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A Method of LPV Model Identification for Control

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Abstract: Nonlinear process identification for control is studied. In identification test, the process is only tested (excited) along its operating-trajectory that includes various working points and transition periods. In model identification, a linear parameter varying (LPV) model is used. First linear models are identified using data sets at various working-points exclusive transition data; then the LPV model is identified by interpolating the linear models using total data. Sufficient conditions for a unique solution in parameter estimation will be given. Simulation study will be used to verify the effectiveness of the method. The identified model is suitable for model predictive control (MPC).

1. INTRODUCTION

Nonlinear MPC has limited industrial applications when compared to linear MPC. The main obstacle is the high cost of modelling and identification of nonlinear processes. Therefore, finding a low cost method in nonlinear process identification is very important for industrial applications of nonlinear MPC.

A theoretically rigorous solution for nonlinear MPC is to use nonlinear models derived from first principles or from nonlinear system identification. At present, this approach is very often not feasible because of its high cost in modelling industrial process units. Developing a first principle model of a given industrial process costs a lot of manpower; the accuracy of first principle models is often not high enough for dynamic control. When system identification is used for modelling, excite the plant to cover the whole operation range is mostly not permitted because of to too large disturbances and of too much production losses.

Although most industrial processes are nonlinear in their operation ranges, not a single process will run chaotically in its whole operation range. Industrial processes are designed to perform certain processing tasks that convert raw materials to certain products. Hence they are operated in certain “orderly” ways. The orderly way of an industrial process can be expressed by the so-called *operating-trajectory*. This concept can be used for both batch processes and continuous processes. For a batch process, its operating trajectory is its serial and/or parallel operations (called batch program) carried out to produce a product. For a continuous process, its operating trajectory consists of its typical working points and related transition periods.

This work will focus on continuous processes and the study of batch processes will be carried out elsewhere. Examples of continuous nonlinear processes are: lubricate oil units that produce products of different viscosities; polymer plants that produce different product grades; electrical power plants that operate on different loads. In general, the control of this class of processes has two requirements: 1) good regulation (disturbance reduction) and optimization of the process operation at each working points and 2) fast transition control when the process changes working point. In general, processes behave so differently at different working points (grades or loads) that an MPC using a single linear model do not perform well for this class of processes.

In identification for control, it is sufficient to have a model that can approximately represent the process behaviour in a thin envelop covering its operating trajectory. Therefore, the idea of our method is to only test and identify process models alone their operating trajectories. This will result in a low cost testing method and simple and reliable identification computations. A simple linear parameter varying (LPV) model structure is used in model identification. Linear local models are identified using data sets at corresponding working-points; then the LPV model is obtained by interpolating the linear models using total data.

The terminology of LPV was first introduced in Shamma and Athans (1991) in the study of gain scheduling control. Applications of LPV (or gain scheduling) control have been reported in the control of electro-mechanical systems; see Rugh and Shamma (2000). The work on LPV model identification can be found in (Nemani, Ravikanth and Bamieh, 1995), Bamieh and Giarre (2002), Verdult and Verhaegen (2002, 2005) and Wei (2006). A common approach of current LPV methods is to parametrize the parameters of the transfer function (or state space) model as a nonlinear function of the scheduling parameter.

In Section 2, the identification method is outlined and some analysis is given; in Section 3, a simulation study is used to demonstrate the method; Section 4 is the conclusion.

2. PLANT TESTS AND MODEL IDENTIFICATION

One convenient way to represent an operating-trajectory model is to use the so called LPV model. Given a multi-input single-output (MISO) LPV system, denote the m inputs as $u_1(t)$, ..., $u_m(t)$ at time t and output as $y(t)$. Assume that the inputs and output data are generated by a sampled LPV system:

$$y(t) = G_1(q,w)u_1(t) + ...+ G_m(q,w)u_m(t) + v(t) \tag{1}$$

where

$$G_i(q,w) = \frac{B_i(q,w)}{A_i(q,w)} = \frac{[b_1^i(w)q^{-1} + ... + b_n^i(w)q^{-n}]q^{-d_i}}{1 + a_1^i(w)q^{-1} + ... + a_n^i(w)q^{-n}}$$

is the transfer function from $\underline{u}_i(t)$ to $y(t)$ which is stable, d_i is the delay from ith input to the output, q^{-1} denotes unit delay operator and $v(t)$ is the unmeasured output disturbance. Here we assume that the disturbance $v(t)$ is a stationary stochastic process with zero mean and bounded variance.

