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Iterative Feedback Tuning for Hamiltonian Systems ⋆

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Abstract: This paper is concerned with iterative feedback tuning for Hamiltonian systems. Hamiltonian systems have a property called variational symmetry which can be used to estimate the input-output mapping of the variational adjoint for certain input-output mappings of the systems. Here this property is utilized for estimating the gradient of an optimal control type cost function with respect to the design parameters of the controllers. This allows one to obtain an iterative feedback tuning algorithm for Hamiltonian systems which generates the optimal parameters by iteration of experiments. The proposed algorithm requires less number of experiments to estimate the gradient and can be used with the iterative learning control proposed previously. Furthermore, numerical simulations demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

In the research area on control of physical systems, most of the existing results focus on feedback stabilization and related topics such as trajectory tracking control, output feedback control and so on. They utilize phyisical properties such as passivity and symmetry for control effectively (van der Schaft [2000], Ortega et al. [2002], Fujimoto et al. [2003b]). In those methods, a precise model of the plant is requred. However, it is quite difficult to construct a precise model for a given plant and it is always requred to adjust the design parameters when we design a control system. Hence it is desired to adjust/generate a feedback controller or feedforward input by automatic learning. For this purpose, several methods are proposed. In control engineering, iterative learning control (Arimoto et al. [1984], Moore [1993]) and iterative feedback tuning (Hjalmarsson [2002], De Bruyne et al. [1997]) are well known. The former method is to generate a feedforward input to achieve a given desired trajectory by iteration of experiments and the latter adjusts the design parameter of the feedback controller via experiments. This paper is concerned with iterative feedback tuning for physical systems described by Hamiltonian equations.

The authors have developped an iterative learning control method for Hamiltonian systems (Fujimoto and Sugie [2003]). The conventional iterative learning control methods rely on the problem setting of trajectory tracking control, and they are not applicable to other problems such as trajectory generation. On the other hand, the authors' former result is based on a special property of the plants Hamiltonian systems called variational symmetry, and it is applicable to wilde class of problems described by cost functions of optimal control type. The purpose of the paper is to employ the variational symmetry to obtain a iterative feedback tuning control algorithm.

There are many results reported on iterative feedback tuning. A common control strategy for an iterative feedback tuning problem is to select a cost function as optimal control and to adjust parameters of the feedback controller so that the cost function decreases. In this approach, the gradient of the cost function with respect to the parameter is estimated using input-output data. However this method requres a number of experiments in order to execute one step optimization in the gradient method compared with iterative learning control in which one step requres only one experiment. It is also noted that the number of parameters to adjust is finite in iterative feedback tuning whereas the feedforward input to be optimized is an infinite dimensional signal in iterative learning control.

The present paper is devoted to iterative feedback tuning for Hamiltonian systems based on variational symmetry. First of all, a version of variational symmetry of Hamiltonian systems which can be used to estimate the gradient of a cost function of optimal control type. Next a novel iterative feedback tuning method is constructed based on it. The proposed method requres less number of experimens compared with the existing results. A numerical simulation of a 3 mass-spring systems demonstrates the effectiveness of the proposed method. Since the proposed algorithm is based on variational symmetry which can be used for iterative learning control as well, it can be used for simultaneous learning control with both iterative feedback tuning and iterative learning control.

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2. PRELIMINARIES

This section briefly refers to preliminary backgrounds.

2.1 Variational symmetry

Our plant is a Hamiltonian system with dissipation Σ with a controlled Hamiltonian $H(x,u,t)$ as $(x^1,y)=\Sigma(x^0,u)$:

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}},\ x(t^0)=x^0 \\ y &= -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \\ x^1 &= x(t^1) \end{cases} \tag{1}$$

with $u,y\in L_2^m(t^0,t^1)$. Here the structure matrix $J\in\mathbb{R}^{n\times n}$ and the dissipation matrix $R\in\mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements such as friction of mechanical systems and resistance of electric circuits. For this system, the following theorem holds. Here the mapping $u\mapsto y$ is denoted by Σ^{x^0} or sometimes just Σ when no confusion arises.

