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A Fast Iterative Learning Control Approach to Rejection of Periodic Disturbances

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Abstract:
Like the well-known RC (Repetitive Control) and AFC (Adaptive Feedforward Cancellation) methods, the new iterative learning control (ILC) method developed recently for the rejection of periodic disturbances also utilize the steady-state behavior of the closed-loop system and requires a settling time + some periods for the satisfactory rejection of periodic disturbances. However, in case of the new ILC method, an iterative update scheme is needed to achieve successful rejection of periodic disturbances ever in the presence of aperiodic disturbances and plant uncertainties. In this paper, we introduce the concept of dwell time instead of settling time to accelerate the rejection speed of the new ILC method. The effectiveness and practicality of the proposed method is demonstrated through mathematical performance analysis as well as various simulation results.

Keywords: Disturbance rejection; Periodic disturbance; Iterative learning control; Dwell time.

1. INTRODUCTION

Many engineering applications often face unavoidable disturbances which can degrade system performance seriously. In particular, periodic disturbances are inherent in rotating machinery. For an example, in a data storage system, there are periodic disturbances due to the eccentricity of tracks on a disk. This periodic disturbance generally occurs at frequencies that are integer multiples of the frequency of disk rotation and can be a considerable source of tracking error. Since the fundamental period of the periodic disturbances is known, much control effort has been usually expended to compensate for these periodic disturbances.

Recently, numerous control design methods have been developed specifically for eliminating periodic disturbances. Generally these methods generate the control input, whereby the system asymptotically tracks the periodic disturbance in the output. One of them is the RC (Repetitive Control) mehtod which is based on the well-known internal model principle Hara et al. [1988]. It was applied to practical disk drive systems and has proved its usefulness in improvement of tracking performance as well as elimination of harmonic components in spectrum of position error Kempf et al. [1993], Onuki et al. [2001], Fujimoto et al. [2004]. However, it is commonly known that the RC method usually tends to amplify the effect of non-repeatable disturbances whose frequencies are located near those of periodic disturbances. Therefore, design engineers should consider some tradeoff between system stability and tracking performance as an important factor Moon et al. [1998], Doh et al. [2006].

On the other hand, the AFC (Adaptive Feedforward Concellation) method considered in Messner et al. [1994]-Zhang et al. [1997] simply rejects sinusoidal disturbances at the input of the plant by adding the negative of their values at all times. It is shown in Bodson et al. [1994] that it is equivalent in some sense to the well-known internal model principle. So one can design an AFC controller easily by using the well-developed linear control theory. Unfortunately, the AFC method requires intensive computation when rejecting periodic disturbances with many harmonic components. Also, it need be designed carefully, considering some tradeoff between convergence rate and system stability.

Some ILC (Iterative Learning Control) methods to periodic disturbance rejection were recently proposed in Takaishi [2006], Ha et al. [2005], and Sidman [1991] for a class of linear systems and in Han et al. [1998]-Kim et al. [2000] for a class of nonlinear systems. Different from the conventional ILC methods Arimoto et al. [1985]-Moore [1993], these new ILC methods do not require the resetting of the initial conditions at each iteration. Like the AFC method, these new ILC methods also estimate the magnitude and phase of the harmonic components but the estimation carries out off-line at every fundamental period. So they do not raise such destabilization problem involved inevitably in the actual implementation of the RC and AFC methods. Furthermore, they are less sensitive to the effect of noise and other aperiodic disturbances. As will be shown in Section 3, the RC and AFC methods as well as the new ILC methods also need a settling time + one period for the satisfactory rejection of the periodic disturbances. Different from the RC and AFC methods, however, these new ILC methods have to perform some iterative update schemes in order to reject the periodic disturbances successfully even in the presence of aperiodic disturbances and plant uncertainties. In the paper, we introduce the concept of dwell time instead of settling time. Thereby, we attempt to accelerate the disturbance rejection speed of these new ILC methods in such nonideal environment. The effectiveness and practicality of the proposed method is demonstrated through mathematical performance analysis as well as various simulation results.