The variable $w(t)$ will be called *working-point variable* which determines the working point of the process operation. It is a measured variable from the process or can be calculated from measurable process variables. Examples of working point variables are: load of a power plant, air feed rate of an air separation process, product viscosity of a lubricate oil unit, and product grade of a polymer unit.

Remark 1: In the literature the working-point variable $w(t)$ is often call scheduling variable and is denoted as $p(t)$. We call it working-point variable due to two reasons: 1) it better represent the process control environment; and 2) the word *scheduling* can be misleading because scheduling is an important layer in total plant control systems in process industries which consist of regulatory control layer, advanced control layer, real-time optimization layer, scheduling layer and planning layer.

Denote the parameter vector of the model $G_1(q),...,G_m(q)$ as

$$[a_1^1(w),...,a_n^1(w),b_1^1(w),...,b_n^1(w),......,a_1^m(w),...a_n^m(w),b_1^m(w),...,b_n^m(w)]^T \tag{2}$$

In Bamieh and Giarre (2002) and Wei (2006), the authors parametrize each parameter in vector $\theta(w)$ as a nonlinear function of the working-point variable $w(t)$ and use recursive least-squares method to estimate model parameters. For large scale industrial control, this approach has some difficulties. Long test in the whole process operation range is necessary which is often not permitted. Recursive least-squares method uses ARX (or equation error) models. It is well known that an ARX model with low order has large bias for industrial data due to high level unmeasured disturbances. Use high order ARX model will leads to too many parameters.

Here we will propose a simpler method: we only test and identify the LPV model alone its operating-trajectory. Assume that the process operating-trajectory is determined by the working-point variable $w(t)$ which is in the range

$$w(t) \in [w_{lo}, w_{hi}] \tag{3}$$

where w_{lo} and w_{hi} are low and high limits of $w(t)$. Then (1), (2) and (3) define an operating-trajectory model of an industrial process.

Identification Test

The test used here only covers the trajectory of the process operation.

1) **Working point test.** At each working point, perform a normal identification test for linear model identification using small test signals; see, e.g., Zhu (2001). The test can be in open loop or in closed-loop. Proper test signals (excitation) should be used during the tests.

2) **Transition test.** Perform normal transition control (manually or automatically); add small test signals to the inputs during the transition periods.

Industrial experiences have shown that working point tests can be applied without problems in linear MPC control. Transition tests do not add additional production cost than normal transition control because the only difference is the addition of small test signals. Therefore, this test approach is low cost.

This test approach is very similar to that of Banerjee and Arkun (1998) for the identification of nonlinear ARX models, except that they do not use test signals during transition periods.

LPV Model Identification

1) Identify linear models for each working-point

Each linear model is identified using the data at each working point. Several linear identification methods can be used here: prediction error method class, subspace method (Ljung, 1999) and ASYM (Zhu, 2001). For closed-loop test, one needs to use a method that gives unbiased estimate for closed-loop data.

Without loss of generality, assume that the process has 3 working points at

$$w_1 < w_2 < w_3$$

Denote the 3 identified linear working-point models as

$$\begin{aligned} y(t) &= \hat{G}_1^1(q)u_1(t) + ...+ \hat{G}_m^1(q)u_m(t), \quad \text{for } w = w_1 \\ y(t) &= \hat{G}_1^2(q)u_1(t) + ...+ \hat{G}_m^2(q)u_m(t), \quad \text{for } w = w_2 \\ y(t) &= \hat{G}_1^3(q)u_1(t) + ...+ \hat{G}_m^3(q)u_m(t), \quad \text{for } w = w_3 \end{aligned} \tag{4}$$

The variable w is dropped in the models because they are linear at their working-points.