Theorem 1. (Fujimoto and Sugie [2003]) Consider the Hamiltonian system (1). Fréchet derivative $\mathrm{d}\Sigma(x^0,u)(\cdot)$ of $\Sigma(x^0,u)$ is described by a Hamiltonian system. Suppose that there exists a nonsingular matrix $N\in\mathbb{R}^{n\times n}$ satisfying

$$NJ=-JN,\quad NR=RN \tag{2}$$

$$\begin{pmatrix} N & 0\\ 0 & I\end{pmatrix}\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}=\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\begin{pmatrix} N & 0\\ 0 & I\end{pmatrix}. \tag{3}$$

Then a state-space realization of $(\mathrm{d}\Sigma(\cdot))^*$ coincide with a time-reversal version of that of $\mathrm{d}\Sigma(\cdot)$ and they are described by Hamiltonian systems

$$\mathrm{d}\Sigma(x^0,u)\ :\ (x_v^0,u_v)\mapsto(x_v^1,y_v)$$

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, & x(t^0)=x^0 \\ \dot{x}_v &= (J-R)\dfrac{\partial H_v(x_v,u_v,x,u,t)}{\partial x_v}^{\mathrm{T}}, & x_v(t^0)=x_v^0 \\ y_v &= -\dfrac{\partial H_v(x_v,u_v,x,u,t)}{\partial u_v}^{\mathrm{T}} \\ x_v^1 &= x_v(t^1) \end{cases}$$

$$(\mathrm{d}\Sigma(x^0,u))^*\ :\ (x_a^1,u_a)\mapsto(x_a^0,y_a)$$

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \quad x(t^0)=x^0 \\ \dot{x}_a &= -(J-R)\dfrac{\partial H_v(x_v,u_a,x,u,t)}{\partial x_v}^{\mathrm{T}}, \\ y_a &= -\dfrac{\partial H_v(x_v,u_a,x,u,t)}{\partial u_a}^{\mathrm{T}} \\ x_v(t^1) &= -(J-R)Nx_a^1 \\ x_a^0 &= -N^{-1}(J-R)^{-1}x_v(t^0) \end{cases}$$

with a Hamiltonian

$$H_v(x_v,u_v,x,u,t)=\frac{1}{2}\begin{pmatrix}x_v\\u_v\end{pmatrix}^{\mathrm{T}}\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\begin{pmatrix}x_v\\u_v\end{pmatrix}.$$

Suppose moreover that, for two inputs $v,w\in L_2^m(t^0,t^1)$, the corresponding state trajectories $\phi(t),\psi(t)\in\mathbb{R}^n$, $t\in(t^0,t^1)$ satisfy

$$\mathcal{R}\left(\left.\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\right|_{\substack{x=\phi\\u=v}}\right)=\left.\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\right|_{\substack{x=\psi\\u=w}}. \tag{4}$$

Here $\mathcal{R}$ is a time reversal operator on (t^0,t^1).

$$(\mathcal{R}(u))(t):=u(t^1-t),\quad t\in(t^0,t^1)$$

Then the following relationship holds.

$$\mathcal{S}\ (\mathrm{d}\Sigma(\phi(t^0),v))^*=(\mathrm{d}\Sigma(\psi(t^1),w))\ \mathcal{S}$$

Here the operator $\mathcal{S}:\mathbb{R}^n\times L_2^m(t^0,t^1)\to\mathbb{R}^n\times L_2^m(t^0,t^1)$ is defined by

$$\mathcal{S}(x^0,u):=(-(J-R)Nx^0,\mathcal{R}(u)).$$

2.2 Iterative learning control

Based on the property in Theorem 1, an iterative learning control algorithm was derived. We employ a cost function (functional) of optimal control type $\Gamma(u,y)$. The gradient method implies that, if we can obtain the gradient $\nabla\Gamma^u(u)$ of $\Gamma^u(u):=\Gamma(u,\Sigma(u))$, then the gradient method implies that the input u should be updated as follows in order to minimize the cost function.

$$u_{(i+1)}=u_{(i)}-K_{(i)}\ \nabla\Gamma^u(u_{(i)}),\quad i=0,1,2\ldots$$

Here a positive constant $K_{(i)}$ is called a step parameter and the subscript $(\cdot)_{(i)}$ denotes the data in the i-th step of the gradient method. Further, this gradient can be decomposed as

$$\nabla\Gamma^u(u)=\nabla_u\Gamma(u,y)+(\mathrm{d}\Sigma(u))^*\nabla_y\Gamma(u,y)$$

All terms except the variational adjoint $(\mathrm{d}\Sigma(u))^*$ are known. In this way, when we want to solve an optimal control, we need to construct a variational adjoint of the plant in order to estimate the gradient of the cost function.