2. MAIN RESULTS

The proposed learning control scheme is depicted in Fig. 1, where $K_c(s)$ is a stabilizing controller designed so as to stabilize the closed-loop system and to satisfy design specifications. We assume that there are two classes of bounded disturbances : (i) specific periodic disturbance $d_P(t)$ to reject and (ii) other periodic and aperiodic disturbances $d_N(t)$. We also assume that the plant is a linear system with Laplace transfer function $P(s)$. And $\hat{d}_P(t)$ is the estimate of $d_P(t)$ estimated off-line and is used as the feedforward term which is updated once at each iteration rather than continuously in time.

Since $d_P(t)$ is periodic with a known period T, it can be expressed as the following Fourier series representation :

$$d_P(t) = \sum_{i=1}^{N} Re\left[c_i e^{-j\omega_i t}\right] \quad (1)$$

where $\omega_i \triangleq 2\pi n_i/T$ and $c_i \in \mathbb{C}$, $i = 1, 2, \cdots, N$. Here, N and the n_i are nonnegative integers. In this context, we can also express $\hat{d}_P(t)$ as follows :

$$\hat{d}_P(t) = \hat{d}_P^{\,k}(t) \triangleq \sum_{i=1}^{N} Re\left[\hat{c}_{i,k} e^{-j\omega_i t}\right], \quad \text{if } t \in [t_k, t_{k+1}), k = 0, 1, 2, \cdots \quad (2)$$

where $t_0 \triangleq 0$ and $\hat{c}_{i,k} \in \mathbb{C}$, $i = 1, 2, \cdots, N$. Let

$$\Delta\hat{d}_P^{\,l}(t) \triangleq \sum_{i=1}^{N} Re\left[\Delta\hat{c}_{i,l} e^{-j\omega_i t}\right] v(t - t_l), \quad l = 0, 1, 2, \cdots, \quad (3)$$

$$v(t) \triangleq \begin{cases} 1, & t > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where $\Delta\hat{c}_{i,l} \triangleq \hat{c}_{i,l} - \hat{c}_{i,l-1}$, $l = 0, 1, 2, \cdots$, but $\hat{c}_{i,-1} \triangleq c_i$. Then, $\hat{d}(t)$ can be written in the following form :

$$\hat{d}_P(t) = \sum_{l=0}^{\infty} \Delta\hat{d}_P^{\,l}(t). \quad (5)$$

On the other hand, from Fig. 1 and (5), we can easily derive the following expression of the position error $e(t)$:

$$E(s) = S(s)R(s) + H(s)D_N(s) - \sum_{l=0}^{\infty} H(s)\Delta\hat{D}_P^{\,l}(s) \quad (6)$$

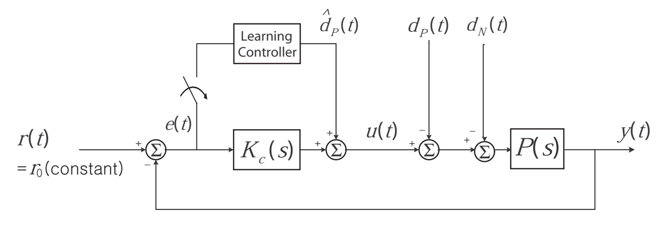

Fig. 1. Blockdiagram representation of the proposed learning control scheme

where $S(s) \triangleq \frac{1}{1+P(s)K_c(s)}$, $H(s) \triangleq \frac{P(s)}{1+P(s)K_c(s)}$, $E(s) \triangleq \mathcal{L}[e(t)]$, $R(s) \triangleq \mathcal{L}[r(t)]$, $D_N(s) \triangleq \mathcal{L}[d_N(t)]$, and $\Delta\hat{D}_P^{\,l}(s) \triangleq \mathcal{L}[\Delta\hat{d}_P^{\,l}(t)]$. We denote the nominal value of $H(j\omega)$ by $\hat{H}(j\omega)$. Also, we define a kind of periodic impulse response $h_N^{-1}(t)$ by

$$h_N^{-1}(t) \triangleq \frac{2}{T}\sum_{i=1}^{N} Re\left[\frac{e^{-j\omega_i t}}{H^*(j\omega_i)}\right]. \quad (7)$$

We also define

$$\gamma_N^{-1} \triangleq \sup_{0\le t\le T} |h_N^{-1}(t)|. \quad (8)$$

Now, we are ready to describe precisely the update scheme of our off-line learning control algorithm. In what follows, the subscript k is used to denote the iteration number and z^* denotes the conjugate of a complex number z.