2) Obtain LPV model by interpolation

Instead of identifying a full LPV model in (1) and (2), we use an approximation model to model the process in the operating-trajectory as follows

$$\begin{aligned} y(t) &= \alpha_1(w)[\hat{G}_1^1(q)u_1(t)+...+\hat{G}_m^1(q)u_m(t)] \\ &+\alpha_2(w)[\hat{G}_1^2(q)u_1(t)+...+\hat{G}_m^2(q)u_m(t)] \\ &+\alpha_3(w)[\hat{G}_1^3(q)u_1(t)+...+\hat{G}_m^3(q)u_m(t)]+v(t) \end{aligned} \tag{5}$$

where $\alpha_1(w)$, $\alpha_2(w)$ and $\alpha_3(w)$ are weights which are functions of the working point variable $w(t)$.

Assume that the process parameters such as gains and time constants vary as monotone functions of $w(t)$ between each two neighbouring work-points, the simplified model (5) can be a good approximation of the original model (1) and (2) alone the operating-trajectory.

Remark 2. Several methods developed in the literature also use local models. Johansen and Foss (1998) developed a so-called operating-regime-based identification method. In their approach, first principle modelling and system identification are combined to solve nonlinear process identification problems. First, the selection of operating regimes using several characteristic variables is performed. Then local (linear or nonlinear) models are estimated using globally tested data. The global nonlinear model is obtained by calculating the weights. Their aim is to obtain a global nonlinear model and the concept of working-point variable is not used explicitly. In Banerjee and Arkun (1998), the authors suggested the use of testing data sets from various working points and transition periods; in model identification, they used local linear models to identify nonlinear ARX models.

A good way to determine the weights is to estimate them from total testing data which should include transition periods. The weight functions $\alpha_1(w)$, $\alpha_2(w)$ and $\alpha_3(w)$ can be parametrized as cubic splines, polynomials, or piece-wise linear function. The estimation method is illustrated using cubic splines.

Denote a set of knots $\{k_1, k_2, \ldots, k_m\}$ for the working-point variable $w(t)$ which are real numbers and satisfy

$$k_1 = k_{\min} < k_2 < \ldots < k_m = k_{\max} \tag{6}$$

A cubic spline function for $\alpha_1(w)$ is given as

$$\alpha_1(w) = \beta_1^1 + \beta_2^1 w + \sum_{j=2}^{m-1} \beta_{j+1}^1 \, | w - k_j |^3 \tag{7}$$

where $[\beta_1^1, \beta_2^1, ..., \beta_m^1]$ are the parameters to be estimated. Here m can be called the order of the cubic splines. It can be verified that the function (7) is a smooth function. The same can be defined for $\alpha_2(w)$ and $\alpha_3(w)$. Assume that, for the moment, all three weighting functions use the same knots as in (6). The knots should span in the process operation range, namely

$$k_1 = k_{\min} = w_{\mathrm{lo}} \text{ and } k_m = k_{\max} = w_{\mathrm{hi}} \tag{8}$$

A good way to knots distribution is to use different knots for each weighting functions except at the high/low limits; and distribute the knots (nearly) uniformly in range $[w_{\min}, w_{\max}]$. The order of the cubic splines m depends on the number of working points and the amount of data. Higher order can be used when there are more working points and more transition data.