Now, Theorem 1 implies that the variational adjoint $(\mathrm{d}\Sigma(u))^*$ can be approximated by

$$\begin{aligned}(\mathrm{d}\Sigma(u))^*(v) &= \mathcal{R}(\mathrm{d}\Sigma(u))\mathcal{R}(v)\\ &= \frac{1}{\epsilon}\mathcal{R}\left(\Sigma(u+\mathcal{R}(\epsilon v))-\Sigma(u)\right)+\frac{o(\epsilon)}{\epsilon}.\end{aligned} \tag{5}$$

Here $o(\cdot)$ denotes a term satisfying

$$\lim_{\epsilon\to 0}\frac{o(\epsilon)}{\epsilon}=0.$$

The right hand side of Equation (5) can be estimated by using two set of experiments since it is a function of two Σ's. Thus optimal control problem for Hamiltonian systems to obtain optimal feedforward input can be solved by iteraion of experiments via variational symmetry. The purpose of the present paper is to extend this idea to iterative feedback tuning, as precisely described in the following sections.

3. ITERATIVE FEEDBACK TUNING BASED ON VARIATIONAL SYMMETRY

This section provides an iterative feedback tuning algorithm based on variationlal symmetry for Hamiltonian systems. Whereas iterative learning control produces an optimal feedforward input based on the input-output data of experiments, iterative feedback tuning is an algorithm to adjust finite number of parameters of a feedback controller. Since the number of parameters are finite, we can construct an learning algorithm based on gradient method in whcich a set of data for a finite number of experiments is required to update the estimation for the parameters. In the conventional iterative feedback tuning algorithm, e.g. (Hjalmarsson [2002]), the estimation of the gradient of a given cost fucntion needs $s+1$ experiments where s denotes the number of parameters to be tuned. On the other hand, the proposed algorithm based on variational symmetry requres only 3 experiments to estimate the gradient for any number of parameters contained in the Hamiltonian function.

3.1 Variational symmetry

Let us consider a feedback system of a Hamiltonian system with a generalized canonital transformation (Fujimoto et al. [2003b]). Since a generalized canonical transformation is a set of feedback and coordinate transformations preserving the Hamiltonian structure in Equation (1), the feedback system has the form as in Equation (1) as well. Therefore the system parameters of the closed loop system $H(x,u)$, J and R depend on the the parameters of the feedback controller to be adjusted. For simplicity, let us suppose that only the Hamiltonian function $H(x,u)$ depends on the tuning parameter $\rho \in \mathbb{R}^s$. The case where the other system parameters J and R also depend on the tuning parameter will be considered later.

Consider a feedback system (1) with a Hamiltonian $H(x,u,\rho)$ where $\rho \in \mathbb{R}^s$ is the tuning parameter. Namely, the dynamics is written as

$$\dot{x} = (J - R)\frac{\partial H(x,u,\rho)}{\partial x}^{\mathrm{T}}. \tag{6}$$

For this dynamics, let us construct the following input-output map

$$\Sigma_\rho^{x^0,u} : u_\rho \mapsto y_\rho \begin{cases} \dot{x} \;=\; (J-R)\dfrac{\partial H(x,u,u_\rho)}{\partial x}^{\mathrm{T}}, \quad x(t^0) = x^0 \\ y_\rho \;=\; -\dfrac{\partial H(x,u,u_\rho)}{\partial u_\rho}^{\mathrm{T}} \end{cases}$$

with $u_\rho, y_\rho \in L_2^s(t^0,t^1)$. Since this map Σ_ρ is a Hamiltonian system in the form (1), Theorem 1 implies that it has variational symmetry.

$$(\mathrm{d}\Sigma_\rho^{x^0,u}(u_\rho))^* = \mathcal{R}\;(\mathrm{d}\Sigma_\rho^{\xi^0,w}(w_\rho))\;\mathcal{R}$$

Here ξ^0, w, w_ρ are selected in such a way that the condition (4) holds. In order to describe the true dynamics of the closed loop system, we need to select $u_\rho \in L_2^s$ as constant with respect to time. To this end, let us introduce a (0-order) holder

$$\mathcal{H} : \mathbb{R}^s \to L_2^s(t^0,t^1) :$$
$$(\mathcal{H}(\rho))(t) \equiv \rho, \quad \forall t \in (t^0,t^1)$$

Then, clearly, the composition map $\Sigma \circ \mathcal{H}(\rho)$ describe the dynamics in Equation (6). For this map, let us define the following operator

$$\Sigma_{\mathcal{H}} := \mathcal{H}^* \circ \Sigma_\rho \circ \mathcal{H}$$

Then we can prove the variational symmetry of $\Sigma_{\mathcal{H}}$.

