Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Practical Robust Control for Flexible Joint Robot Manipulators

Je S. Yeon * **Jong H. Park** ** **Sang-Hun Lee** ***

* *Dept. of Mechanical Engineering, Hanyang University, 17 Haengdang-dong, Sungdong-ku, Seoul, 133-791, Korea. (Tel: +82-1-2297-3786; e-mail: mydoban@hanyang.ac.kr)*
** *School of Mechanical Engineering, Hanyang University, 17 Haengdang-dong, Sungdong-ku, Seoul, 133-791, Korea. (Tel: +82-1-2220-0435; e-mail: jongpark@hanyang.ac.kr)*
*** *Electro-Mechanical Research Institute, Hyundai Heavy Industries Co., Ltd., Yongin, Gyeonggi, 446-716,Korea. (Tel: +82-31-289-5270; e-mail: mrshlee@hanyang.ac.kr)*

Abstract: In this paper we proposed a practical robust control using transformed dynamics. The dynamic model of the flexible manipulator can be split up into two subsystems, however the transformed dynamics is made into one with the singular perturbation standard form. The proposed controller has simple structure, more easy tuning factor, and control forms having direct relation with control performance. The design procedure consists of two parts. A model based computed torque control part, and robust control part to maintain the tracking performance using the nonlinear H-infinity control. The designed robust control is applied to a 6-DOF robot manipulator with joint flexibilities. The proposed robust controller has better tracking performance and advantage in its application.

1. INTRODUCTION

To manufacture high quality products, we need high accuracy robot manipulator. However, industrial robots are complex structures with many variable parts such as electronics cables and tool kits, also there are nonlinear friction forces, variable viscose values and stiffness coefficients. In other words, an industrial robot has many uncertainties such as unmodelled dynamics, parameter variations and external disturbances. For these reasons, a model based control has a limits of tracking performance. Therefore, robust control must be considered.

A recursive design is applied to the design of a stabilizing controller for a class of nonlinear systems. Every system in the class is a series connection of a finite number of nonlinear subsystems which are individually stabilizable. Interesting progress in the recursive design has been achieved in adaptive control of feedback linearizable systems. If the linearized system is linear with respect to the parameters, the recursive design can be used to develop an adaptive control [1]. However this design is not suitable to multi links industrial robot manipulator.

Since many systems inherently have uncertainties such as parameter variations, external disturbances, and unmodelled dynamics, robust control can be considered in the recursive design. To design robust controllers, it is usual to use Lyapunov's second method, as proceeded in the existing results [2,3]. However, a difficulty of using Lyapunov's second method is that a Lyapunov function for control design is required.

Another robust control, which has attracted attention of many researchers, is H_∞ control. Although the nonlinear H_∞ control is derived by the L_2-gain analysis based on the concept of energy dissipation [4,5], its applications are not easy to implement due to the difficulty of obtaining of solution to Hamilton Jacobi inequality (HJ inequality). The H_∞ control problem in nonlinear systems reduces to the solution to HJ inequality. Many methods have been proposed in recent papers [6,7,8,9].

In recursive design of the robust control for robot manipulators with joint flexibilities, a fictitious control is designed as if the link dynamics had independent control. As the robust control, the nonlinear H_∞ control is used. The solution to the HJ inequality can be obtained through a more tractable nonlinear matrix inequality (NLMI) method due to the fact that the matrices forming the NLMI are bounded [9,10]. The control for the joint dynamics, the second subsystem, is designed recursively to satisfy the stability and robustness of the overall system by Lyapunov's second method. Finally, the saturation-type control input of a recursive robust control becomes the function of angular velocity error and bound function denoted the preceding inequality [11,12,13]. Thus, the designer must chose between robust range and tracking accuracy.