Now, the weighting functions $\alpha_1(w)$, $\alpha_2(w)$ and $\alpha_3(w)$ will be estimated using total testing data. Assume that the test data set is in one piece obtained by moving the process passing all the working points w_1, w_2 and w_3. This is only for notation convenience. It is also possible to use different pieces of transition test data from discontinuous transition tests. Denote the data set as

$$Z^N = \{u_1(t), ..., u_m(t), y(t), w(t)\ t = 1, 2, ..., N\} \tag{9}$$

Simulate the three working point models using test data Z^N as:

$$\begin{aligned} \hat{y}^1(t) &= \hat{G}_1^1(q)u_1(t)+...+\hat{G}_m^1(q)u_m(t) \\ \hat{y}^2(t) &= \hat{G}_1^2(q)u_1(t)+...+\hat{G}_m^2(q)u_m(t) \\ \hat{y}^3(t) &= \hat{G}_1^3(q)u_1(t)+...+\hat{G}_m^3(q)u_m(t) \end{aligned} \tag{10}$$

Denote the parameter vector as the weighting functions as

$$\theta = [\beta_1^1, \beta_2^1, ..., \beta_m^1, \beta_1^2, \beta_2^2, ..., \beta_m^2, \beta_1^3, \beta_2^3, ..., \beta_m^3]^T \tag{11}$$

Note that the superscripts in (10) and (11) are used for numbering and they are not used as powers. Then the parameters of weighting functions can be determined by minimizing the output error loss function:

$$\hat{\theta} = \min_{\theta} \sum_{t=1}^{N} [e_{OE}(t)]^2 \tag{12}$$

where $e_{OE}(t)$ is the output error of model (5)

$$e_{OE}(t) = y(t) - [\alpha_1(w)\hat{y}^1(t) + \alpha_2(w)\hat{y}^2(t) + \alpha_3(w)\hat{y}^3(t)] \tag{13}$$

Now, the data of working-point variable $w(t)$ is used.

Denote the data vectors related to cubic splines weighting functions as

$$\begin{aligned}\varphi^1(t) &= [1 \quad w(t) \quad |w(t)-k_2^1|^3 \quad \ldots \quad |w(t)-k_{m-1}^1|^3] \\ \varphi^2(t) &= [1 \quad w(t) \quad |w(t)-k_2^2|^3 \quad \ldots \quad |w(t)-k_{m-1}^2|^3] \\ \varphi^3(t) &= [1 \quad w(t) \quad |w(t)-k_2^3|^3 \quad \ldots \quad |w(t)-k_{m-1}^3|^3]\end{aligned} \tag{14}$$

Then the output error can be written as

$$e_{OE}(t) = y(t) - [\varphi^1(t)\hat{y}^1(t) \quad \varphi^2(t)\hat{y}^2(t) \quad \varphi^3(t)\hat{y}^3(t)]\theta \tag{15}$$

Because the output error $e_{OE}(t)$ is linear in the weighting function parameters, the optimization in (12) is a linear least-squares problem and has the following solution:

$$\hat{\theta} = [\Phi^T\Phi]^{-1}\Phi^T Y \tag{16}$$

where

$$Y = [y(1), y(2), \ldots, y(N)]^T \tag{17a}$$

and

$$\Phi = \begin{bmatrix} \varphi^1(1)\hat{y}^1(1) & \varphi^2(1)\hat{y}^2(1) & \varphi^3(1)\hat{y}^3(1) \\ \varphi^1(2)\hat{y}^1(2) & \varphi^2(2)\hat{y}^2(2) & \varphi^3(2)\hat{y}^3(2) \\ \vdots & \vdots & \vdots \\ \varphi^1(N)\hat{y}^1(N) & \varphi^2(N)\hat{y}^2(N) & \varphi^3(N)\hat{y}^3(N) \end{bmatrix} \tag{17b}$$

Some conditions of the data are needed so that data matrix Φ has a full column rank and (16) has a unique solution. They are listed here:

A1: During the test, the working-point variable $w(t)$ takes a number of distinct values in the range $[w_{\min}, w_{\max}]$ that is greater than 3 times the order of cubic splines.
A2: The true process is given in (1) and it is stable for all values of $w(t)$ and the three identified working-point models are also stable.
A3: Denote n as the highest order of the working-point models. The inputs $u_1, u_2, \ldots, u_m$ are persistent exciting with an order higher than $2n$ and not linearly dependent.