Theorem 2. Consider the Hamiltonian system (1) and suppose that the assumptions (2), (3) and (4) in Theorem 1 hold. Then the following equation holds.

$$(\mathrm{d}\Sigma_{\mathcal{H}}^{x^0,u}(\rho))^* = \mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho) \tag{7}$$

Proof. Proof is obtained from direct calculation under the assumptions (2), (3) and (4).

$$\begin{aligned}(\mathrm{d}\Sigma_{\mathcal{H}}^{x^0,u}(\rho))^* &= (\mathrm{d}(\mathcal{H}^* \circ \Sigma_\rho^{x^0,u} \circ \mathcal{H}(\rho)))^* \\ &= (\mathcal{H}^*\;\mathrm{d}\Sigma_\rho^{x^0,u}(\mathcal{H}(\rho))\mathcal{H})^* \\ &= \mathcal{H}^*\;(\mathrm{d}\Sigma_\rho^{x^0,u}(\mathcal{H}(\rho)))^*\;\mathcal{H} \\ &= \mathcal{H}^*\;\mathcal{R}\;(\mathrm{d}\Sigma_\rho^{\xi^0,w}(\mathcal{H}(\rho)))\;\mathcal{R}\;\mathcal{H} \\ &= \mathcal{H}^*\;(\mathrm{d}\Sigma_\rho^{\xi^0,w}(\mathcal{H}(\rho)))\;\mathcal{H} \\ &= \mathrm{d}(\mathcal{H}^* \circ \mathrm{d}\Sigma_\rho^{\xi^0,w} \circ \mathcal{H}(\rho)) \\ &= \mathrm{d}\Sigma_\rho^{\xi^0,w}(\rho)\end{aligned}$$

Here the fourth equality follows from Theorem 1 and the fifth one is implied by

$$\begin{aligned}&\mathcal{R}\;\mathcal{H} = \mathcal{H} \\ &\mathcal{H}^*\mathcal{R} = \mathcal{H}^*\mathcal{R}^* = (\mathcal{R}\;\mathcal{H})^* = \mathcal{H}^*.\end{aligned}$$

This proves the theorem. □

As in the previous results, this property is called *variationla symmetry.* It can be utilized to derive iteration algorithm for iterative feedback tuning problems.

3.2 Iterative feedback tuning

This subsection is devoted to iterative feedback tuning based on variational symmetry characterized in Theorem 2. Before stating the result, the following property is exhibited.

Lemma 3. $\mathcal{H}^*$ is characterized by the following equation.

$$\mathcal{H}^*(y) = \int_{t^0}^{t^1} y(t)\mathrm{d}t$$

Proof. The adjoint $\mathcal{H}^*$ satisfies the following equations for arbitrary $\rho \in \mathbb{R}^s$ and $y \in L_2^s$.

$$\begin{aligned}\langle \mathcal{H}^* y, \rho\rangle_{\mathbb{R}^s} &= \langle y, \mathcal{H}\rho\rangle_{L_2^s} \\ &= \int_{t^0}^{t^1} \rho^{\mathrm{T}} y(t)\mathrm{d}t \\ &= \langle \int_{t^0}^{t^1} y(t)\mathrm{d}t, \rho\rangle_{\mathbb{R}^s}\end{aligned}$$

Since the above equation holds for arbitrary ρ and y, the lemma is true. □

The investigation given in the previous section derives that any cost function of the input and output of the operator $\Sigma_{\mathcal{H}}$ can be minimized by only using input-output data as in iterative learning control case.

Theorem 2 and Lemma 3 implies that the closed loop system (6) should be rewritten by $\Sigma_{\mathcal{H}}(\rho)$

$$\Sigma_{\mathcal{H}} : \rho \mapsto \eta : \begin{cases} \dot{x} \;=\; (J-R)\dfrac{\partial H(x,u,\rho)}{\partial x}^{\mathrm{T}} \\ \eta \;=\; -\displaystyle\int_{t^0}^{t^1} \dfrac{\partial H(x,u,\rho)}{\partial \rho}^{\mathrm{T}} \mathrm{d}t \end{cases} \tag{8}$$

In order to utilize Theorem 2 for iterative feedback tuning, the cost function to be minimized shoud have a form $\Gamma(\rho,\eta)$.

