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Multi-objective preview control of active vehicle suspensions ⋆

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Abstract: This paper develops the LMI-based solution for the design of a discrete-time multi-objective preview controller, and considers it for an active vehicle suspension system. A quarter car model, which captures many features of real structures, is used in this study and hence the look-ahead preview control is considered. To provide ride comfort for a wide range of road irregularities, H_∞ norm is used as a comfort measure, while generalized H_2 is used to care for the constraints on suspension working space, tire deflection and actuator saturation. Moreover, to ensure desired stability margins for the feedback part of the system, pole location constraints are considered in the design. The effects of inclusion of preview information in control law on ride comfort, ride safety, working space and power requirements, for various road profiles, are examined. The results demonstrate the effectiveness of the preview-included multi-objective design to pure feedback scheme.

1. INTRODUCTION

A vehicle suspension system, apart from its static task of carrying body, has to provide as much comfort as possible for the passengers, while it achieves an allowable level for suspension stroke and tire deflection as measures of packaging and ride safety. These design requirements are highly conflicting, for example enhancing ride comfort calls for larger suspension stroke and smaller damping of wheel-hop mode (Chen, H. and Guo, K. [2005]) and hence leads to a degradation in ride safety. Therefore, the design of a vehicle suspension calls for a trade-off between these conflicting objectives. To manage this trade-off, many semi-active / active vehicle suspension have been proposed, which have improved system performance to a considerable extent (Hrovat, D. [1997]).

An interesting control scheme considered for active vehicle suspension design, is to include a feedforward term (or preview control) in feedback controller. This scheme involves the acquisition and use of information concerning the road profile ahead of vehicle to 'prepare' the system for oncoming disturbance (Hac, A. [1992]). This scheme promises more improvements in system performance, compared to its pure feedback counterpart. The Notion of preview since its emergence by Bender (Bender, E.K. [1968]) has been followed by many researchers. This scheme has been considered for both fully-active and slow-active suspension systems on quarter-, half- or full-car models and enhanced vertical vibration isolations have been reported for all cases, when road surface preview data contribute in control law. Interested readers are referred to (Tomizuka, M. [1976], Hac, A. [1992], Foag, W. [1990], Roh, H.S. and Park, Y. [1999], Marzbanrad, J., Ahmadi, G., Zohoor, H. and Hojjat, Y. [2004]) to see the main relations and results for fully-active systems and to (Pilbeam, C. and Sharp, R.S. [1993], Prokop, G. and Sharp, R.S. [1995]) for slow-active systems.

In one hand, as the best knowledge of the authors, all of the preview based active suspension systems reported in the literature, utilized LQ-based optimization approaches.
But in the other hand, controller design for a vehicle suspension is by its nature a multi-objective one (Abdellahi, E., Mehdi, D. and M'Saad, M. [2000]). It is well-known that ride comfort is judged by body (vertical/rotational) acceleration and calls for its minimization, whereas suspension stroke, tire deflection and control signal are required to be kept within allowed bounds rather than to be minimized. Multi-objective design (Scherer, C.W., Gahinet, P. and Chilali, M. [1997]) offers a very flexible and powerful design framework, in which control objectives are specified as different channels of the system and each channel is handled with an appropriate norm independently.
It has been shown (Wang, J. and Wilson, D. A. [2001], Sun, P.Y. and Chen, H. [2003], Gao, H., Lam, J. and Wang, C. [2006]) that this design scheme is more promising for active vehicle suspensions.
Therefore, the main goal of this paper is to investigate multi-objective preview controllers for vehicle suspension. As difference with the above multi-objective designs, where ride comfort is characterized by H_2-norm, in this study to provide ride comfort for a wide range of road irregularities, H_∞ norm is used as a comfort measure. Similarly generalized H_2 (GH_2) norm is used to care for the constraints on suspension working space, tire deflection and actuator saturation. All these objectives are more improved by utilizing preview information of road profile ahead of vehicle. Moreover, to ensure desired stability margins for the feedback part of the system, pole location constraints are considered in the design. Clearly, extension of multi-objective design to the preview included case is some part of this work.

To show the effectiveness of the proposed approach also a discrete-time pure feedback multi-objective controller is designed for the system, and the system performance using both controllers are compared.

