,2)=\begin{bmatrix}100\\ 75\end{bmatrix}. \tag{18}$$

By the LMI toolbox in the Matlab, solutions of (13) can be obtained as follows:

$$\bar{K}_{1,t,1}=[\,2.055\ \ -1.919\,],\ \bar{K}_{1,t-1,1}=[\,-43.253\ \ -10.955\,],$$
$$\bar{K}_{1,t,2}=[\,0.363\ \ -0.608\,],\ \bar{K}_{1,t-1,2}=[\,-37.886\ \ -10.477\,],$$
$$\bar{K}_{2,t,1}=[\,-3.664\ \ 10.119\,],\ \bar{K}_{2,t-1,1}=[\,-0.211\ \ -0.140\,],$$
$$\bar{K}_{2,t,2}=[\,-8.982\ \ 1.925\,],\ \bar{K}_{2,t-1,2}=[\,0.164\ \ 0.102\,],$$
$$\bar{K}_{3,t,1}=[\,0.135\ \ 0.204\,],\ \bar{K}_{3,t-1,1}=[\,-12.649\ \ -9.725\,],$$
$$\bar{K}_{3,t,2}=[\,0.211\ \ -0.185\,],\ \bar{K}_{3,t-1,2}=[\,-15.149\ \ -8.502\,],$$
$$\bar{P}_{1,1}=\begin{bmatrix}113.512 & 9.945\\ 9.945 & 7.776\end{bmatrix},\quad \bar{P}_{1,2}=\begin{bmatrix}99.275 & 11.969\\ 11.969 & 7.201\end{bmatrix},$$
$$\bar{P}_{2,1}=\begin{bmatrix}24.254 & 1.503\\ 1.503 & 0.872\end{bmatrix},\quad \bar{P}_{2,2}=\begin{bmatrix}21.669 & 1.467\\ 1.467 & 1.002\end{bmatrix},$$
$$\bar{P}_{2,1,1}=\begin{bmatrix}3.143 & 0.219\\ 0.219 & 0.109\end{bmatrix},\quad \bar{P}_{2,2,1}=\begin{bmatrix}1.581 & 0.102\\ 0.102 & 0.276\end{bmatrix},$$
$$\bar{P}_{2,1,2}=\begin{bmatrix}9.465 & 0\\ 0 & 9.465\end{bmatrix},\quad \bar{P}_{2,2,2}=\begin{bmatrix}2.054 & 0.130\\ 0.130 & 0.120\end{bmatrix},$$
$$W_{1,1}=\begin{bmatrix}77.035 & 1.923\\ 0.768 & 6.274\end{bmatrix},\quad W_{1,2}=\begin{bmatrix}87.760 & 2.274\\ 7.964 & 2.387\end{bmatrix},$$
$$W_{2,1}=\begin{bmatrix}2.288 & 0.148\\ 1.466 & 0.089\end{bmatrix},\quad W_{2,2}=\begin{bmatrix}1.657 & 0.119\\ 0.566 & 0.043\end{bmatrix},$$
$$W_{3,1}=\begin{bmatrix}23.859 & 1.495\\ 1.420 & 0.903\end{bmatrix},\quad W_{3,2}=\begin{bmatrix}21.044 & 1.350\\ 1.377 & 1.034\end{bmatrix}.$$

Fig. 1 and Fig. 2 show the state trajectory according to t and k. As shown in the figures, the 2-D LPV system is asymptotically stablized by the designed parameter-dependent controller.

5. CONCLUSION

In this paper, we proposed the parameter-dependent state feedback controller for the 2-D discrete LPV system with the FM first model. To find the stabilizing conditions of the system, we first transformed the FM first model to the Roesser model, and then using the PDLF, the relaxation technique and constraint elimination, the conditions for stabilization were reduced to an LMI term. Numerical examples verified that the designed controller was valid and the state asymptotically converged to the origin. As further works, we shall handle the 2-D discrete LPV system with the FM second model and the output feedback case.

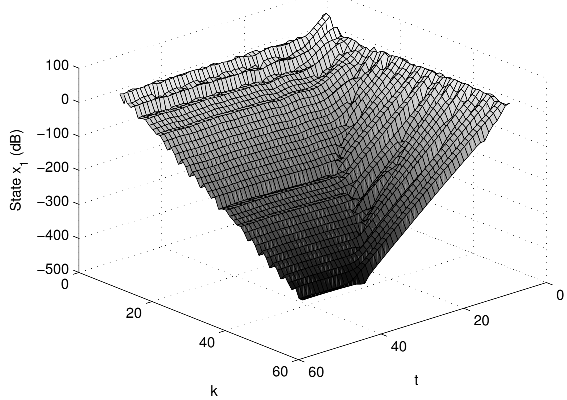

Fig. 1. The state trajectory of x_1 with respect to t and k.

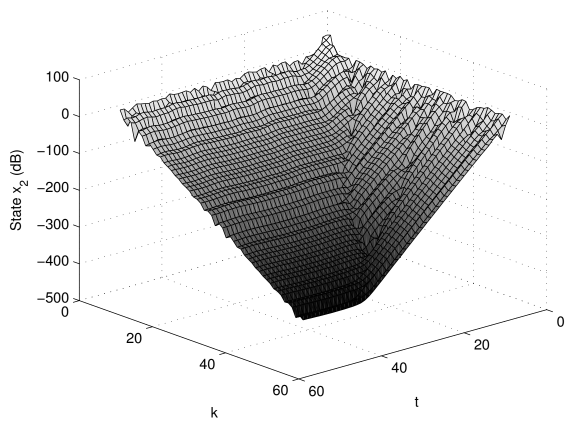

Fig. 2. The state trajectory of x_2 with respect to t and k.

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