Step 1) Set $k = 0$, $t = 0$ and $\hat{d}_P(t) = \hat{d}_P^{\,0}(t)$ (initial guess).

Step 2) Wait for some dwell time τ_k. Let $t_{k+1} \triangleq t_k + \tau_k + T$.

Step 3) Save the time history of the position error $e(t)$ on the time interval $[t_k + \tau_k, t_{k+1}]$ by letting $e_{ss}^{\,k}(\bar{t}) \triangleq e(\bar{t}+t_k+\tau_k)$ for each $\bar{t} \in [0, T]$. Then, determine $\hat{d}_P^{k+1}(t)$ by

$$\hat{d}_P^{k+1}(t) \triangleq \hat{d}_P^{\,k}(t) + \frac{1}{k+1}\int_0^T e_{ss}^{\,k}(\tau)\hat{h}_N^{-1}(t-\tau)d\tau. \quad (9)$$

Step 4) At $t = t_{k+1}$, set $\hat{d}_P(t) = \hat{d}_P^{k+1}(t)$ and increase k by one. Then jump to Step 2.

For better understanding of the update scheme stated above, we depict its timing diagram in Fig. 2. The kth estimate $\hat{d}_P^{\,k}(t)$ is determined by the $(k-1)$th estimate $\hat{d}_P^{k-1}(t)$ and the steady-state response of the closed-loop system with the feedforward term $\hat{d}_P^{k-1}(t)$. At the kth update time t_k, we then wait for the dwell time τ_k after which the learning systems is ready for data acquisition.

For further discussion on the proper choice of the dwell times $\tau_k, k = 0, 1, \cdots$, we now investigate the convergence performance of the proposed algorithm. For this aim, we need the following Lemma 1.

Lemma 1. The position error $e(t)$ can be divided into three components: steady-state response, transient response, and aperiodic-disturbance response as follows:

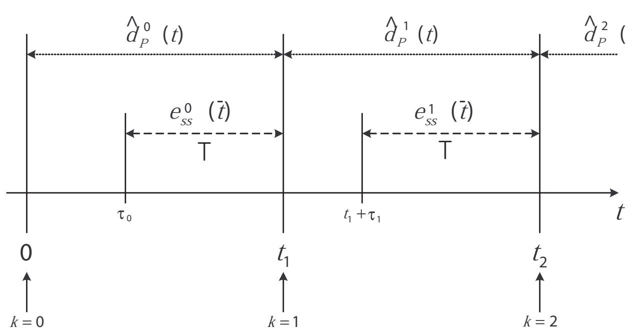

Fig. 2. Timing diagram of the proposed update scheme

$$e(t) = e_{ss}(t) + e_{tr}(t) + v_N(t) \tag{10}$$

where

$$\begin{aligned} e_{ss}(t) \triangleq & \int_0^\infty s(\tau) r(t-\tau) d\tau \\ & - \sum_{l=0}^{\infty} \left[\sum_{i=1}^{N} Re \left[H^*(j\omega_i) \Delta \hat{c}_{i,l} e^{-j\omega_i t} \right] \right] v(t - t_l), \end{aligned} \tag{11}$$

$$\begin{aligned} e_{tr}(t) \triangleq & - \int_t^\infty s(\tau) r(t-\tau) d\tau \\ & + \sum_{l=0}^{\infty} \left[v(t-t_l) \int_{t-t_l}^{\infty} h(\tau) \Delta \hat{d}_P^{\,l}(t-\tau) d\tau \right], \end{aligned} \tag{12}$$

$$v_N(t) \triangleq \int_0^t h(\tau) d_N(t-\tau) d\tau. \tag{13}$$

□

We also need to make the following assumptions.