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There are two ways to obtain preview information, one using a "look-ahead" sensor and the other by estimating road profile from the response of front wheel. In this paper a quarter car model, described in section 2, is used and therefore the look-ahead preview control is considered. Section 2 also describes the control problem and gives the framework for the design. section 3 derives LMI-based solution for discrete-time multi-objective preview design. This section is followed by the design of both above mentioned controllers. In section 4 some numerical simulation is carried out to compare both controllers and section 5 contains concluding remarks.

2. PROBLEM FORMULATION

A 2-DOF vehicle suspension representing a quarter-car model, shown in Fig. 1, will be used in this study. This model is widely used in the active vehicle suspension studies and captures major characteristics of a real suspension system.

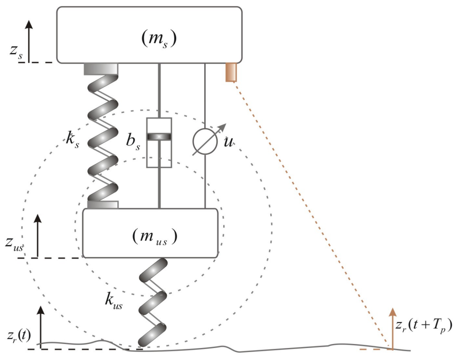

Fig. 1. active suspension system with preview

2.1 System Description

The nomenclature used and parameter values, taken from (Chen, H. and Guo, K. [2001]), are given in Table 1. According to the variables defined on figure 1, the equations governing the motions of sprung and unsprung mass are given by:

$$\begin{cases} m_s\ddot{z}_s + b_s(\dot{z}_s - \dot{z}_{us}) + k_s(z_s - z_{us}) = u, \\ m_{us}\ddot{z}_{us} + b_s(\dot{z}_{us} - \dot{z}_s) + k_s(z_{us} - z_s) + \\ \qquad k_{us}(z_{us} - z_r) = -u \end{cases}$$

Choosing the set of state variables as:

$$\begin{aligned} x_1(t) &= z_{us} - z_r, \\ x_2(t) &= \dot{z}_{us}, \\ x_3(t) &= z_s - z_{us}, \\ x_4(t) &= \dot{z}_s \end{aligned}$$

the state space description of the system is obtained as:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -k_{us}/m_{us} & -b_s/m_{us} & k_s/m_{us} & b_s/m_{us} \\ 0 & -1 & 0 & 1 \\ 0 & b_s/m_s & -k_s/m_s & -b_s/m_s \end{pmatrix} x(t) + \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} w(t) + \begin{pmatrix} 0 \\ -u_{max}/m_{us} \\ 0 \\ u_{max}/m_s \end{pmatrix} u(t)$$

where the control input u is defined as u_f/u_{max}, with u_{max} being the normalizing factor and u_f actuator real force., and $w = \dot{z}_r$ (ground vertical velocity) is considered as disturbance input.

Model parameters	symbol	values	unit
sprung mass	m_s	320	kg
suspension stiffness	k_s	18000	N/m
suspension damping rate	b_s	1000	N/(m/sec)
Wheel assembly mass	m_{us}	40	kg
tire stiffness	k_{us}	200000	N/m
max control signal	u_{max}	1000	N

Table 1. Nomenclature and parameter values in a quarter car model (Chen, H. and Guo, K. [2001])

2.2 Derivation of design framework

Generalized H_2 norm measures the peak amplitude of the output signal over all unit energy inputs

In this section control problem for above system is formulated and the design framework is obtained. This framework except for the inclusion of preview information and related blocks, will be used for pure feedback multi-objective design as well.
In designing the control law for a suspension system, the following requirements are taken into consideration:

(1) Ride Comfort: Ride comfort of a vehicle, also known as vibration isolation ability, is judged by the RMS value of the acceleration, sensed by vehicle passengers. This is a widely used measure for ride comfort. To design a system to perform satisfactorily for a wide range of road irregularities (not just white noises), calls for minimizing the H_∞ norm of the transfer function from road disturbance to body acceleration. Recall that H_∞ norm of a system is its worst case output energy (RMS value).
(2) Ride safety: Firm uninterrupted contact of wheels to road against road disturbances (good road holding) is necessary for vehicle handling and leads to ride safety. In practical vehicle system, there are many forces acting on the wheel that can lift it off the road. However as in (Chen, H. and Guo, K. [2005], Gordon, T. and Milsted, M. [1991]), we rely on the idea that for ride safety, the dynamic tire load should not exceed the static one, i.e.,
$$k_{us}(z_{us}(t) - z_r(t)) < (m_s + m_{us})g, \quad \forall t \geq 0$$
(3) Suspension deflection limit: Suspension systems are placed between the chassis and wheel assembly, hence structural features of a vehicle impose a hard limit on the suspension stroke. Hitting the deflection limit not only results in deterioration of ride comfort, but also even may cause structural damage. Thus it is important that the suspension stroke should not exceed a prespecified limit,
$$|z_s(t) - z_{us}(t)| < SS_{max}, \quad \forall t \geq 0$$
(4) Control signal: control signal is generated by hydraulic actuator and because of its saturation is bounded. it is assumed that normalized control signal is bounded as
$$|u(t)| < 1, \quad \forall t \geq 0$$

To achieve the best possible ride comfort, it is required to minimize RMS body acceleration, while suspension deflection, tire deflection and control signal are allowed to vary freely within their prespecified bounds. Therefore, we divide the controlled outputs to two parts of to-be-minimized (z_1) and to-be-constrained (z_2) as

$$z_1 = \ddot{z}_s$$

$$z_2 = \begin{pmatrix} \dfrac{z_s - z_{us}}{SS_{max}} \\ \\ \dfrac{k_{us}(z_{us} - z_r)}{(m_s + m_{us})g} \\ \\ u \end{pmatrix}$$

Remembering that GH_2 norm is defined as L_2-L_∞ induced norm (or 'energy to peak' norm) (Scherer, C.W. and Weiland, S. [2000])

$$\|T_2\|_{GH_2} := \|T_2\|_{2,\infty} := \sup_{0 \le w \le \infty} \frac{\|z_2\|_\infty}{\|w\|_2} \tag{1}$$

in order to constrain the outputs z_2 within their given bounds, GH_2 norm of closed loop system from disturbance to this outputs should be less than a positive scalar γ_2. This guarantees that output ∞ norm (its max.) not to exceed a given maximum. The design scheme is depicted in Figure 2. As it can be seen, road irregularities T_p time units ahead of t (T_p is referred to as preview time) also contribute in control signal. They construct the feedforward component of the control law, while the states of the suspension system form the feedback part.

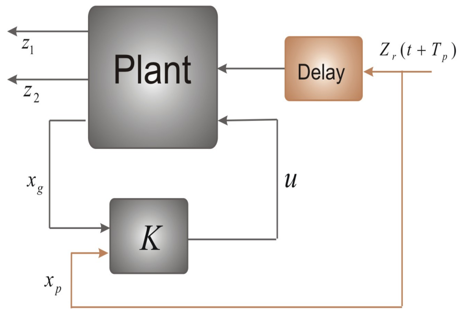

Fig. 2. Multi-objective preview design framework

3. CONTROLLER DESIGN

In this section we derive a multi-objective preview solution to the problem formulated in section II, based on LMI optimization.

For a generality, it is assumed that the plant to be controlled, $G(s)$, is described by the following discrete-time state-space realization:

$$\begin{aligned} x_g(k+1) &= A_g x_g(k) + B_{g1} w(k) + B_{g2} u(k), \\ z_1(k) &= C_{g1} x_g(k) + D_{g11} w(k) + D_{g12} u(k), \\ z_2(k) &= C_{g2} x_g(k) + D_{g21} w(k) + D_{g22} u(k) \end{aligned} \tag{2}$$

where k is a counter for the samples and denotes the time kT_s, with T_s being the sampling time. $x_g(k) \in \mathbb{R}^n$ is the state vector of the plant, $w(k) \in \mathbb{R}^{m_1}$ is exogenous input, $u(k) \in \mathbb{R}^{m_2}$ is the control input, $z_1(k) \in \mathbb{R}^{p_1}$ is the to-be-minimized output vector and $z_2(k) \in \mathbb{R}^{p_2}$ is the to-be-constrained output vector. Without loss of generality, in the following it is assumed that $m_1 = m_2 = 1$.

