Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

The T-S Fuzzy Design and Implementation for Nonlinear Motion Systems with Control Redundancy

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Abstract: The linear matrix inequality (LMI) method is generally adopted to obtain the Takagi-Sugeno (T-S) fuzzy control design with guaranteed system stability only. However, in real implementations, control specifications, like the damping ratio, the natural frequency or the bandwidth, are crucial to system design. In this paper, a feasible T-S fuzzy control design approach is proposed for the nonlinear motion systems with control redundancy, like in robots and satellites with input freedom to obtain the desirable position and orientation. In this paper, by applying the T-S fuzzy modeling and the inverse of the input matrix, a feasible T-S fuzzy controller is obtained to directly meet both system stability and design specifications. The proposed controller design method has been successfully applied to an omindirectional mobile robot (ODMR) system. Both simulation and experimental results indicate that the obtained system performance and stability directly meet the control design specifications. Results also indicate that the effect of the Coriolis force is effectively suppressed by the proposed approach and it leads to improved accuracy in both tracking and contouring precision especially in high-speed motions.

1. INTRODUCTION

This paper addresses the tracking control problem for a class of nonlinear control redundancy systems using the Takagi-Sugeno (T-S) fuzzy models. A large class of control redundancy systems such as robotic hands, space robots, wheeled vehicles, and mobile robots are common in many industrial applications (McClamroch and Wang, 1988; Mochiyama et al., 1999). They tend to have more large load-carrying capacity and higher kinematics accuracy. However, its nonlinear model; mainly due to the coordinate transform, centrifugal, Coriolis and gravity terms, requires a suitable control design to achieve desirable motion accuracy especially under high-speed operations.

The T-S fuzzy control has been successfully applied to nonlinear systems (Wang et al, 1996; Ma et al, 1998). In general, the T-S fuzzy control is designed through linear matrix inequality (LMI) optimization procedures to obtain guaranteed stability only. Available T-S fuzzy design procedures cannot lead to a systematic design results with satisfactory performance in some crucial specifications, like the limited rise time or constrained overshoot. Although some efforts have been put in T-S fuzzy design procedures to meet performance indices (Chen et al, 2003; Chang and Shing, 2004), their approaches are either difficult to obtain the controller due to the added constraints in the LMI procedure, or the obtained control gains are unreasonably large to be implemented (Wu et al, 2006).

In this paper, we will introduce a fuzzy path tracking controller method solving by system specifications for the nonlinear redundancy system, which is modeled by a T-S fuzzy states and tracking errors construct. Thus, the present design not only meets the system specifications with desirable responses, but also is simpler with easier implementation. An example is provided to control an omindirectional mobile robot (ODMR) system (Watanabe et al, 1998) to compare the present control with a well-tuned PID control and other T-S fuzzy controllers. Simulation results indicate that under different speed operations, the responses of the proposed approach are met well with desirable specifications. The experimental results can verify that the effect of Coriolis force can be reduced by proposed method of this paper.

2. THE T-S FUZZY MODELING WITH CONTROL REDUNDANCY

In real motion systems like the robots and satellites, their multiple control inputs with redundancy provide the design freedom. Thus, those mechanisms usually perform functions and precision to meet desirable position, velocity, and orientation in real applications. Moreover, nonliearity due to the mechanical dynamics and coordinate transform is unavoidable in modeling and control design for those motion systems. Although general T-S fuzzy controllers are adopted to cope with those nonlinear systems by applying the LMI approach to obtain guaranteed stability, they are difficult to meet the system specifications like the damping ratio, natural frequency or bandwidth in real applications. The proposed T-S fuzzy system with control redundancy considered in this paper to meet both system stability and control performance is defined as

$Rule^i$:
IF $z_1(t)$ is $M_{i\ell}$ … and $z_{n_x}(t)$ is M_{in_x}
THEN $\dot{x}(t) = \boldsymbol{A}_i x(t) + \boldsymbol{B}_i u(t)$ and $y(t) = \boldsymbol{C}_i x(t)$ (1)

where $x(t) \in \Re^{n_x}$ is the state vector; $u(t) \in \Re^{n_u}$ is the control input vector with redundancy as $n_u > n_x$; $y(t) \in \Re^{n_y}$ is output vector. The M_{ij} are fuzzy sets, $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{C}_i)$ are the i-th matrices of adequate dimension with subsystem of the system (1). Besides, $z_1(t)$, $z_2(t)$, …, $z_{n_x}(t)$ are the premise variables of the fuzzy model and they are the functions of state variables. The pair $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ are assumed to be controllable.