A1) All poles of $S(s)$ and $H(s)$ are in the left-half complex plane and $S(0) = 0$.

A2) The learning system is initially relaxed at $t = 0$.

By A1), we see that the impulse responses $s(t) \triangleq \mathcal{L}^{-1}[S(s)]$, $h(t) \triangleq \mathcal{L}^{-1}[H(s)]$ satisfy the following inequalities for some positive constants $\alpha_s, \alpha_h, \beta_s, \beta_h$:

$$|s(t)| \le \alpha_s e^{-\beta_s t}, |h(t)| \le \alpha_h e^{-\beta_h t}, t \ge 0. \tag{14}$$

Now, we are ready to state our main result.

Theorem 1. Further, suppose that (i) the time average of $v_N e^{j\omega_i t}$ is zero, $i = 1, 2, \cdots, N$. (ii) there is no modelling uncertainty such as $\hat{H}(j\omega_i) = H(j\omega_i)$, $i = 1, 2, \cdots, N$, and (iii) the dwell times τ_k, $k = 0, 1, \cdots$ are chosen so as to satisfy

$$\tau_k \ge \begin{cases} \max\left\{ \dfrac{1}{\beta_s} \ln \dfrac{r_0 \alpha_s}{{\beta_s}^2 \epsilon_k}, \dfrac{1}{\beta_h} \ln \dfrac{\delta_0 \alpha_h}{{\beta_h}^2 \epsilon_k}, 0 \right\}, & \text{if } k = 0 \\ \max\left\{ \dfrac{1}{\beta_h} \ln \dfrac{\delta_k \alpha_h}{{\beta_h}^2 \epsilon_k}, 0 \right\}, & \text{otherwise} \end{cases} \tag{15}$$

where ϵ is a positive constant and

$$\delta_k \triangleq \sup_{0 \le t \le T} |\Delta \hat{d}_P^{\,k}(t)|, \tag{16}$$

$$\epsilon_k \triangleq \frac{\epsilon}{1 + e^{-\beta_s k T} - e^{-\beta_s (k+1) T} - e^{-\beta_h (k+1) T}}. \tag{17}$$

Then, the update scheme described by Steps 1) - 4) assures that

$$\lim_{k \to \infty} |\hat{c}_{i,k} - c_i| \le \frac{2\epsilon}{|H(j\omega_i)| T}, \quad i = 1, 2, \cdots, N, \tag{18}$$

$$\lim_{k \to \infty} \sup_{t_k \le t \le t_{k+1}} |d_P(t) - \hat{d}_P^{\,k}(t)| \le \gamma_N^{-1} \epsilon, \tag{19}$$

$$\begin{aligned} \lim_{k \to \infty} \sup_{t_k + \tau_k \le t \le t_{k+1}} |e(t)| \le & \ \epsilon \left[\frac{2N}{T} + \frac{\beta_h}{1 - e^{-\beta_h T}} \right] \\ & + \lim_{k \to \infty} \sup_{t_k + \tau_k \le t \le t_{k+1}} |v_N(t)|. \end{aligned} \tag{20}$$

□

The proofs of Lemma 1 and Theorem 1 are omitted because of limited space.

Note from (15) that larger difference between $\hat{d}_P^{k-1}(t)$ and $\hat{d}_P^{\,k}(t)$ requires the next kth dwell time to be chosen larger. Thus, the dwell time should be the minimum time after which the transient response of position error becomes sufficiently small. When the information of the parameters $\delta_0, \alpha_s, \alpha_h, \beta_s, \beta_h$ is not available, the dwell time can be chosen simply as the settling time, say, t_s of the closed-loop system. In particular when $d_N(t) = 0$, $t \ge 0$, then the estimation error $d_P(t) - \hat{d}_P(t)$ is reduced near zero after $t = \tau_0 + T$, as is implied in the proof of Theorem 1. However, the iteration number k more than 1 is usually needed in the presence of the aperiodic disturbances d_N. Also, note from Theorem 1 that the effect of the aperiodic disturbance on the estimation error tends to disappear as $k \to \infty$. This quite desirable feature is due to our unique choice of the update gain as $\frac{1}{k+1}$ in (9).