Notice the special structure of the data matrix Φ. From linear algebra, we know that Φ will have full column rank if the two matrixes

$$\begin{bmatrix} \varphi^1(1) & \varphi^2(1) & \varphi^3(1) \\ \varphi^1(2) & \varphi^2(2) & \varphi^3(2) \\ \vdots & \vdots & \vdots \\ \varphi^1(N) & \varphi^2(N) & \varphi^3(N) \end{bmatrix} \tag{18a}$$

and

$$\begin{bmatrix} \hat{y}^1(1) & \hat{y}^2(1) & \hat{y}^3(1) \\ \hat{y}^1(2) & \hat{y}^2(2) & \hat{y}^3(2) \\ \vdots & \vdots & \vdots \\ \hat{y}^1(N) & \hat{y}^2(N) & \hat{y}^3(N) \end{bmatrix} \tag{18b}$$

both have full column ranks. Because different knots are used for each weighting functions, the first matrix will have full column rank if condition A1 holds. The second matrix will have full column rank if conditions A2 and A3 hold. Hence, we have:

Theorem 1. Given the LPV model (5) and assume that the same knots are used for all the weighting functions. Then data matrix Φ has full column rank and (16) has a unique solution if conditions A1 to A3 hold.

Remark 3. If the linear models are dynamically different, meaning that their time constants (or poles and zeros) are different, then, the same knots can be used for all the weighting functions. This is because that, under the condition, the matrix in (18b) will always have a full column rank and hence the data matrix in (17b) will always have a full column rank.

Trivial Interpolation
Assume that no tests in transition periods are permitted due to economical considerations. The best we can do is to let the weightings equal to the distances between the current working-point and the working point of the linear models. Then the weights can be given directly without estimation:

$$\alpha_1(w) = \begin{cases} 1 & w < w_1 \\ \dfrac{w_2 - w}{w_2 - w_1} & w_1 \le w \le w_2 \\ 0 & w > w_2 \end{cases} \tag{19a}$$

$$\alpha_2(w) = \begin{cases} 0 & w < w_1 \\ \dfrac{w - w_1}{w_2 - w_1} & w_1 \le w \le w_2 \\ \dfrac{w_3 - w}{w_3 - w_2} & w_2 < w \le w_3 \\ 0 & w > w_3 \end{cases} \tag{19b}$$

$$\alpha_3(w) = \begin{cases} 0 & w < w_2 \\ \dfrac{w - w_2}{w_3 - w_2} & w_2 \le w \le w_3 \\ 1 & w > w_3 \end{cases} \tag{19c}$$

Although very simple, the LPV model given in (19) can give reasonably good approximation of the nonlinear process alone its operating-trajectory, at least much better than an averaging linear model.

The Use of Two Working Point Variables
Up to now, only one working-point variable is used to explain the ideas. Although seems over simplified, the LPV model with one working-point variable can properly represent many industrial processes for control purpose. If not enough, a second working-point variable can be introduced, which does not bring any challenge in our approach. We believe that the use of one or two working point variables will be sufficient to model most of industrial

processes along their operating-trajectories, although this may not be enough to provide a global nonlinear model.

Assume that two working point variables are used in the LPV model

$$w_1(t) \in [w_{1,lo}, w_{1,hi}], \quad w_2(t) \in [w_{2,lo}, w_{2,hi}] \tag{20}$$

The LPV model (5) can be easily modified to incorporate the two working-point variables

$$\begin{aligned} y(t) = {} & \alpha_1(w_1,w_2)[\hat{G}_1^1(q)u_1(t) + ...+ \hat{G}_m^1(q)u_m(t)] \\ & + \alpha_2(w_1,w_2)[\hat{G}_1^2(q)u_1(t) + ...+ \hat{G}_m^2(q)u_m(t)] \\ & + \alpha_3(w_1,w_2)[\hat{G}_1^3(q)u_1(t) + ...+ \hat{G}_m^3(q)u_m(t)] + v(t) \end{aligned} \tag{21}$$

where $\alpha_1(w_1,w_2)$, $\alpha_1(w_1,w_2)$ and $\alpha_1(w_1,w_2)$ are bivariate cubic spline functions. Then, the same algorithm just introduced can be used to solve the parameter estimation of the weighting functions. Note that more working point tests and transition tests may be needed for LPV models with two working point variables.

