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Application of a Game-theoretic Multi-loop Control System Design With Robust Performance

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Abstract: The game-theoretic view of control system design for multi-loop systems is extended in this work to ensure a closed-loop system with robust stability. The control system design is modeled as a differential cooperative game to incorporate interactions between the multiple loops of the control system. A robust stability indicator is formulated as an additional cost function. The developed approach is applied to a reverse osmosis desalination plant with different constraint settings on the control signals. The solution of the game provides Pareto-optimal sets, depending on the control signal constraints. Single points are chosen from the Pareto-optimal sets resulting in controller parameters leading to a reverse osmosis system with optimal performance concerning the error convergence, control effort and robust stability.

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1. INTRODUCTION

Standard techniques for controller tuning of multi-loop systems assume that the control loops can be adjusted individually by loop decoupling, thereby neglecting the interactions of the different control loops. A detailed literature research leads to the conclusion that there is no good method for simultaneous tuning of several controllers that significally improved performance compared to a single loop controller, see for example in Brosilow and Joseph [1999]. According to Johnson and Mohradi [2005], the disadvantages of using multi-loop PID or PI controllers are the lack of interaction consideration and the existence of few powerful tools for its design.

This was the motivation to develop a new approach of controller parameter tuning in multi-loop control structures. In Wellenreuther et al. [2006a], Wellenreuther et al. [2006b], Gambier et al. [2006] and Gambier et al. [2007] the method, based on game theory was proposed for continuous time systems as well as for discrete time systems. Thereby, the control system is viewed as a differential cooperative game where the controllers represent the players. A cost function is assigned to each controller, such that the control system design consists in minimizing jointly all indices. This leads to a multi-objective optimization (MOO) problem that has to be solved (see Rusnak [2005]). The approach was modified in Wellenreuther et al. [2007], in order to add constraints to the cost functions.

Since every model of a physical system is involved with uncertainties as a result of several reasons (see Skogestad and Postlethwaite [1996]), it is useful to include a robust stability analysis in the presented control system design. Thus, the solution of the differential cooperative game should ensure robust stability for a given uncertainty.

Game theory deals with objectives, which are in conflict, as for example robust stability and performance, and tries to find a good trade-off between the conflicting participants.

The idea to use game theory to solve control theoretic problems is not new. According to Basar and Olsder [1999], differential game theory can be viewed as a child of the parents game theory and optimal control theory. For example, a game theoretic approach to design controllers for safety specifications is given in Tomlin et al. [2000]. Lygeros et al. [1997] uses ideas of game theory, to treat the control system design process as a two player zero-sum game between the controller of a player and the disturbance generated by the actions of the other player.

The description of the game-theoretic framework for a multi-loop control system design is given in Section 2 including the consideration of robust stability. The application to a reverse osmosis desalination plant is considered in Section 3. The paper is completed with simulations, presented in Section 4 and a conclusion in Section 5.

2. GAME-THEORETIC FRAMEWORK

To include control loop interactions in multi-loop systems, the control system design is considered here as a differential game between i players with $i = 1, ..., N$ on the time period $[t_0, T]$. The strategies of the players are defined as

$$u_i(t) = \int_{t_0}^{T} c_i(t) e_i(t - \tau) d\tau \tag{1}$$

with

$$\mathcal{L}\{c_i(t)\} = C_i(s) = Q_i(s)/P_i(s). \tag{2}$$

Q_i and P_i are polynomials of the controller C_i. The strategies u_i of the players belong to the strategy sets $U_i = \{u_i | u_i \text{ is given by } (1)\}$.

The differential game can now be described as the evolution of the errors e_i with

$$e_i^{(n)} = f_i(e_i^{(n-1)}, ..., \dot{e}_i, u_1, ..., u_N) \tag{3}$$

and initial condition

$$e_i(t_0) = e_{i0} \tag{4}$$

as well as a cost J_i with

$$J_i = g_{i0}(e_{iT}). \tag{5}$$

The errors e_i belong to the set $E_i = \{e_i | e_i \text{ as solution of } (3)\}$. Function f_i is defined on $f_i : E_i \times U_1 \times \ldots \times U_N \to \mathbb{R}$ and

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function g_{i0} on $g_{i0} : E_i \rightarrow \mathbb{R}^+$.