The dynamics of the proposed T-S fuzzy redundancy model is then represented as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[\boldsymbol{A}_i x(t) + \boldsymbol{B}_i u(t)] \tag{2a}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\boldsymbol{C}_i x(t) \tag{2b}$$

where $h_i(z(t))$ is the normalized weighting of the i-th rule and it is calculated by the membership values and $z(t) = [z_1(t)\ z_2(t) \cdots z_{n_x}(t)]$.

Denote the integral error $\dot{e}(t)$ for the path tracking controller.

$$\dot{e}(t) = r(t) - y(t) = r(t) - \sum_{i=1}^{r} h_i(z(t))\boldsymbol{C}_i x(t) \tag{3}$$

and $r(t) \in \Re^{n_y}$ denotes the reference command state.

Since the T-S fuzzy controller shares the same fuzzy sets with its T-S fuzzy model as in (2), the present state feedback rules of the controller are as follows:

$Rule^i$:
IF $z_1(t)$ is $M_{i\ell}$ … and $z_{n_x}(t)$ is M_{in_x}
THEN $u(t) = \boldsymbol{F}_i x(t) + \boldsymbol{G}_i e(t)$ (4)

In addition, the present modeling and control include both the system states and the path tracking error, the overall fuzzy controller for the control redundancy system is obtained as

$$u(t) = \sum_{i=1}^{r} h_i(z(t))[\boldsymbol{F}_i x(t) + \boldsymbol{G}_i e(t)] \tag{5}$$

By substituting (3) and (5) into (2), it yields the closed-loop modeling with the augmented system as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\tilde{\boldsymbol{A}}_{ij}\,\tilde{x}(t) + \boldsymbol{N}\,r(t) \text{ and} \tag{6a}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\tilde{\boldsymbol{C}}_i\,\tilde{x}(t) \tag{6b}$$

where

$$\tilde{\boldsymbol{A}}_{ij} = \begin{bmatrix} \boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{F}_j & \boldsymbol{B}_i\boldsymbol{G}_j \\ -\boldsymbol{C}_i & 0 \end{bmatrix},\ \boldsymbol{N} = \begin{bmatrix} 0 \\ \boldsymbol{I}_{3\times 3} \end{bmatrix},\ \tilde{x}(t) = \begin{bmatrix} x(t) \\ --- \\ e(t) \end{bmatrix} \text{ and}$$

$$\tilde{\boldsymbol{C}}_i = [\boldsymbol{C}_i\ \ 0]$$

The schematic diagram of the overall T-S fuzzy path control system is shown in Fig. 1.

3. THE T-S FUZZY DESIGN WITH SYSTEM SPECIFICATIONS

By regarding the closed-loop system as an augmented form in (6), applications of the LMI method in general T-S fuzzy control design usually leads to a common positive definite matrix $\boldsymbol{P}$ and corresponding feedback gains $\boldsymbol{F}_i$ and $\boldsymbol{G}_i$ for each rule to guarantee the stability of the whole system only. However, if the system performance constraints are also included in the design, procedures thus become more complicated. A new design approach which solves feedback gains for the T-S fuzzy path tracking control is proposed here that the characteristic equation associated with the reference/output map for each subsystem is obtained as

$$\det[s\boldsymbol{I} - \tilde{\boldsymbol{A}}] = s^2 - (\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{F}_j)s + \boldsymbol{B}_i\boldsymbol{G}_j = 0 \tag{7}$$

Then, $\boldsymbol{F}_j$ and $\boldsymbol{G}_j$ can be directly determined to characterize the performance specifications for each subsystem as

$$s^2 + 2\zeta_{ki}\omega_{ki}s + \omega_{ki}^2 = 0,\ k = 1\cdots n_x \tag{8}$$

where ζ_{ki} and ω_{ki} are the damping ratio and natural frequency of the characteristic equation associated to the reference/output map corresponding to k-th states. Note that the damping ratio ζ_{ki} and natural frequency ω_{ki} can be determined according to specifications of the maximum overshoot and the response time.