3. SIMULATION

Given a first order process, the transfer function in the continuous-time is

$$G(s,w) = \frac{K(w)}{\tau(w)s+1} \tag{22}$$

where

$$K(w) = 0.6 + w^2, \quad \tau = 3 + 0.5w^3, \quad w \in [1,4] \tag{23}$$

In the operation range $w \in [1,4]$ process gain changes nonlinearly more than 10 times and time constant changes 10 times. Thus a linear model cannot give good approximation of the process behaviour in the whole operation trajectory.

Here, we will show how well the proposed LPV model can approximate the process over the whole operation trajectory. For generating input-output data, the process is simulated at a sampling time of 1 second. First, noise-free data are used to test the approximation capability of the proposed method; then noisy data are used to check the noise-reduction capability of the method.

In order to obtain linear local models, the process (21) and (22) will be tested at three working-points:

$$w_1 = 1, \; w_2 = 2.25, \; w_3 = 4$$

The following test simulation is performed:

- First period: 500 seconds, at working-point $w = 1$
- Second period: 1500 seconds, working point w varies linearly in time from 1 to 2.25
- Third period: 500 seconds, at working point $w = 2.25$
- Forth period: 1500 seconds, working point w varies linearly in time from 2.25 to 4
- Fifth period: 500 seconds, at working point $w = 4$.

The input is a GBN (generalized binary noise) signal with average switch time of 35 seconds; the output signal is noise-free. Note that the five test periods do not need to be performed continuously as the identification method applies equally well to data with discontinuous periods.

Three first order output-error (linear) models are identified using the data at working-points $w = 1$ (first period), $w = 2.25$ (third period) and $w = 4$ (fifth period). The three obtained models are perfect because the data are noise-free. Denote the LPV model based on the two working-point models as

$$y(t) = \alpha_1(w)\hat{G}^1(q)u(t) + \alpha_2(w)\hat{G}^2(q)u(t) + \alpha_3(w)\hat{G}^3(q)u(t) + v(t) \tag{24}$$

where $\hat{G}^1(q)$, $\hat{G}^2(q)$ and $\hat{G}^3(q)$ are the three output-error models. Then, weighting functions $\alpha_1(w)$, $\alpha_2(w)$ and $\alpha_3(w)$ are estimated using the total data set. The knots for the three weighing functions are the same and the order of cubic splines is 7.

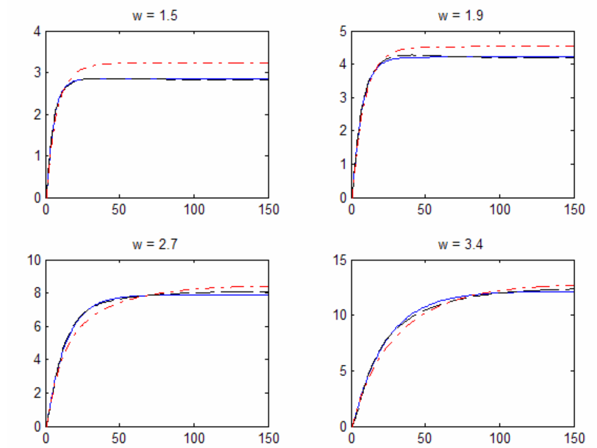

Figure 1. Step responses of the LPV model. Blue solid lines, true step responses; black dashed lines, LPV model; red dashdot lines, trivial interpolation