Finally, we address the issue of practical implementation of the proposed iterative learning control algorithm. Let T_s be a sampling time. For computational simplicity, we assume that $T = MT_s$ for a positive integer and $M >> 1$. Then, we can show that the iteration equation in (9) can be well approximated by

$$\begin{aligned} \hat{d}_P^{k+1}(nT_s) = & \ \hat{d}_P^{\,k}(nT_s) \\ & + \frac{1}{2(k+1)} \sum_{l=0}^{M-1} \left[e_{ss}^{\,k}(lT_s) h_N^{-1}((n-l)T_s) \right. \\ & \left. + e_{ss}^{\,k}((l+1)T_s) h_N^{-1}((n-l-1)T_s) \right], \\ & n = 0, 1, \cdots. \end{aligned} \tag{21}$$

Thus, the proposed learning control algorithm can be converted into the form of a FIR filter, where $h_N^{-1}(mT_s), m = 0, 1, \cdots, (M-1)$ can be pre-computed and stored in the microprocessor memory.

3. SIMULATION RESULTS

In this section, we present some simulation results to illuminate further the effectiveness of the proposed learning control method. For fair comparison of the proposed ILC method with the previously known RC and AFC methods, we have assumed in our simulation that $P(s)$ and $K_c(s)$ are given by

$$P(s) = \frac{75}{s^2 + 60s + 150000}, \tag{22}$$

$$K_c(s) = 5.4 \times 10^6 \frac{3.87(s + 1364)(s + 9425)}{(s + 942)(s + 87965)}, \tag{23}$$

as in the DVD-ROM system considered in [Doh et al., 2006].

Then, the 98% settling time of the time responses of position error to a step disturbance input is $t_s = 0.0024s$. Other data used in simulation are as follows :

$$\delta_0 = 0.05 \ , \ \hat{d}_P^{\,0}(t) = 0 \ , \ T = 0.025s \ , \ N = 3 \ , \ n_i = i \ , \tag{24}$$

$$c_1 = 0.03-0.01j\,,\; c_2 = 0.015+0.006j\,,\; c_3 = -0.003+0.002j. \tag{25}$$

Based on the simulation data, we can calculate or choose the following values:

$$\beta_s = 10200s^{-1}\,,\; \alpha_s = 24000\,,\; \beta_h = 7600s^{-1}\,,\; \alpha_h = 0.0058\,. \tag{26}$$

In the following simulation results, all of three methods were turned on equally at the third cycle (i.e., $t_0 = 0.075$s, as indicated by $\triangle$ in Fig. 3 - Fig. 8).

Observe from Fig. 3, 4, and 5 that in the ideal case (no plant uncertainty and no noise), the RC and AFC method as well as the proposed ILC method need at least $(t_s + T)$ time (=0.1024s here) in order to reject the periodic disturbances effectively. This is mainly because all of three methods get the information of the periodic disturbance basically from the steady-state responses of the closed-loop system. Particularly, in the case of the RC method, some tracking error still exists even after $(t_s + T)$ time. This is due to the low-pass filter needed for the stability of the closed-loop system.

Next, we consider the case that $d_N(t)$ is the white noise and there is no plant uncertainty. Comparing the simulation results in Fig. 6 and 7 with the one in Fig. 8, the RC and AFC methods are more sensitive to the white noise. This is mainly because the RC and AFC methods usually destabilize the closed loop system. It also can be observed from Figs. 8 that as the iteration number is increased, the estimation errors converge to zero even in the presence of $d_N(t)$, while the position error does not due to the effect of $d_N(t)$. This results can be explained well by Theorem 1.