A typical performance index applied to control problems is the integral square error (ISE) over the complete time interval with $t_0 = 0$ and $T = \infty$, which is now used for the costs J_i as

$$J_i = \int_0^\infty e_i^2(t)dt. \qquad (6)$$

The terminal state e_{iT} as well as the cost functions J_i depend on the strategies u_i of the players i. In contrast, the players strategies u_i depend on the controller parameters Q_i and P_i as well as the control system structure and the reference signals r_i.

In a cooperative differential game involving N players, each player wants to minimize his cost $J_i = g_{i0}(e_T)$ through the selection of his control strategies u_i.

A minimization of multiple costs J_i with given reference signals r_i and a given control structure leads to an optimization of the controller parameters Q_i and P_i.

2.1 *Constrained strategy sets*

Since every control signal cannot be followed by the physical system, the controls u_i of the multi-loop control structure are limited around an operating point in a predefined range of u_{limit_i}:

$$|u_i| \leq u_{limit_i}. \qquad (7)$$

These constraints are considered in the game theoretic control system design, yielding to an optimization of (6) subject to (7). In terms of the differential game, constraints on the control signals imply limitations on the players' strategy sets U_i.

2.2 *Solution of the game*

According to Neumann and Morgenstern [2004], the solution of a cooperative game is a set of solutions. All nondominated solutions, also called Pareto-optimal solutions, are part of this set, called Pareto-optimal set.

To obtain a Pareto-optimal set for the described game, all cost functions J_i have to be optimized by tuning the controller parameters Q_i and P_i. Optimizing more than one cost function is known to be a multi-objective optimization (MOO) problem that has to be solved. The genetic algorithm of Pohlheim [2000], which is already used in previous works (see Wellenreuther et al. [2006a], Wellenreuther et al. [2006b], and Gambier et al. [2007]) is applied to solve the MOO problem. If the reader is interested to know more about GA's, he is referred for example to Holland [1992] or Beasly et al. [1993].

In the present paper, only two constraints are considered during the optimization. First, the chosen parameter sets have to ensure, that the final closed-loop system is stable, which is done during the evaluation of the cost functions J_i. And second, the resulting control strategies have to satisfy their predefined limits.

A range for each controller parameter of Q_i and P_i must be specified at the beginning. The values for the starting population are selected from this range. The final solution of the GA is a Pareto-optimal set for the costs J_i providing the controller parameters.

The costs J_i could be part of a solution concept for cooperative games, named the core, known to be the most attractive solution concept in cooperative game theory. In Aumann [1961], the core is defined to be the subset of outcomes from which there is no tendency to move away - the equilibrium states. Hence, the core collects cost sets J_1, ..., and J_N (also called imputations) that are not dominated. All possible cost sets are imputations where none of the players gets less than he would get if he plays alone.

For two player games the set of imputations coincides with the core and thus with the obtained Pareto-optimal set.

So far, the new method in the game-theoretic framework provides controller parameters for multi-loop systems ensuring a stable closed loop system with optimal performance concerning the error convergence and additionally having regard to constraints on the control strategies. Robust stability with respect to model uncertainties is not yet considered in the design.

2.3 *Robust Stability Consideration*

Modeling physical systems can lead to substantial differences between the model and the physical system, since no capable mathematical model exists, that describes a physical process exactly, (Skogestad and Postlethwaite [1996], and Manoso et al. [1997]). This problem is called the robustness problem. The robustness problem is solved first by characterizing the uncertainty and incorporating it into the mathematical model. If the system remains stable for all perturbed plants about the nominal model, up to the worst-case model uncertainty. In the literature, uncertainty is distinguished between two main classes: parametric uncertainty and uncertainty caused by unmodeled dynamics (Balas et al. [1996], Skogestad and Postlethwaite [1996]). In the case of parametric uncertainty, the structure of the model, including the order, is known, but some parameters are uncertain. This type of uncertainty can be modeled as inverse additive uncertainty (Becerra [1999]). In contrast, unmodeled dynamics occur due to the high frequency plant behaviour, which is often uncertain since only the low order nominal model describing the low-mid frequency range behaviour of the plant is available. One common approach to model this type of uncertainty is to use a multiplicative uncertainty model (Skogestad and Postlethwaite [1996]).

The singular value analysis, identified as σ and a generalization of the Nyquist criterion, is a popular general way to analyse the robust stability of multi-input/multi-output systems.