The gradient of this cost function

$$\Gamma^\rho(\rho) := \Gamma(\rho, \Sigma_{\mathcal{H}}(\rho))$$

with respect to ρ is given as follows.

$$\begin{aligned}\langle \nabla\Gamma^\rho, \mathrm{d}\rho\rangle_{\mathbb{R}^s} &= \langle \nabla_\rho \Gamma(\rho,\eta),\ \mathrm{d}\rho\rangle_{\mathbb{R}^s} + \langle \nabla_\eta\Gamma(\rho,\eta),\ \mathrm{d}\eta\rangle_{L_2^s} \\ &= \langle \nabla_\rho\Gamma(\rho,\eta) + (\mathrm{d}\Sigma_{\mathcal{H}}(\rho))^* \nabla_\eta\Gamma(\rho,\eta),\ \mathrm{d}\rho\rangle_{\mathbb{R}^s}\end{aligned}$$

If the assumption in Theorem 2 holds, then the gradient $\nabla\Gamma^\rho$ is given by

$$\begin{aligned}\nabla\Gamma^\rho(\rho) &= \nabla_\rho\Gamma(\rho,\eta) + (\mathrm{d}\Sigma_{\mathcal{H}}^{x^0,u}(\rho))^*\nabla_\eta\Gamma(\rho,\eta) \\ &= \nabla_\rho\Gamma(\rho,\eta) + (\mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho))\ \nabla_\eta\Gamma(\rho,\eta) \qquad (9)\end{aligned}$$

Here, the partial gradients $\nabla_\rho\Gamma(\rho,\eta)$ and $\nabla_\eta\Gamma(\rho,\eta)$ are known (can be obtained by experiments). The Fréchet derivative $\mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho)$ can be obtained as welll by an approximation

$$\begin{aligned}\mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho)(\nu) &= \frac{1}{\epsilon}\mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho)(\epsilon\nu) \\ &= \frac{1}{\epsilon}\left(\Sigma_{\mathcal{H}}^{\xi^0,w}(\rho+\epsilon\nu) - \Sigma_{\mathcal{H}}^{\xi^0,w}(\rho)\right) + \frac{o(\epsilon)}{\epsilon} \qquad (10)\end{aligned}$$

as in Equation (5) Once we can obtain the gradient estimation for the cost function $\Gamma(\rho,\eta)$ based on Equations (9) and (10), the gradient method suggests the following parameter update law

$$\begin{aligned}\rho_{(i+1)} &= \rho_{(i)} - K_{(i)}\nabla\Gamma^\rho(\rho_{(i)}) \\ &= \rho_{(i)} - K_{(i)} \times \\ &\quad \left(\nabla_\rho\Gamma(\rho_{(i)},\eta_{(i)}) + \mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0_{(i)},w_{(i)}}(\rho_{(i)})\ \nabla_\eta\Gamma(\rho_{(i)},\eta_{(i)})\right) \\ &\approx \rho_{(i)} - K_{(i)} \times \left(\nabla_\rho\Gamma(\rho_{(i)},\eta_{(i)}) + \frac{1}{\epsilon_{(i)}}\times\right. \\ &\quad \left.\left(\Sigma_{\mathcal{H}}^{\xi^0_{(i)},w_{(i)}}(\rho_{(i)}+\epsilon_{(i)}\nabla_\eta\Gamma(\rho_{(i)},\eta_{(i)})) - \Sigma_{\mathcal{H}}^{\xi^0_{(i)},w_{(i)}}(\rho_{(i)})\right)\right)\end{aligned}$$

where $K_{(i)} > 0$ is the step parameter of the gradient method and the subsucript $(\cdot)_{(i)}$ denotes the data in the i-th step of iteration. In each step, we need two more experiments in order to produce the input-output map of the operator $\Sigma_{\mathcal{H}}^{\xi^0_{(i)},w_{(i)}}$. Therefore, the concrete iterative feedback tuning algorithm reduces to

$$\begin{cases} x^0_{(3i+1)} = \xi^0_{(i)} \\ u_{(3i+1)} = w_{(i)} \\ \rho_{(3i+1)} = \rho_{(3i)} \end{cases}$$
$$\begin{cases} x^0_{(3i+2)} = \xi^0_{(i)} \\ u_{(3i+2)} = w_{(i)} \\ \rho_{(3i+2)} = \rho_{(3i)} + \epsilon_{(i)}\nabla_\eta\Gamma(\rho_{(3i)},\eta_{(3i)}) \end{cases} \qquad (11)$$
$$\begin{cases} x^0_{(3i+3)} = x^0_{(0)} \\ u_{(3i+3)} = u_{(0)} \\ \rho_{(3i+3)} = \rho_{(3i)} - K_{(i)}\times \\ \qquad \left(\nabla_\rho\Gamma(\rho_{(3i)},\eta_{(3i)}) + \dfrac{1}{\epsilon_{(i)}}(\eta_{(3i+2)} - \eta_{(3i+1)})\right) \end{cases}$$

Here the condition $\xi^0_{(i)}$ and $w_{(i)}$ are chozen such that it satisfies the condition (4) with the trajectory derived by the pair $x^0_{(3i)} = x^0_{(0)}$ and $u_{(3i)} = u_{(0)}$ with $\rho_{(3i)}$. How to select $\xi^0_{(i)}$ and $w_{(i)}$ is discussed in (Fujimoto et al. [2003a]) and a concrete algorithm is given for mechanical systems in the following section.