4. CONCLUSION

We have presented a learning control method, in which the update time is adjusted adaptively at each iteration. Its excellent estimation performance has been demonstrated by both mathematical analysis and simulation results. However, we have considered only the case of no plant uncertainty. Nonetheless, it can be shown with much more complicated mathematical analysis that if $|1 - H(j\omega_i)/\hat{H}(j\omega_i)| < 1, i = 1, 2, \cdots, N$, the proposed method still can provide good estimation performance, although the iteration number k need be increased more than 1, as in the presence of $d_N(t)$. On the other hand, the estimation speed of the RC and AFC methods are not affected by plant uncertainty. Nonetheless, the RC and AFC methods can destabilize the closed-loop system. Therefore, the stabilizing controller $K_c(s)$ has to be used mainly to restabilize the destabilized closed-loop system. In fact, the situation becomes worse in the case of lightly damped processes. In contrast, the proposed ILC method does not destabilize the closed-loop system and hence $K_c(s)$ can be used to achieve better control performance such as fast settling time for lightly damped processes or effective attenuation of aperiodic disturbances. Finally, it also should be noted that the proposed method for disturbance rejection can be directly extended with slight modification to asymptotic tracking.

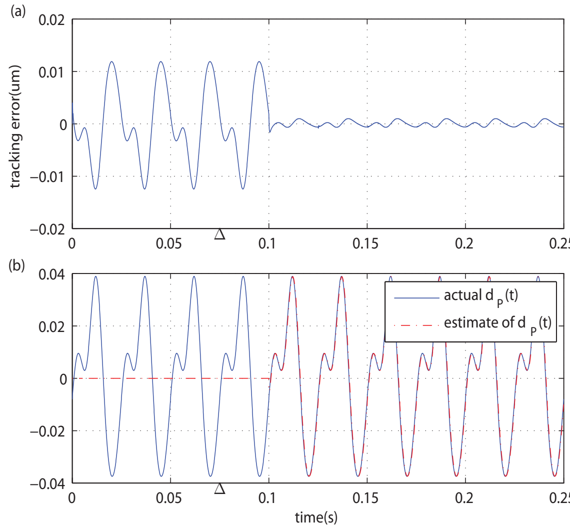

Fig. 3. Estimation performance of the repetitive control in case of $d_N(t) = 0$ and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimate $\hat{d}_P(t)$ and actual $d_P(t)$

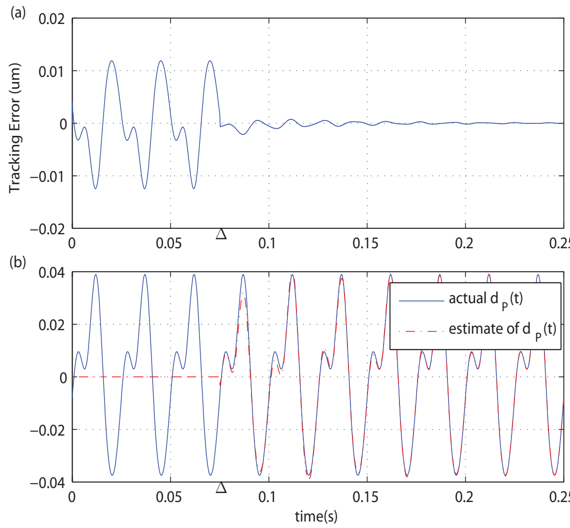

Fig. 4. Estimation performance of the AFC method in case of $d_N(t) = 0$ and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimate $\hat{d}_P(t)$ and actual $d_P(t)$

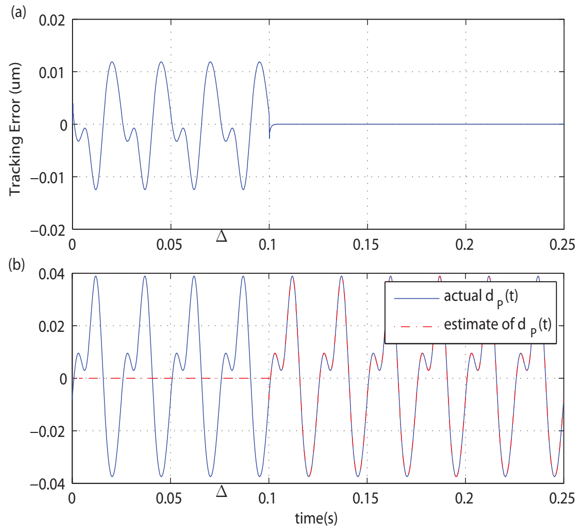

Fig. 5. Estimation performance of the proposed control in case of $d_N(t) = 0$ and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimate $\hat{d}_P(t)$ and actual $d_P(t)$