The structured singular value μ of a transfer function matrix M, where M represents a known linear system, is defined as $\mu(M) = 1/\sigma(M)$ subject to the singular value. It was developed to analyse the effects of parametric uncertainties and unmodelled dynamics to the stability and the performance of multi-loop systems. The structured singular value μ is defined on finding the smallest structured perturbation Δ (measured in terms of $\sigma(\Delta)$) which makes $\det(I - M\Delta) = 0$, then $\mu(M) = 1/\sigma(\Delta)$.

The peak of the frequency response of the general structured singular value μ delivers, dependent on the structure of the perturbation, the size for the perturbation where the closed loop system remains stable. A value of $\mu = 1$ represents a perturbation with $\sigma(\Delta) = 1$. If smaller perturbations makes the system unstable, the value of μ is larger than 1 and if the value of μ is smaller than 1, larger perturbations are permitted.

A robust stability theorem for block-diagonal perturbations is given in Skogestad and Postlethwaite [1996]:

Theorem 1. Assume that the nominal system M and the perturbations Δ are stable. Then the $M\Delta$-system is stable for all allowed perturbations with $\bar{\sigma}(\Delta) \leq 1, \forall\omega$, if and only if $\mu(M(j\omega)) < 1, \forall\omega$.

To calculate the structured singular value μ, the μ-Analysis and Synthesis Toolbox, available for Matlab, is used. Considering the robust stability analysis during the differential cooperative game, modeling the control system design, a cost function J_μ for the system is defined as

$$J_\mu = \mu(M). \tag{8}$$

The value of the robust stability cost J_μ depends on the players' control strategies u_i, given through the controller parameters Q_i and P_i. Considering the cost J_μ of (8) with regard to the solution of the game, an additional tradeoff between the robust stability and the performance of the system subject to constraints on the control strategies has to be met.

3. APPLICATION

The presented differential cooperative game description is applied to a reverse osmosis (RO) desalination plant. The RO system accomplishes the requirements of being a multi-loop system with control loop interactions.

3.1 Example Description

The ultimate purpose of a RO desalination process is producing a constant quantity of water with an acceptable purity. Several papers were published, for example Assef et al. [1995], Riverol and Pilipovik [2005] or Robertson et al. [1996], where RO system identification is considered as a two-input/two-output (TITO) system. The controlled output variables are the permeate flux (F) and the permeate conductivity (C). The system interaction can be written as

$$\begin{bmatrix} F \\ C \end{bmatrix} = \begin{bmatrix} G_{p11} & G_{p12} \\ G_{p21} & G_{p22} \end{bmatrix} \begin{bmatrix} P \\ pH \end{bmatrix} \tag{9}$$

belonging to the control structure, displayed in Fig. 1.
The process transfer functions, used in this work, are chosen from Robertson et al. [1996]:

$$\frac{F}{P} = G_{p11} = \frac{B_{11}}{A_{11}} = \frac{0.002(0.056s+1)}{(0.003s^2+0.1s+1)} \tag{10}$$

$$\frac{F}{pH} = G_{p12} = \frac{B_{12}}{A_{12}} = 0 \tag{11}$$

$$\frac{C}{P} = G_{p21} = \frac{B_{21}}{A_{21}} = \frac{-0.51(0.35s+1)}{(0.213s^2+0.7s+1)} \tag{12}$$

$$\frac{C}{pH} = G_{p22} = \frac{B_{22}}{A_{22}} = \frac{-57(0.32s+1)}{(0.6s^2+1.8s+1)} \tag{13}$$

In words, a change in the transmembrane pressure (P) effects the permeate flux as well as it has a negative effect on the permeate conductivity (C). Changing the pH has no effect on the permeate flux (F), as a result of (11), but a negative effect in the permeate conductivity (C). The control structure reflects the triangular (asymmetric) dependency in such a way that the upper control loop acts as a disturbance on the lower control loop. Thus, the control loops of the multi-loop system interact only in one-way.
The control system design with optimal performance concerning the error convergence and the robustness is now implemented using the proposed approach in the game theoretic framework.