To design a multi-objective controller for the system depicted in Figure 2, the preview information and time delay between sensing and excitation should be absorbed by the plant. For this purpose in the following, as in (Tomizuka, M. [1976]), also used in (Prokop, G. and Sharp, R.S. [1995], Takaba [2003], Roh, H.S. and Park, Y. [1999]), a state-augmentation technique is used. Let $x_p(k)$ denote the vector which represents the preview information which is available for control, namely:

$$x_p(k) = \begin{pmatrix} w(k) \\ w(k+1) \\ \vdots \\ w(k+N_p) \end{pmatrix}$$

where $N_p = T_p/T_s$. It can be easily seen that:

$$x_p(k+1) = A_p x_p(k) + B_{p1} w(k + N_p + 1) \tag{3}$$

where

$$A_p = \left(\begin{array}{c|ccc} 0 & & & \\ \vdots & & \mathbf{I} & \\ 0 & & & \\ \hline 0 & 0 & \cdots & 0 \end{array}\right), B_{p1} = \left(\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline 1 \end{array}\right)$$

Considering the augmented state vector as:

$$x(k) = \begin{pmatrix} x_g(k) \\ x_p(k) \end{pmatrix},$$

the equations for the augmented system, by combinig (2) and (3) is obtained as:

$$\begin{aligned} x(k+1) &= Ax(k) + B_1 w(k+N_p+1) + B_2 u(k), \\ \begin{pmatrix} z_1(k) \\ z_2(k) \end{pmatrix} &= Cx(k) + D \begin{pmatrix} w(k+N_p+1) \\ u(k) \end{pmatrix} \end{aligned} \tag{4}$$

where

$$A = \left(\begin{array}{c|cc} A_g & B_{g1} & \mathbf{0} \\ \mathbf{0} & & A_p \end{array}\right), B_1 = \begin{pmatrix} \mathbf{0} \\ B_{p1} \end{pmatrix}, B_2 = \begin{pmatrix} B_{g2} \\ \mathbf{0} \end{pmatrix},$$

$$C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \left(\begin{array}{c|cc} C_{g1} & D_{g11} & \mathbf{0} \\ C_{g2} & D_{g21} & \mathbf{0} \end{array}\right),$$

$$D = \left(\begin{array}{c|c} D_{11} & D_{12} \\ D_{21} & D_{22} \end{array}\right) = \left(\begin{array}{c|c} \mathbf{0} & D_{g12} \\ \mathbf{0} & D_{g22} \end{array}\right)$$

Now the control problem is to find a controller such that the H_∞ norm of the closed loop system from $w(k + N_p + 1)$ to $z_1(k)$ is minimized, while GH_2 norm from $w(k + N_p + 1)$ to $z_2(k)$ is kept less a prespecified positive number γ_2. Henceforth, we deal with a discrete-time multi-objective control problem and what follows is in fact an extension of the (Chilali, M. and Pascal, G. [1996], Scherer, C.W., Gahinet, P. and Chilali, M. [1997]) to discrete-time case with preview. Consider again the system described by (4) and let the controller K to be designed is represented by:

$$u(k) = Kx(k)$$

Then closed loop system has a state space realization with the following matrices:

$$\begin{aligned} \mathcal{A} &= A + B_2 K \\ \mathcal{B} &= B_1 \\ \mathcal{C}_1 &= C_1 + D_{12}K, \mathcal{D}_1 = D_{11} \\ \mathcal{C}_2 &= C_2 + D_{22}K, \mathcal{D}_2 = D_{21} \end{aligned} \tag{5}$$

Let us denote the channels from disturbance to the output z_1 and to the output z_2 as T_1 and T_2 respectively.

- H_∞ control of channel T_1

It is known [from (Zhou, K., Khargonekar, P. P., Stoustrup, J. and Niemann, H. H. [1995])with slight modification] that above discrete time system is quadratically stable and $\|T_1(z)\|_\infty < \gamma_1$ if and only if there exists some P such that:

$$P \succ 0$$

$$\begin{pmatrix} P & 0 & \mathcal{A}P & \mathcal{B} \\ * & \gamma_1 I & \mathcal{C}_1 P & \mathcal{D}_1 \\ * & * & P & 0 \\ * & * & * & \gamma_1 I \end{pmatrix} \succ 0 \tag{6}$$

where * represents the transpose of the elements across the diagonal.