In the control redundancy systems, the control inputs are more than degrees of freedom. The generalized inverse $\boldsymbol{B}_i^+$ exists and its rank is in full row. Consequently, the $\boldsymbol{F}_i$ and $\boldsymbol{G}_i$ can be directly acquired to meet system specifications as

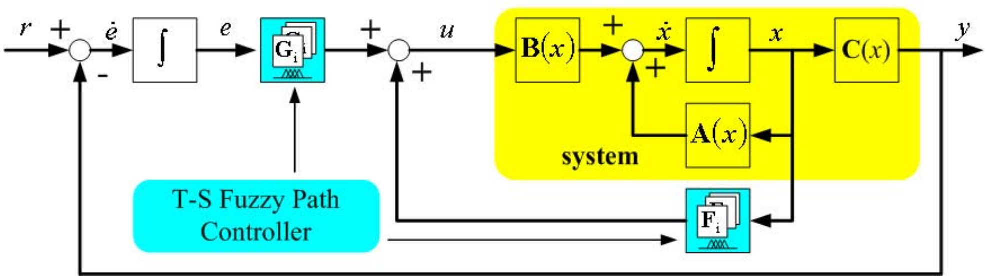

Fig. 1. The schematic diagram of the T-S fuzzy path control system

$$\boldsymbol{G}_j = \boldsymbol{B}_i^+ \mathrm{diag}\left(\omega_{ki}^2\right),\ k = 1\cdots n_x \tag{9}$$

$$\boldsymbol{F}_j = \boldsymbol{B}_i^+\left[\mathrm{diag}\left(-2\zeta_{ki}\omega_{ki}\right) - \boldsymbol{A}_i\right],\ k = 1\cdots n_x \tag{10}$$

After getting the feedback gains for each rule, the stability conditions of the T-S fuzzy path control system (6) are referred to (Wang *et al*, 1996). The T-S fuzzy path control system (6) is globally asymptotically stable if there exists a common positive definite matrix $\boldsymbol{P}$ such as

$$\widetilde{\boldsymbol{A}}_{ij}\boldsymbol{P} + \boldsymbol{P}\widetilde{\boldsymbol{A}}_{ij}^{\mathrm{T}} < 0 \tag{11}$$

In the T-S fuzzy system, the overall system is not guaranteed stable even each subsystem is stable. Hence, when the closed-loop subsystem has been decided by the feedback gains for each rule, it needs to check the global stability condition (11). If it is not satisfied, it is necessary to redesign the T-S system by relaxing system specifications.

4. THE T-S FUZZY MODELING OF THE ODMR

Consider a four-wheel ODMR system as shown in Fig. 2 and as formulated in (Watanabe *et al*,1998). The dynamic equations can be described by the following equations which be represented in the absolute coordinate system with the state variables $x(t) = [\dot{x}_{\mathrm{w}}(t)\ \ \dot{y}_{\mathrm{w}}(t)\ \ \dot{\varphi}(t)]^{\mathrm{T}}$ and the control input variables $u(t) = [u_1(t)\ \ u_2(t)\ \ u_3(t)\ \ u_4(t)]^{\mathrm{T}}$.

$$\dot{x}(t) = \boldsymbol{A}(x)x(t) + \boldsymbol{B}(x)u(t) \tag{12a}$$

$$y(t) = \boldsymbol{C}\,x(t) \tag{12b}$$

where

$$\boldsymbol{A}(x) = \begin{bmatrix} a_1 & -a_2\dot{\varphi}(t) & 0 \\ a_2\dot{\varphi}(t) & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix},$$

$$\boldsymbol{B}(x) = \begin{bmatrix} b_1\beta_1(t) & b_1\beta_2(t) & b_1\beta_3(t) & b_1\beta_4(t) \\ b_1\beta_4(t) & b_1\beta_1(t) & b_1\beta_2(t) & b_1\beta_3(t) \\ b_2 & b_2 & b_2 & b_2 \end{bmatrix},\ \boldsymbol{C} = \boldsymbol{I}_{3\times 3},$$

$$a_1 = -2c/\left(mr^2 + 2I_{\mathrm{w}}\right), \qquad a_2 = 2I_{\mathrm{w}}/\left(mr^2 + 2I_{\mathrm{w}}\right),$$