3.2 Game-theoretic control system design for the RO process

The control system design of the two-input/two-output system in Fig. 1 is considered as a differential game between two players i with $i = 1, 2$ on the time period $[t_0, T]$.
The strategies of the players are defined as

$$u_i(t) = \int_{t_0}^{T} c_i(t) e_i(t-\tau) d\tau \tag{14}$$

with

$$\mathcal{L}\left\{c_i(t)\right\} = C_i(s) = \frac{Q_i}{P_i} = \frac{K_{P_i}s + K_{P_i}/K_{TI_i}}{s}. \tag{15}$$

Q_i and P_i are polynomials and contain the proportional and integral controller parameters of C_i in Fig.1. The strategies u_i of the players belong to the strategy sets $U_i = \{u_i | u_i \text{ is given by (14)}\}$.
The differential game can now be described as the evolution of the errors e_i with

$$e_1^{(3)} = f(\ddot{e}_1, \dot{e}_1, u_1, u_2), \tag{16}$$

$$e_2^{(8)} = f(e_2^{(7)}, \ldots, \dot{e}_2, u_1, u_2), \tag{17}$$

and initial condition

$$e_i(t_0) = e_{i0} \tag{18}$$

as well as the costs J_i with

$$\begin{aligned} J_i &= \int_0^\infty e_i^2(t) dt \\ &= \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} e_i(s) e_i(-s) ds \end{aligned} \tag{19}$$

and

$$J_\mu = \mu(M). \tag{20}$$

The errors e_i belong to the set

$$E_i = \{e_i | e_i \text{ as solution of (16) and (17)}\}.$$

Function f_i is defined on $f_i : E_i \times U_1 \times U_2 \to \mathbb{R}^+$. Equations (19) are solved according to Aström [1970].
For shortage of space, the polynomials $A_{ij}(s)$, $B_{ij}(s)$, $P_{ij}(s)$, $Q_{ij}(s)$, $e_i(s)$ and $r_i(s)$ are abbreviated in the following as A_{ij}, B_{ij}, P_{ij}, Q_{ij}, e_i, and r_i with $j = 1, 2$.
According to the presented game description, the error signal $e_1(s)$ of the first player is

$$e_1(s) = \frac{A_{11} r_{01}}{A_{11} P_1 + B_{11} Q_1}. \tag{21}$$

For the second player, the error signal e_2 is

$$e_2(s) = \frac{A_{21} A_{22} (A_{11} P_1 + B_{11} Q_1) r_{02} - B_{21} Q_1 A_{11} A_{22} r_{01}}{A_{21} (A_{11} P_1 + B_{11} Q_1)(A_{22} P_2 + B_{22} Q_2)} \tag{22}$$

The cost function J_μ, concerning the robust stability needs a computation of G_{ro}, see Fig. 1. The structure of G_{ro} depends on the class of uncertainty and how the uncertainties are introduced to the control structure. In this work, only parametric uncertainties are considered. For multi-loop systems, particularly multi-input/multi-output (MIMO) systems, the consideration

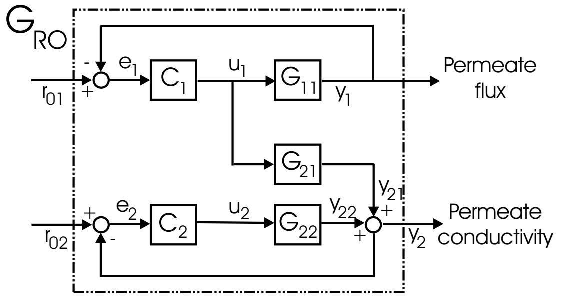

Figure 1. Control structure of the RO process

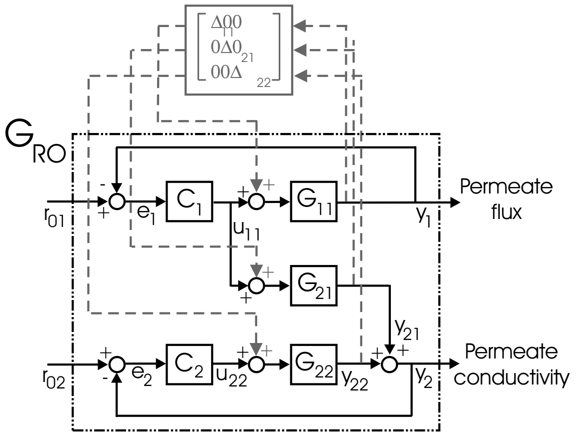

Figure 2. Control structure of the RO process, where the uncertain blocks Δ_{11}, Δ_{21}, and Δ_{22} are pulled out and placed inside a matrix block

of parametric uncertainty is very important, since it emerges the coupling between the uncertain transfer function elements (Skogestad and Postlethwaite [1996]). Thus, the parametric uncertainties are modeled as inverse additive uncertainties, see Fig. 2. To distinguish between what is known and what is uncertain, the uncertainties Δ_{11}, Δ_{21}, and Δ_{22} are pulled out and placed inside a matrix block. The computation of G_{ro}, needed for the computation of the cost function J_μ, is done with the Matlab program $sysic$, which is a simple linear system interconnection program, writing the loop equations of the interconnections.