- GH_2 control of channel T_2

GH_2 norm, given by 1, for a discrete time system satisfies:

$$\|T_2\|_{GH_2} = \lambda_{max}^{1/2} \left\{ \frac{1}{2\pi} \int_0^{2\pi} T_2(e^{j\theta}) T_2^*(e^{j\theta}) d\theta \right\}$$

It is well-known that this norm can be computed as (Note that in the case of preview $\mathcal{D}_2 = 0$ and it is dropped in the relation)

$$\|T_2\|_{GH_2} = \lambda_{max}^{1/2} (\mathcal{C}_2 P_0 \mathcal{C}_2)^T$$

where P is the solution of Lyapunov equation

$$P_0 = \mathcal{A} P_0 \mathcal{A}^T + \mathcal{B}\mathcal{B}^T$$

It can be readily verified that $\|T_2\|_{GH_2} < \gamma_2$ if and only if there exists a symmetric matrix P such that

$$\begin{aligned} &\lambda_{max}^{1/2} (\mathcal{C}_2 P \mathcal{C}_2^T) < \gamma_2, \\ &P - \mathcal{A} P \mathcal{A}^T + \mathcal{B}\mathcal{B}^T > 0 \end{aligned}$$

These inequalities can be restated as the following LMI's

$$\begin{aligned} &\begin{pmatrix} P & \mathcal{A}P & \mathcal{B} \\ P\mathcal{A}^T & P & 0 \\ \mathcal{B}^T & 0 & I \end{pmatrix} \succ 0 \\ &\begin{pmatrix} \gamma_2 I & \mathcal{C}_2 P \\ P\mathcal{C}_2^T & P \end{pmatrix} \succ 0 \end{aligned} \tag{7}$$

As usual in multi-objective design framework (Scherer, C.W., Gahinet, P. and Chilali, M. [1997]), we have used the same decision matrix P of H_∞ design for the above constraint.

- Pole location constraint

In order to ensure desired stability margins for the feedback part of the system, the poles of the closed loop system are constrained to be inside the disc with radius $\rho = exp(-\alpha T_s)$ centered at the origin. This ensures a minimum damping coefficient of α. All eigenvalues of $\mathcal{A}_{fb}$ lie in a disk with radius ρ centered at the origin if and only if there exists a matrix P_{fb} satisfying (Chilali, M. and Pascal, G. [1996])

$$\begin{pmatrix} -\rho P_{fb} & \mathcal{A}_{fb} P_{fb} \\ * & -\rho P_{fb} \end{pmatrix} \prec 0 \tag{8}$$

where $\mathcal{A}_{fb}$ and P_{fb} denote the part of matrices $\mathcal{A}$ and P respectively which correspond to the feedback.

After substitution of calligraphic matrices in the above LMI's (6-8) with their values of (5) and change of matrix variables as:

$$Y = KP \tag{9}$$

the above inequalities are satisfied if and only if there exist $P = P^T \succ 0$ (, P_{fb}) and Y (, Y_{fb}) such that:

$$P \succ 0 \tag{10a}$$

$$\begin{pmatrix} P & 0 & AP + B_2 Y & B_1 \\ * & \gamma_1 I & C_1 P + D_{12} Y & D_{11} \\ * & * & P & 0 \\ * & * & * & \gamma_1 I \end{pmatrix} \succ 0 \tag{10b}$$

$$\begin{pmatrix} P & AP + B_2 Y & B_1 \\ * & P & 0 \\ * & 0 & I \end{pmatrix} \succ 0 \tag{10c}$$

$$\begin{pmatrix} \gamma_2 I & C_2 P + D_{22} Y \\ * & P \end{pmatrix} \succ 0 \tag{10d}$$

$$\begin{pmatrix} -\rho P_{fb} & A_g P_{fb} + B_{g2} Y_{fb} \\ * & -\rho P_{fb} \end{pmatrix} \prec 0 \tag{10e}$$

Now we can formulate discrete-time multi-objective control design for a given γ_2 by the following optimization problem in LMI's:

$$\min_{\gamma_1, P, Y, P_{fb}, Y_{fb}} \gamma_1, \text{subject to LMI's}(10) \tag{11}$$

given the solution of the above LMI problem, K is obtained by 9.