$$a_3 = -4cL^2/\left(4I_{\mathrm{w}}L^2 + I_{\mathrm{v}}r^2\right),$$

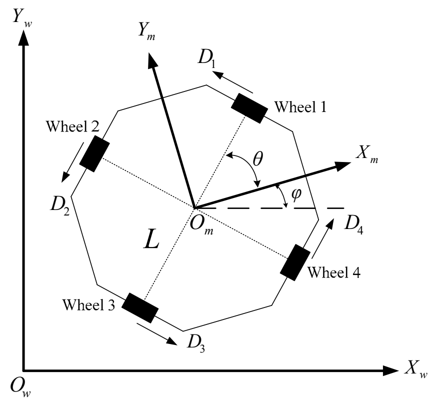

Fig. 2. A four-wheel ODMR model

$$b_1 = \sqrt{2}\,kr/2\left(mr^2 + 2I_{\mathrm{w}}\right), \qquad b_2 = krL/\left(4I_{\mathrm{w}}L^2 + I_{\mathrm{v}}r^2\right),$$

$$\beta_1(t) = -\sin\varphi(t) - \cos\varphi(t), \qquad \beta_2 = \sin\varphi(t) - \cos\varphi(t),$$

$$\beta_3(t) = \sin\varphi(t) + \cos\varphi(t), \qquad \beta_4(t) = -\sin\varphi(t) + \cos\varphi(t)$$

There exists the nonlinearities caused by the coordinate transform and the Coriolis force in modeling as in (12) and they can be approximated by four linearized subsystems around 0, $\pi/2$, π and $3\pi/2$ rad. for $\varphi(t)$ with the present T-S fuzzy modeling. Moreover, considering each linearized model around three rotational speeds $\dot{\varphi}(t) = -d$, 0 and d, total 12 rules are required to construct the T-S fuzzy model for the ODMR. By applying the proposed kinematics inversion with the following trigonometric fundamental identities as

$$u(t) = \overline{\boldsymbol{B}}\,\overline{u}(t) \tag{13}$$

where

$$\overline{\boldsymbol{B}} = \begin{bmatrix} \dfrac{\beta_1(t)}{4b_1} & \dfrac{\beta_4(t)}{4b_1} & \dfrac{1}{4b_2} \\ \dfrac{\beta_2(t)}{4b_1} & \dfrac{\beta_1(t)}{4b_1} & \dfrac{1}{4b_2} \\ \dfrac{\beta_3(t)}{4b_1} & \dfrac{\beta_2(t)}{4b_1} & \dfrac{1}{4b_2} \\ \dfrac{\beta_4(t)}{4b_1} & \dfrac{\beta_3(t)}{4b_1} & \dfrac{1}{4b_2} \end{bmatrix} \text{ and } \overline{u}(t) = [\overline{u}_1(t)\ \overline{u}_2(t)\ \overline{u}_3(t)]^{\mathrm{T}}$$

the nonlinear term of the coordinate transform in the model is thus eliminated. By substituting (13) into (12a), a dynamic ODMR model can be simply obtained as

$$\ddot{x}_w(t) = a_1\dot{x}_w(t) - a_2\dot{\varphi}(t)\dot{y}_w(t) + \overline{u}_1(t) \tag{14}$$

$$\ddot{y}_w(t) = a_2\dot{\varphi}(t)\dot{x}_w(t) + a_1\dot{y}_w(t) + \overline{u}_2(t) \tag{15}$$

$$\ddot{\varphi}_w(t) = a_3\dot{\varphi}(t) + \overline{u}_3(t) \tag{16}$$

Therefore, the present ODMR nonlinear model is further linearized around three rotational rates, $\dot{\varphi}(t) = -d$, 0, and d, with the fuzzy sets shown in Fig. 3. Then, the simplified dynamic ODMR model in (14)-(16) can be constructed simply by the following three linear subsystems:

Rule^i :
IF $\dot{\varphi}(t)$ is about M^i
THEN $\dot{x}(t) = \boldsymbol{A}_i\,x(t) + \boldsymbol{B}_i\,\overline{u}(t)$ and $y(t) = \boldsymbol{C}_i\,x(t)$,
$i = 1, 2, 3$ (17)

where

$$\boldsymbol{A}_1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix},\ \boldsymbol{A}_2 = \begin{bmatrix} a_1 & -a_2d & 0 \\ a_2d & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} a_1 & a_2d & 0 \\ -a_2d & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix},\ \boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}_3 = \boldsymbol{B}_{\mathrm{c}} = \boldsymbol{I}_{3\times 3},$$

$$\boldsymbol{C}_1 = \boldsymbol{C}_2 = \boldsymbol{C}_3 = \boldsymbol{C} = \boldsymbol{I}_{3\times 3}$$