3.3 Game solution

The solution of the game provides a Pareto-optimal set. The selection of a parameter set from the Pareto-optimal set is done with no predefined choice in this paper. For the solution of the game, it is primary necessary to satisfy all constraints and belonging to the Pareto-optimal set. The required decision maker, choosing a single parameter set from control theoretic view is still an open question.

Controllers were obtained, using the GA, where the parameter vector χ for the controllers are of the form

$$\chi = [K_{P1},\ K_{P1}/K_{TI1},\ K_{P2},\ K_{P2}/K_{TI2}], \qquad (23)$$

with proportional (K_{Pi}) and integral (K_{Pi}/K_{TIi}) parameters. The controller parameters are listed in Table 1. Games (A) and (B) are results obtained in Wellenreuther et al. [2007], where only J_1, and J_2 were optimized (those for the error signals) subject to predefined constraints on the control signals. In contrast, during the course of games (C) and (D), the cost J_μ is considered.

To be able to determine a possible relationship between constraint settings on the control signals and how robustly stable the final system is, the constraints for games (A) and (C) were chosen to be larger ($u_{limit_i} = 2 \cdot u_{i_{set}}$) than those for games (B) and (D) with $u_{limit_i} = 0.1 \cdot u_{i_{set}}$, subject to $u_{i_{set}}$, the corresponding control signals u_i to the set points of y_i.

Table 1. Controller and optimization parameters

	K_{P1}	K_{TI1}	K_{P2}	K_{TI2}
$Game(A)$	425	0.03993	-0.48898	0.49514
$Game(B)$	501.78	0.04303	-0.071875	4.12175
$Game(C)$	450.74	0.14631	-9.156	0.025833
$Game(D)$	450.77	0.155144	-1.1444	0.003107

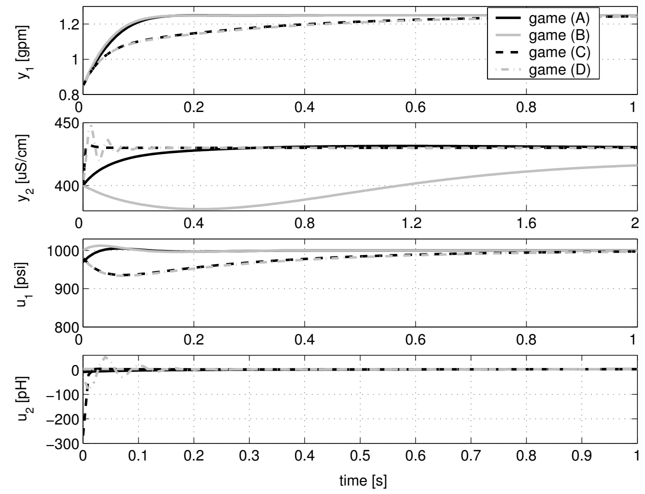

Figure 3. Responses to changes in the permeate flux y_1 and the conductivity y_2 for games $(A)-(D)$ of the nominal model

4. SIMULATION RESULTS AND COMPARISON

The operating point of the plant is given by a permeate flux of $0.85[gpm]$ $(0.2m^3/h)$ and a conductivity of $400[\mu S/cm]$.

Fig.3 shows the responses for the outputs (flux and conductivity) and the control signals (pressure and pH) of the nominal system for the different games $(A) - (D)$ to a change in the set point of the flux, from 0.85 $[gpm]$ to 1.25 $[gpm]$, as well as a change in the set point of the conductivity from $400[\mu S/cm]$ to $430[\mu S/cm]$.

Concerning the responses of the flux (y_1), games (A) and (B) already reach the set point after 0.2 minutes, in contrast to games (C) and (D), reaching the set point not until the first minute. All responses for the conductivity (y_2), except for game (B), reach the set point within 0.4 minutes. Concerning the control signal amplitudes, those for games (A) and (B) show very similar behaviour. In the figure concerning the control signal amplitudes u_2 (pH), the difference between the larger constraint settings of game (C), accepting a large negative overshoot, and the narrower constraint setting of game (D) is traceable.