4. APPLICATION TO THE PROBLEM AND SIMULATION

Considering that all the modal frequencies of the system is less than 12 Hz, sampling time is set 10 msec. First we consider the case of without preview. The approach described above, apart from the inclusion of preview related parts, is used to design a pure feedback multi-objective controller for the system. A γ_2 value of 2 was assumed and the controller was obtained by solving the above LMI optimization, using Matlab LMI toolbox (Gahinet, P., Nemirovski, A., Laub, A.J. and Chilali, M. [1995]). One can handle each of constrained outputs separately with a different $\gamma_{2,i}$ (or separately but with identical $\gamma_{2,i}$ in the case of normalized outputs as in this paper). However, whether this will increase or decrease the conservativeness of the design is beyond the scope of this paper.

Remark 1. Noting the definition of GH_2 norm of (1), to determine a suitable value for γ_2, a priori knowledge on road disturbances may be useful. For a given road disturbance energy, a greater γ_2 value will allow for more variation of constrained outputs. Considering normalized constrained outputs implies that γ_2 value be less than the inverse of the worst case disturbance energy. In this study it is considered as the only controller design parameter.

Now, we consider the case with preview. It is assumed that a preview sensor is mounted in the front bumper of vehicle and could capture road irregularities $L_p = 2$ meter ahead of tire. Clearly, the longer the preview length, the better the performance is improved, however to meet a realistic situation this assumption is made. This assumptions, when car travels with a speed of 20 m/s, will lead to $N_p = 10$ The design of preview included controller is firstly performed with $\gamma_2 = 2$, however we will use it as a design parameter. Clearly, a possible violation of constraints happens when the system runs in extreme case (Sun, P.Y. and Chen, H. [2003]). For suspension systems this is for example when the vehicle travels over a bump. Therefore, to check for fulfillment of system constraints, we consider the case of shock input.

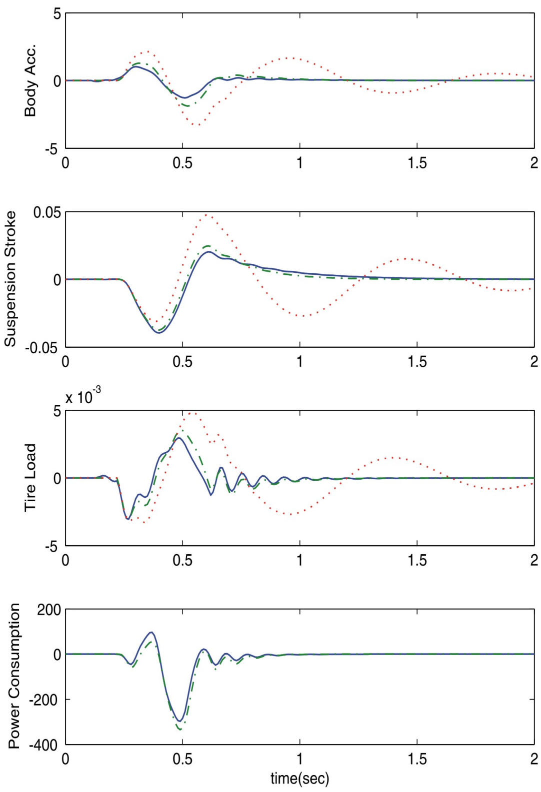

Fig. 3. Bump response. preview-included multi-objective with $\gamma_2 = 2$ (–), pure feedback multi-objective with $\gamma_2 = 2$ (-.),passive (..)

Shocks are discrete events of relatively short duration and high intensity, for example, an isolated bump or pothole in an otherwise smooth road surface. Such a disturbance can be described as:

$$z_r(t) = \begin{cases} \dfrac{H}{2}(1 - \cos(\dfrac{2\pi V(t - T_p)}{L})), & T_p \leq t \leq T_p + \dfrac{L}{V} \\ 0, & Otherwise \end{cases}$$

where H and L are the height and the length of the bump. We choose $H = 0.06m$, $L = 5m$ and the vehicle forward velocity as $V = 12.5m/s(= 45Km/h)$. Figure 3 shows the bump response of closed loop systems using both pure feedback and preview included controllers as well as that of passive system. It can be easily seen that in presence of preview ride comfort, compared to pure feedback case, is considerably improved. When preview information is used by the controller, the control system lifts the wheel over the bump, and thereby reduces the forces transmitted to the body.