3.3 Mechanical systems

Let us consider a simple mechanical system of the form

$$\begin{aligned} J &= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \\ R &= \begin{pmatrix} 0 & 0 \\ 0 & K_{\mathrm{D}} \end{pmatrix} \\ x &= \begin{pmatrix} q \\ p \end{pmatrix} \\ H(x,u,\rho) &= \frac{1}{2}p^{\mathrm{T}}M(q)\ p + \frac{1}{2}q^{\mathrm{T}}K_{\mathrm{P}}\ q - u^{\mathrm{T}}q \end{aligned}$$

Here $\rho := \mathrm{vec}(K_{\mathrm{P}})$. This system can be obtained by applyin the following PD feedback to a simple mechanical system without dissipation.

$$u = \bar{u} - K_{\mathrm{P}}\ q - K_{\mathrm{D}}\ \dot{q} \qquad (12)$$

Here the PD feedback gains K_{P} and K_{D} are selected such that the feedback system is asymptotically stable. The feedback system is depitcted in Figure 1. In the figure, q^r and $\dot{q}^r$ are reference signals such that the internal states q and $\dot{q}$ will track them.

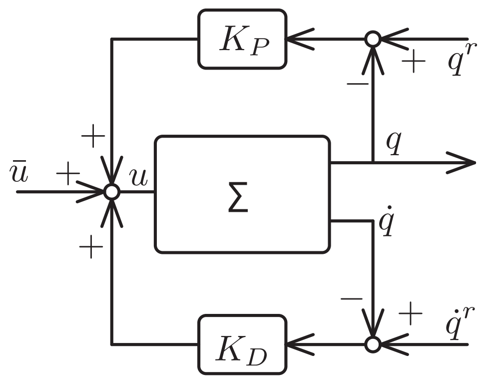

Fig. 1. Feedback system

In order to produce the trajectory of $\mathrm{d}\Sigma_{\mathcal{H}}^{\xi^0,w}$ satisfying the condition (4), we select the reference signals as

$$\begin{aligned} q^r &= \mathcal{R}(q_{old}) \\ \dot{q}^r &= -\mathcal{R}(\dot{q}_{old}) \end{aligned}$$

with the initial states

$$\begin{aligned} q^r(t^0) &= q_{old}(t^1) \\ \dot{q}^r(t^0) &= -\dot{q}_{old}(t^1) \end{aligned}$$

where q_{old} and $\dot{q}_{old}$ denote the data q and $\dot{q}$ in the previous step of iteration. That is, they are selected such that the two state tajectories $x = (q,p)$ and $x_{old} = (q_{old}, p_{old})$ satisfy the condition (4) for variational symmetry. Here we can regard that the feedforward input $\bar{u} = w$ is selected as follows in this case.

$$\begin{aligned}\bar{u} = w &= K_{\mathrm{P}} q^r + K_{\mathrm{D}} \dot{q}^r \\ &= K_{\mathrm{P}} \mathcal{R}(q_{old}) - K_{\mathrm{D}} \mathcal{R}(\dot{q}_{old})\end{aligned}$$

Using this idea, the tuning algorithm in Equation (11) reduces to

$$\begin{cases} q^0_{(3i+1)} &= q^1_{(3i)} \\ \dot{q}^0_{(3i+1)} &= -\dot{q}^1_{(3i)} \\ \bar{u}_{(3i+1)} &= K_{\mathrm{P}} \mathcal{R}(q_{(3i)}) - K_{\mathrm{D}} \mathcal{R}(\dot{q}_{(3i)}) \\ \rho_{(3i+1)} &= \rho_{(3i)} \end{cases}$$
$$\begin{cases} q^0_{(3i+1)} &= q^1_{(3i)} \\ \dot{q}^0_{(3i+1)} &= -\dot{q}^1_{(3i)} \\ \bar{u}_{(3i+2)} &= K_{\mathrm{P}} \mathcal{R}(q_{(3i)}) - K_{\mathrm{D}} \mathcal{R}(\dot{q}_{(3i)}) \\ \rho_{(3i+2)} &= \rho_{(3i)} + \epsilon_{(i)} \nabla_\eta \Gamma(\rho_{(3i)}, \eta_{(3i)}) \end{cases} \qquad (13)$$
$$\begin{cases} q^0_{(3i+1)} &= q^0_{(0)} \\ \dot{q}^0_{(3i+1)} &= \dot{q}^0_{(0)} \\ \bar{u}_{(3i+3)} &= u_{(0)} \\ \rho_{(3i+3)} &= \rho_{(3i)} - K_{(i)} \times \\ & \quad \left(\nabla_\rho \Gamma(\rho_{(3i)}, \eta_{(3i)}) + \dfrac{1}{\epsilon_{(i)}} (\eta_{(3i+2)} - \eta_{(3i+1)}) \right) \end{cases}.$$