To take into account the corresponding values for the cost functions, especially for the robust stability indicator J_μ, they are listed in Table 2.

An incorporation of the robust stability consideration leads to a cost function J_μ, which is in conflict with the cost functions J_1 and J_2. The values of the cost functions for the player concerning the upper loop, J_1, see Table 2, increase with the additional robust stability cost function J_μ, while the one for the lower loop J_2 decreases. So, a trade-off between all three conflicting cost functions has to be found with respect to the solution of the game.

Games (A) and (B) are not robustly stable at all, compare to J_μ in Table 2, since this property was not considered during their optimization process. However, games (C) and (D), where the parameters are obtained with the presented approach are

Table 2. Cost function values

	J_1	J_2	J_μ
$Game(A)$	0.0180	0.5701	2.0407
$Game(B)$	0.0155	15.3980	11.3822
$Game(C)$	0.048526	0.00057632	0.51452
$Game(D)$	0.051118	0.00041084	0.82884

robustly stable, but for different families of models, depending on the size of the structured singular value μ. For larger constraint settings (game (C)), the resulting control system is more robustly stable compared to smaller constraint settings (game (D)). The worth of the cost concerning J_2 for game (C) degrades about 40 percent compared to game (D) while it is more robustly stable. But the worth of the cost J_1 for game (C) improves only 5 percent compared to game (D). According to Skogestad and Postlethwaite [1996], stability is guaranteed for all perturbations with appropriate structure, and $max\ \sigma\left[\Delta(j\omega)\right] \leq \frac{1}{\mu_{game}}$. For the single games this yields to

$$\frac{1}{\mu_A} \approx 0.49, \quad \frac{1}{\mu B} \approx 0.088$$

and

$$\frac{1}{\mu_C} \approx 1.2065, \quad \frac{1}{\mu_D} \approx 1.943559.$$

If the admissible size of perturbation is exceeded, the stability of the system cannot be guaranteed anymore.

In the following, the RO model is changed in the domain of the different perturbation (uncertainty) sizes in order to see which parameter sets perform better for the whole family of models under the assumption that the perturbations are with appropriate structure. The four different perturbations are, in dependence on $1/\mu_A$, $1/\mu_B$, $1/\mu_C$ and $1/\mu_D$, of the following size and form, where Δ is a block-diagonal matrix:

$$\Delta = diag(\Delta_i)$$

for $i = 1, ..., 4$ with

$$|\Delta_1| = 0.1, \ \ |\Delta_2| = 0.5, \ \ |\Delta_3| = 1.5and \ \ |\Delta_4| = 2.0.$$

The perturbed systems are simulated according to a change in the set point of the permeate flux and a change in the set point of the permeate conductivity with the same sizes as with the nominal system. The effects of the perturbations are shown for all games, but only for the second output y_2, the conductivity. Due to the triangular control structure, the system gets unstable first in the lower control loop concerning the conductivity if the perturbations are too large.

Fig.4 shows the step responses for all games (A)-(D). Game (B), the one with the highest cost function value concerning the robust stability, leads to an unstable closed loop system for the family of models around the nominal system and a perturbation of Δ_1. The step response of game (A) shows a larger and longer overshoot than the nominal system case, but it is still stable. In Fig.5, the representation of game (B) was neglected, since $|\Delta_1| > |\Delta_2|$ and therefore unstable in any case. Game (A) is unstable for a maximum perturbation of size Δ_2. The step responses of game (C) and (D) remain comparatively unchanged due to the extension of the perturbation size from Δ_1 to Δ_2 (compare Fig.4 with Fig.5).

An enlargement of the perturbation from Δ_2 to Δ_3 results in instability in the step responses of game (D), as shown in Fig.6. Finally, Fig.7 shows, that for a perturbation with structure and size of Δ_4, larger than $\frac{1}{\mu_C}$, the system is unstable, too.

Comparing all games with respect to robust performance, the robust stability indicator J_μ is smaller for all games with larger constraints than for games with smaller constraints. The system with the parameters of game (C) and the larger constraint range accepts a larger perturbation Δ before it becomes unstable than the system with the parameters of game (D).