This figure also shows improvements in ride safety and suspension deflection. Finally it reveals that power consumption of system calculated by:

$$P(t) = [-b_s(\dot{z}_s - \dot{z}_{us}) + u(t)](\dot{z}_s - \dot{z}_{us})$$

by introducing the preview to system, is also reduced. This motivates the designer to increase the γ_2 value to improve ride comfort even more. The results of the preview based design with $\gamma_2 = 6$ are shown in Figure 4. It can be seen that constraint outputs of the system using preview don't exceed those of pure feedback system, whereas body acceleration is decreased much more.

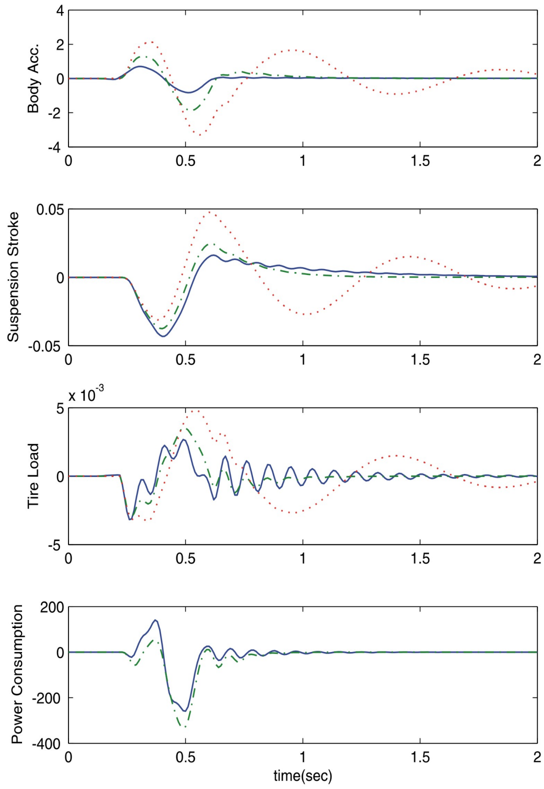

Fig. 4. Bump response. preview-included multi-objective with $\gamma_2 = 6$ (–), pure feedback multi-objective with $\gamma_2 = 2$ (-.),passive (..)

Now, we consider the case of vibration input. Vibration, i.e., consistent road roughness, is typically specified as a random process with a ground displacement power spectral density (PSD)(Hrovat, D. [1997]). However, to assess system performance for this type of road disturbance, we consider the real road profile of Figure 5, which is obtained with vehicle speed of 100 km/h.

RMS values of body acceleration, suspension deflection, relative tire load and power consumption, for 3 suspension systems of this paper, are listed in table II. In presence of preview

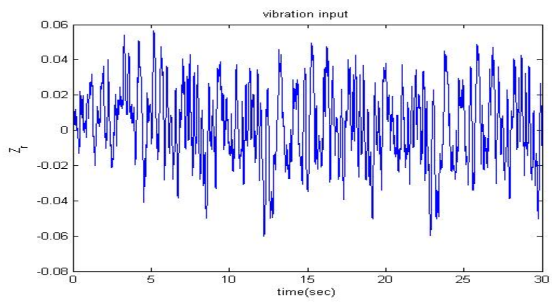

Fig. 5. Vibration input

information, with the cost of an immaterial increase of 15% in suspension stroke compared to pure feedback case, ride comfort is improved 35% more and power consumption decreases 19%.

System	RMS $\ddot{z}_s$	RMS SS	RMS TL	RMS power
Passive	250	110	182	0
Multi-objective	100	100	100	100
Multi-objective preview	65	115	103	81

Table 2. Components of performance index for vibration input

5. CONCLUSION

A discrete-time scheme was proposed to design a multi-objective preview controller for active vehicle suspension. The approach requires regulation of just one parameter, namely γ_2. Under the same value of this parameter, preview based design not only improves ride comfort compared to its pure feedback counterpart, but also respects system constraints better than pure feedback. Therefore, this parameter provides a degree of freedom for the designer to improve ride comfort much more without sacrificing handling constraints.

This study considered a look-ahead preview scheme. Look-ahead preview suffers the drawback of wrong interpretation of pseudo-obstacles, whereas the wheelbase preview is more promising. application of the strategy described in this paper to the half car model with wheelbase preview will be some part of the author's future work.

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