Thus, an iteration algorithm of iterative feedback tuning for Hamiltonian control systems is obtained.

3.4 General case

In the previous sections, we have derived an iterative feedback tuning algorithm. Basically, this algorithm is to adjust the design parameters contained in the Hamiltonian function H of the closed loop system. However, in general, the design parameters to be tuned may not be contained in the Hamiltonian and the matrices J and/or R may depnend on them. For example, for the mechanical systems treated in the previous section, we introduced a PD feedback given in Equation (12). Thought the P feedback gain K_{P} is contained in the Hamiltonian, the D feedback gain K_{D} is not. In fact, the disspation matrix R depends on K_{D}. For the parameters not contained in the Hamiltonian can be tuned via conventional iterative feedback tuning method (Hjalmarsson [2002]).

Suppose that the Hamiltonian contains a tuning parameter $\rho \in \mathbb{R}^s$ and there is another parameter $\kappa \in \mathbb{R}^r$ not in the Hamiltonian. Then the number of experiments required for one step iteration of the gradient method in the conventional iterative feedback tuning is $1 + r + s$, whereas that reuqured for the proposed algorithm is $3 + r$ since additional r experiments are needed to execute conventonal iterative feedback tuning method in addition to the 3 step given in the algorithm (13). Therefore the proposed algorithm requres less number of experiments when $s > 2$. This analysis is summarized in Table 1. Furthermore, since the iterative learning control in the authors former result depends on the very same property, variational symmetry of Hamiltonian systems, the proposed algorithm can be applied to simultaneous learning control with iterative feedback tuning and iterative learning control. In this combined approach, the number of iteration is also reduced.

Table 1. The required number of experimtns in 1 step parameter estimation

	Existing algorithm	Proposed algorithm
Number of parameter included in Hamiltonian system (s)	$1 + s + r$	3
Number of parameter exclude from Hamiltonian system (r)		r
Required number	$1 + s + r$	$3 + r$

4. NUMERICAL EXAMPLE

4.1 Description of the plant

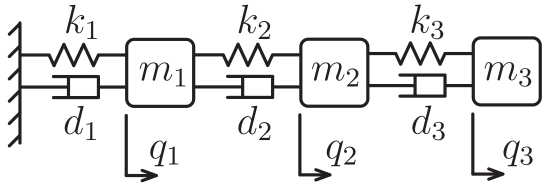

Fig. 2. A mass-spring-damper system

The proposed algorithm is applied to a three degree of freedom mass-spring-damper system depicted in Figure 2. This system can be modeled by a Hamiltonian system in Equation (1) with a Hamiltonian

$$H(q, p, \rho, u) = \sum_{i=1}^{3} \left(\frac{1}{2m_i} p_i^2 + \frac{k_i}{2} q_i^2 \right)$$

with $q = (q_1, q_2, q_3)$, $p = (p_1, p_2, p_3)$ and $\rho = (k_1, k_2, k_3)$. Here k_i's are the spring coefficients and m_i's are the masses. The variables q_i's and p_i's denote the positions and the corresponding momentums. Then the dynamics was described by a Hamiltonian system

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I_3 \\ -I_3 & -\mathrm{diag}(d_1, d_2, d_3) \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix}.$$

Since this system satisfies the conditions (2) and (3) of Theorem 1, we can apply the iterative feedback tuning method characterized in the previous section.

Let us apply the proposed algorithm to the system. Suppose that the spring coefficients are adjustable and tune them by the proposed method. (It is possible to adjust the masses m_i's via the proposed method and the dampings d_i's by the existing methods.) The physical parameters and the design parameters are summarized in Table 2.