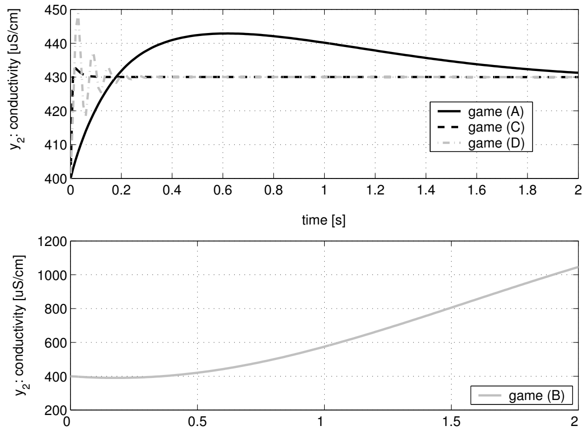

Figure 4. Responses to changes in the permeate flux y_1 and the conductivity y_2 for games $(A) - (D)$ and the perturbation Δ_1

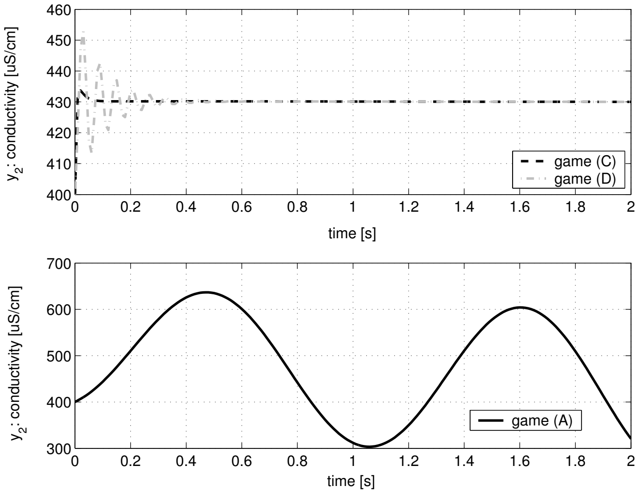

Figure 5. Responses to changes in the permeate flux y_1 and the conductivity y_2 for games (A),(C) and (D) and the perturbation Δ_2

5. CONCLUSIONS

A robust stability consideration, formulated as cost function, was included in an optimal controller parameter tuning method for multi-loop structures in a game-theoretic framework. The presented control design was applied successfully to a reverse osmosis desalination plant. Simulation studies show, that different constraint ranges for the control signals lead to an acceptance of different sizes of block diagonal perturbations (uncertainties) Δ. The conflict between the constrained strategy sets and the robust stability is becoming apparent. Narrower constraints allow only smaller perturbations for robust stability and the other way around.

Although in this work, only parameter uncertainties were considered, it is also possible to include uncertainties caused by unmodelled dynamics.

The fact, that the computational cost of the method increases, in adding the robust stability analysis, is negligible since the controller parameters are tuned offline.

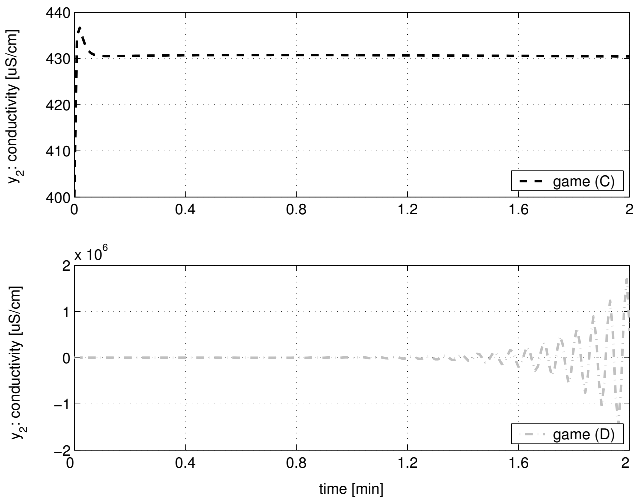

Figure 6. Responses to changes in the permeate flux y_1 and the conductivity y_2 for games (C) and (D) and the perturbation Δ_3

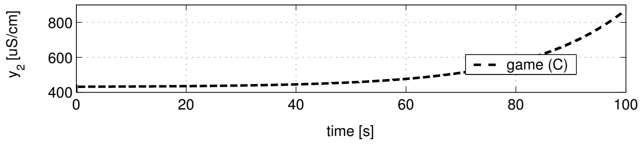

Figure 7. Responses to changes in the permeate flux y_1 and the conductivity y_2 for game (C) and the perturbation Δ_4

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