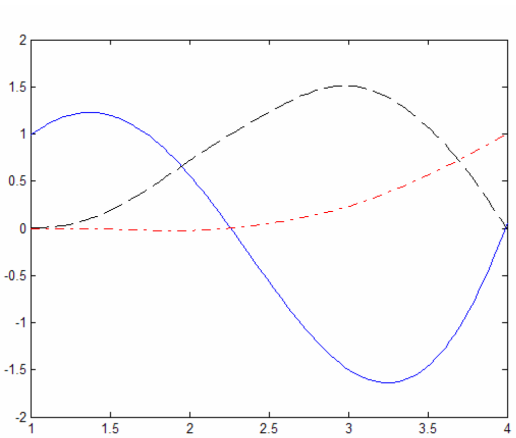

Figure 2. Weighting functions of the LPV model. Blue solid line, weighting 1; black dashed line, weighting 2; red dahsdot line, weighting 3

Then, the simulated output is corrupted by a filtered white noise as

$$v(t) = \frac{c}{1 - 0.9z^{-1}} e(t) \quad (25)$$

where $e(t)$ is a white noise sequence and the constant c is adjusted at the three working points so that the noise is about 3% of the noise-free output in power.

The noisy data are used to obtain an LPV models the same way as before. The estimated LPV model step responses and weighting functions are shown in Figure 3 and Figure 4. One can see that good quality LPV model can be obtained using noisy data.

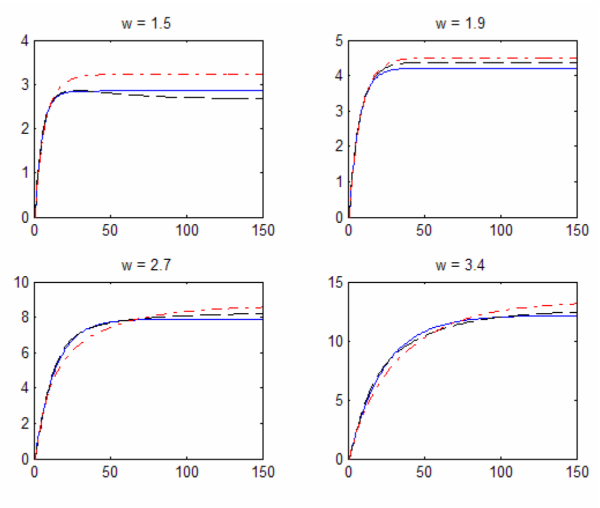

Figure 3. Step responses of the LPV model using noise data. Blue solid lines, true step responses; black dashed lines, LPV model; red dashdot lines, trivial interpolation

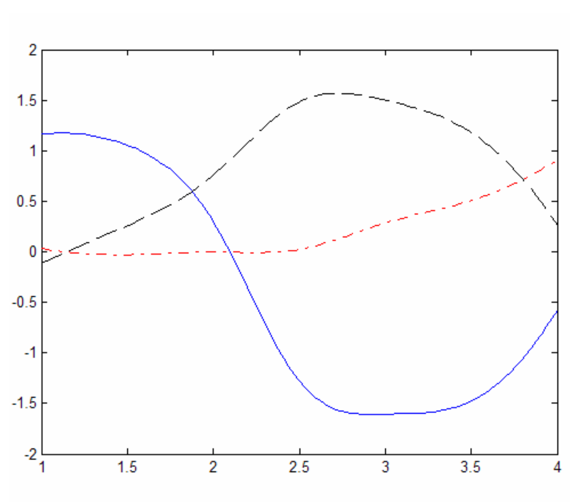

Figure 4. Weighting functions of the LPV model using noise data. Blue solid line, weighting 1; black dashed line, weighting 2; red dahsdot line, weighting 3

4. CONCLUSION

A method of nonlinear model identification for control is proposed. In plant test, working point tests and transition tests are used which can be called low cost tests. In model identification, a simple LPV model is used to represent process behaviour. It consists of linear local models and weighting functions. Sufficient conditions for unique solution of identified model have been derived. The simulation example has shown high approximation power of the proposed LPV model. The advantages of the developed method are: the LPV model can represent a very large class of industrial processes, both continuous and batch; the plant tests are low cost because small test signals are used alone operating-trajectories; the identification method is simple and numerically reliable; and finally, it is easy to assure the stability of the model for stable processes.

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