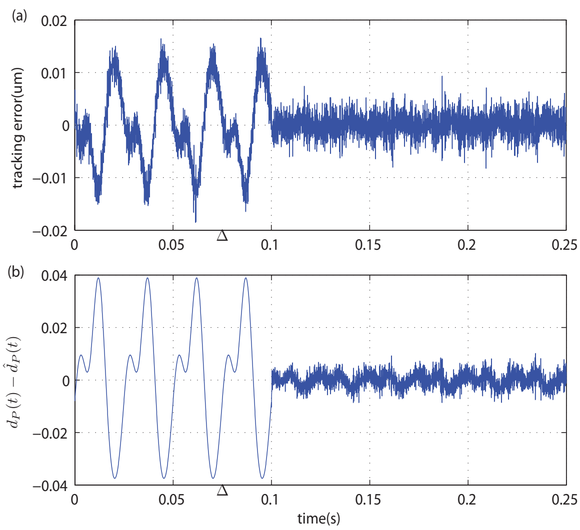

Fig. 6. Estimation performance of the repetitive control in case that $d_N(t)$ is white noise and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimation error $d_P(t) - \hat{d}_P(t)$

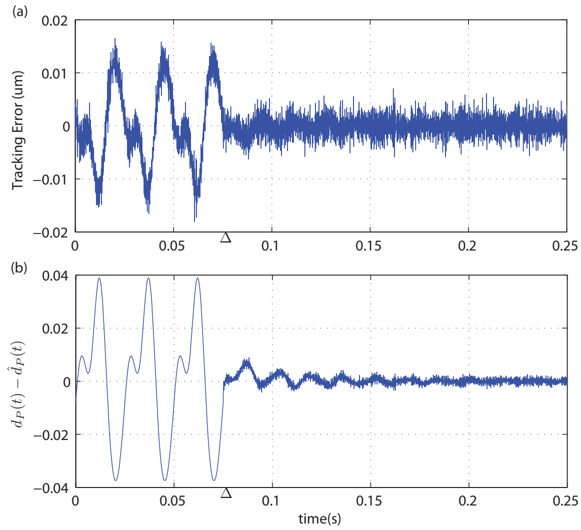

Fig. 7. Estimation performance of the AFC method in case that $d_N(t)$ is the white noise and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimation error $d_P(t) - \hat{d}_P(t)$

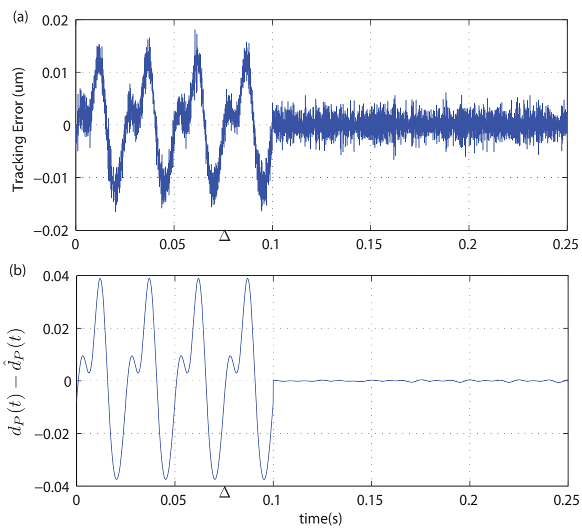

Fig. 8. Estimation performance of the proposed control in case that $d_N(t)$ is white noise and $\hat{P}(j\omega_i) = P(j\omega_i)$, $i = 1, 2, 3$. (a) time history of tracking error $e(t)$, (b) time history of estimation error $d_P(t) - \hat{d}_P(t)$

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