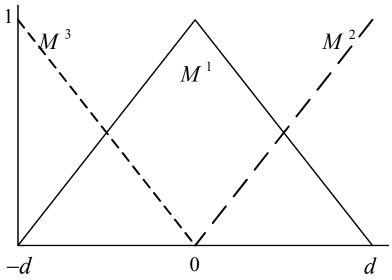

Fig. 3. The membership functions of the T-S fuzzy system.

The real-world ODMR system parameter values are given as follows:

$$m = 16.9\text{kg},\ L = 0.193\text{m},\ r = 0.04\text{m},\ I_v = 0.2518\text{kgm}^2,$$
$$k = 14,\ c = 8.1633\times10^{-4}\text{kgm}^2/\text{s},\ I_w = 1.1\times10^{-4}\text{kgm}^2$$

5. SIMULATION RESULTS

The present T-S fuzzy control design also considers the rotational motion speed $\dot{\varphi}(t)$ operated between $[-1.5\ rad/sec, 1.5\ rad/sec]$. Corresponding design parameters to the system specifications of a prototype 2nd -order system are shown in Table 1. Following the design procedures as in Section 3, the feedback gains in (9)-(10) of the proposed T-S fuzzy controllers are obtained as

$$\boldsymbol{F}_1 = \begin{bmatrix} -16.8 & 0 & 0 \\ 0 & -16.8 & 0 \\ 0 & 0 & -22.1 \end{bmatrix},\quad \boldsymbol{F}_2 = \begin{bmatrix} -16.8 & 0.7515 & 0 \\ -0.7515 & -16.8 & 0 \\ 0 & 0 & -22.1 \end{bmatrix},$$

$$\boldsymbol{F}_3 = \begin{bmatrix} -16.8 & -0.7515 & 0 \\ 0.7515 & -16.8 & 0 \\ 0 & 0 & -22.1 \end{bmatrix}, \text{ and}$$

$$\boldsymbol{G}_1 = \boldsymbol{G}_2 = \boldsymbol{G}_3 = \begin{bmatrix} 196 & 0 & 0 \\ 0 & 196 & 0 \\ 0 & 0 & 256 \end{bmatrix} \tag{18}$$

Table 1. System design specifications

Motion	System Specifications	System Parameters
Translation	$t_r = 0.2\ s$ $M_p = 10\%$	$\omega_{x,y} = 14\ rad/s$ $\zeta_{x,y} = 0.6$
Rotational	$t_r = 0.2\ s$ $M_p = 5\%$	$\omega_\phi = 16\ rad/s$ $\zeta_\phi = 0.7$

5.1 Time response of the T-S fuzzy controller

With a given circular motion command with radius of 2 m and the tangential speed of 0.63 m/sec as shown in Fig. 4, the rotation command with a rate $\dot{\varphi} = 18\ deg/sec$ is given at 10 seconds for another half circular motion. The obtained rise time t_r and the maximum overshoot M_p of the time responses meet the present design specifications well, as shown in Table 1. For another case with a four times faster circular command as V=2.52 m/sec, the present T-S fuzzy control design still achieves satisfactory performance to meet the design specifications well as shown in Table 2.

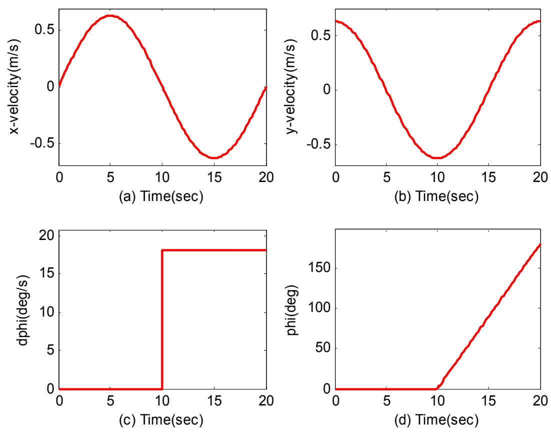

Fig. 4. Reference command

Table 2. Time responses of the rotational moiton

Design specifications		Simulation Results	
		$V = 0.63\ m/s$	$V = 2.52\ m/s$
$\omega_\phi = 16\ rad/s$ $\zeta_\phi = 0.7$	$t_r = 0.2\,s$ $M_p = 5\%$	$t_r = 0.208\,s$ $M_p = 4.6\%$	$t_r = 0.216\,s$ $M_p = 4.6\%$