Then the output $\eta = (\eta_1, \eta_2, \eta_3)$ in Equation (8) is given by

$$\eta_i = -\int_{t^0}^{t^1} \frac{\partial H}{\partial \rho_i} \mathrm{d}t = -\int_{t^0}^{t^1} q_i^2 \mathrm{d}t$$

Here let us take a cost function as

$$\begin{aligned}\Gamma(\eta, \rho) &= \sum_{i=1}^{3} \left(-\frac{\gamma_i}{2} \eta_i + \frac{\gamma_{i+3}}{2} \rho_i^2 \right) \\ &= \sum_{i=1}^{3} \left(\frac{\gamma_i}{2} \int_{t^0}^{t^1} |q_i|^2 \mathrm{d}t + \frac{\gamma_{i+3}}{2} k_i^2 \right)\end{aligned}$$

Table 2. Parameters

parameter	value	
q_1^0, q_2^0, q_3^0	1.0 [m]	Initial position
$\dot{q}_1^0, \dot{q}_2^0, \dot{q}_3^0$	0.0 [m/s]	Initial velocity
m_1	1.0×10^2 [kg]	Mass 1
m_2	1.0 [kg]	Mass 2
m_3	1.0×10 [kg]	Mass 3
$\epsilon_{1(i)}, \epsilon_{2(i)}, \epsilon_{3(i)}$	1.0	A small positive constant
$K_{(i)}$	5.0×10^{-2}	Step parameter
γ_1	5.0×10^6	Coeff. in the cost function
γ_2	1.0×10^4	Coeff. in the cost function
γ_3	5.0×10^4	Coeff. in the cost function
γ_4	1.0×10^{-4}	Coeff. in the cost function
γ_5	1.0×10^{-4}	Coeff. in the cost function
γ_6	1.0×10^{-4}	Coeff. in the cost function
t^0	0.0 [s]	Initial time
t^1	0.50 [s]	Terminal time
d_1, d_2, d_3	4.0×10	Damper coefficints
k_1	5.0×10^3	Initial spring coeff.
k_2	3.0×10^3	Initial spring coeff.
k_3	3.5×10^3	Initial spring coeff.

Here the positive constants γ_i's ($i = 1, 2, \ldots, 6$) are selected appropriately as in Table 2.

4.2 Simulations

Under these circumstances, we executed several simulations. Figures 3 and 4 show the result. Figure 3 depicts the hisotry of the cost function along the iteration. Since the cost function Γ decreases monotonically, we can conclude that the learning procedure works well.

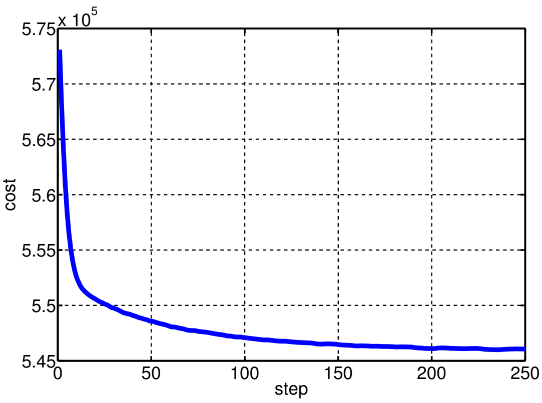

Fig. 3. History of the cost function Γ

Figure 4 shows the responses of the displacement q_3's during the learning. It depicts the renponses for $8k$-th ($k = 0, 1, 2, \ldots$) experiments. The response is oscillatory in the beginning (the thin dashed lines), and then converges to a rather smooth trajectory (the thick solid line) eventually.

5. CONCLUSION

This paper propses a new algorithm for iterative feedback tuning. We have shown a version of variational symmetry of Hamiltonian systems which can be used for estimating the gradient of a given cost function. A novel iterative feedback tuning method has been developed based on this property. The proposed method requres less number of experimens compared with the existing results and can be applied to simultaneous learning control with both iterative feedback tuning and iterative learning control. Furthermore, a numerical simulation of a 3 mass-spring system has exhibited the efficacy of the proposed altorithm.

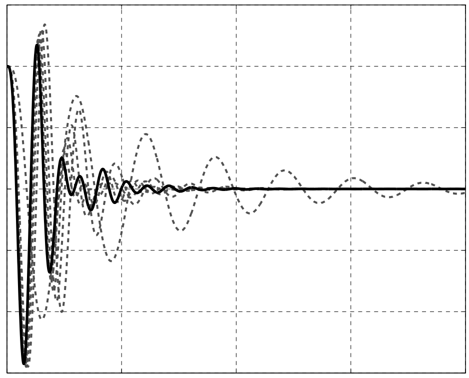

Fig. 4. Resposes of the position q_3's

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