5.2 Motion Accuracy Analysis

The present T-S fuzzy controller is also compared with other controllers implemented as 1) the well-tuned PID controllers (Watanabe *et al*, 1998) and 2) the T-S fuzzy controllers obtained by the LMI-based method (Tanaka *et al*, 1998). The PID servo gains for both the displacement loop and the velocity loop on x and y direction are $K_{\dot{x}p} = K_{\dot{y}p} = 10$ and $K_{\dot{x}i} = K_{\dot{y}i} = 25$, respectively, and the servo gains of the rotation are as $K_{\varphi p} = 2.25$ and $K_{\varphi d} = 3$ (Watanabe *et al*, 1998). In addition, the decay rate 15 is adopted in the LMI-based T-S fuzzy method (Tanaka *et al*, 1998) and the value is around the diagonal elements in $\boldsymbol{F}_i$ obtained by the proposed T-S fuzzy control as in Section 3. Its corresponding gains are obtained as

$$\boldsymbol{F}_{L1} = \begin{bmatrix} -15.21 & 0 & 0 \\ 0 & -15.21 & 0 \\ 0 & 0 & -15.20 \end{bmatrix},$$

$$\boldsymbol{F}_{L2} = \boldsymbol{F}_{L3} = \begin{bmatrix} -15.17 & 0 & 0 \\ 0 & -15.17 & 0 \\ 0 & 0 & -15.20 \end{bmatrix} \tag{19}$$

Table 3. Simulation results of different controllers with relax specifications

	Case 1 (V=0.63 m/s)				Case 2 (V=2.52 m/s)			
	Position error (m)	Rotation error (rad/s)	Contouring error (cm)	Max. Contouring (cm)	Position error (m)	Rotation error (rad/s)	Contouring error (cm)	Max. Contouring (cm)
PID	2.36	1.87	8.9	1.51	34.7	26.4	229	21.1
LMI-based method	12.9	0.29	12.0	3.26	51.3	4.79	87	11.3
Proposed method	10.8	0.75	0.24	0.04	43.8	11.9	7.4	0.63

Table 4. Simulation results of different controllers with stringent specifications

	Case 1 (V=0.63 m/s)				Case 2 (V=2.52 m/s)			
	Position error (m)	Rotation error (rad/s)	Contouring error (cm)	Max. Contouring (cm)	Position error (m)	Rotation error (rad/s)	Contouring error (cm)	Max. Contouring (cm)
PID	1.18	0.99	4.4	0.72	16.9	14.7	112	9.9
LMI-based method	6.5	0.17	6.6	1.69	26.2	3.44	53	7.7
Proposed method	5.4	0.21	0.06	0.01	21.8	3.44	1.82	0.17

- **The relax specifications case** $\omega = 15\ rad/sec$

Simulation results are summarized in Table 3 Results also indicate that the proposed T-S fuzzy controller leads to significant reduction in contouring error while the tracking accuracy for these three controllers are about the same for the lower-speed case. As the speed commands increases, both the tracking error and the contouring error of the PID controller increase dramatically. Compared with the LMI-based T-S fuzzy controller, the proposed T-S fuzzy controller obtained from (18) achieves much better contouring performance under both operation speeds.

- **The stringent specifications case** $\omega = 30\ rad/sec$

To further improve the motion accuracy under high-speed operations, all the PID control gains and the decay rate 15 of the LMI-based method are doubled to achieve better system performance. Also, the proposed T-S fuzzy controller is redesigned by setting the half rise time for the moving velocity and the rotational rate also with double values of (18). Simulation results for all controllers with more stringent specifications are summarized in Table 4. Results indicate that although the present T-S fuzzy design achieves about the same tracking performance with other controllers, it leads to the best contouring performance in both low-speed and high-speed cases.

Simulation results are summarized as in the following:

- The proposed method performs better contouring performance than the PID and LMI-based method because the Coriolis force is effectively eliminated.
- As shown in Table2, results indicate that the present T-S fuzzy control design can directly meet the system design specifications in the time response and the maximum overshoot. For example, the rotation error here can be directly improved by assigning a faster natural frequency with a larger damping ratio. However, the LMI-based T-S method just obtains a set of controller gains merely with guaranteed stability and the decay rate becomes a crucial index to determine the responses.
- As the speed increases, both the tracking errors of the LMI-based and the proposed T-S fuzzy controllers increase around three times. However, the tracking error of the PID controller increases dramatically around 15 times. Moreover, as shown in Table 3 and 4, although the tracking error of the two T-S fuzzy controllers are about the same, the proposed T-S fuzzy method improves more than 90% than the LMI-based T-S fuzzy design in contouring accuracy.

6. EXPERIMENTAL RESULTS

Fig. 5 shows the structure of the experimental setup for the present ODMR. The controller of the ODMR is implemented on TI DSP2812 and each omni-wheel is driven by the Faulhaber 3863A024C DC motor with MCDC3006S motor drivers.

To verify both the tracking and contouring precisions in practice, the final position error is measured with the linear motion command to imply the measurement of the accumulated tracking and contouring errors during the motion. For the translation motion only with a speed 1 *m*/*sec*, both acceleration and deceleration are set for 1 second duration and results listed in Table 5 are summarized as follows:

- When the ODMR is in an open-loop control structure, only each wheel is controlled independently without considering the whole platform. Experimental results indicate that the positional error increases dramatically when the ODMR rotates because the Coriolis force of the robot dynamics becomes significant.
- The PID controller which is applied to the whole ODMR significantly improves the motion accuracy, especially when the rotation motion is included.
- Results also indicate that the proposed T-S fuzzy controller performs the best without rotation. Moreover, the error of the present T-S fuzzy control due to the rotation motion is improved 38% compared to error of the PID control.

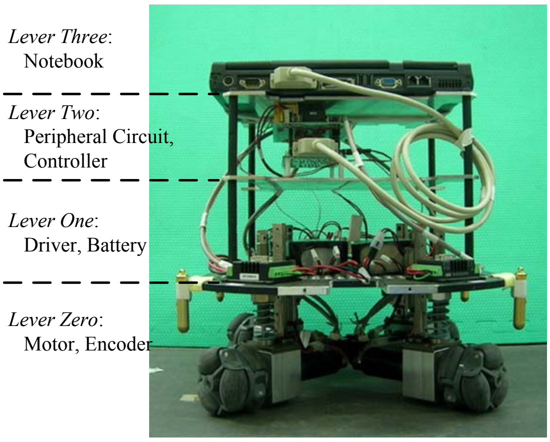

Fig. 5. The experimental setup for the ODMR

Table 5. Experimental results of the ODMR with different controller

Position Error (*cm*) / Case / Method	Linear path w/o rotation	Linear path with $\dot{\varphi} = 60$ *deg/s*
Open-loop control	7.9	30.5
PID control	6.8	13.6
The proposed T-S fuzzy control	5.2	8.4

7. CONCLUSIONS

In this paper, a fuzzy path tracking controller has been developed for nonlinear motion systems with control redundancy. In general, the T-S fuzzy controller obtained by applying the LMI method leads to the guaranteed system stability only and control specifications are usually difficult to meet. In motion systems, because the control inputs may be more than the degrees of freedom, the inverse matrix $\boldsymbol{B}_i^+$ exists and the state and tracking error feedback gains of the T-S fuzzy controllers can be directly assigned according to the desired system specifications.

Furthermore, a feasible T-S fuzzy control design is proposed in this study to achieve both system stability and control specifications for nonlinear motion systems. Simulation results have proven that the present T-S fuzzy controller meets the system specifications properly at different motion speeds. As the speed increases, the improvement of the present T-S fuzzy controller becomes more significant, especially in high-speed motions. Both simulation and experimental results on the ODMR indicate that the proposed T-S fuzzy control design is efficient and feasible in real applications.

ACKNOWLEDGEMENTS

This work was supported by National Science Council of the Republic of China under the Contract NSC 95-2221-E-009 -100 -MY2.

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