In this paper we proposed a practical robust controller which has simple structure, more easy tuning factor, and control forms having direct relation with control performance. Directly we design a robust control using the transformed dynamic equations. The design procedure consists of two parts. A model based computed torque control part, and a robust control part to maintain the tracking performance using the nonlinear H-infinity control. The designed control is applied to a 6-DOF robot manipulator

Fig. 1. A 6-DOF industrial robot.

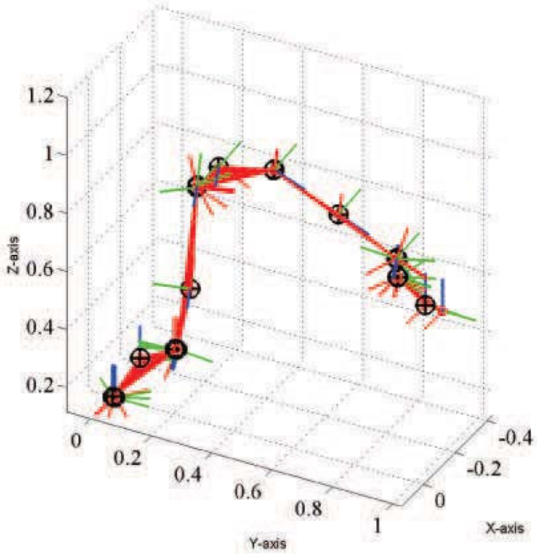

Fig. 2. A SimMechanics model.

with flexible joints. Simulations are performed for this system with inertia and stiffness uncertainties.

This paper is organized as follows. In Sec. 2, the dynamics of flexible joint robot manipulator are presented. In Sec. 3, robust control is designed for the system with uncertainties. In Sec. 4, the simulation is presented with a 6-DOF industrial robot. In Sec. 5, the conclusions are presented.

2. DYNAMICS OF FLEXIBLE JOINT ROBOT MANIPULATORS

The target model of the simulation is a heavy payload industrial robot which handles 165kg load as shown in Fig. 1. It is very difficult to model a multi-links serial robot, because of its complex structure in physical relationship. Thus we used MATLAB/SimMechanics toolbox that makes it easy to design a flexible joint mechanical system. Figure 2 is the SimMechanics model of target robot.

In the flexible manipulator model, the link dynamics is actuated by the spring torque generated by the angular difference between motor and link, and the motor dynamics is actuated by the driving torque.Consider the dynamics of robot manipulators with joint flexibility. The dynamics are

$$M(x_1)\ddot{x}_1 + C(x_1,\dot{x}_1)\dot{x}_1 + G(x_1) = K(x_2 - x_1) \quad (1)$$
$$J\ddot{x}_2 + D\dot{x}_2 + K(x_2 - x_1) = \tau \quad (2)$$

where $x_1 \in R^n$ is the link side angle, $x_2 \in R^n$ is motor side angle, $M(x_1)$ is the positive definite symmetric inertia matrix, $C(x_1,\dot{x}_1)$ represents the centripetal and coriolis torque, $G(x_1)$ represents the gravitational torque, J denotes the diagonal inertia matrix of actuator about their principal axes of rotation multiplied by the square of the respective gear ratios, D is the motor damping constant matrix, and K is the stiffness matrix.

The elastic torques at the joints are

$$h = K(x_2 - x_1). \quad (3)$$

Because the inertia matrices are non-singular, the equations of motion of the flexible system (1) and (2) are changed to the following singular perturbation standard form

$$M(x_1)\ddot{x}_1 + C(x_1,\dot{x}_1)\dot{x}_1 + G(x_1) = h \quad (4)$$
$$K^{-1}J\ddot{h} + K^{-1}D\dot{h} + [JM^{-1} + I]h \quad (5)$$
$$= [JM^{-1}C - D]\dot{x}_1 + JM^{-1}G + \tau$$

If $K^{-1} \to 0$, (1) and (2) become the equations of a quasi-steady state system such as

$$[M(x_1) + J]\ddot{x}_1 + [C(x_1,\dot{x}_1) + D]\dot{x}_1 + G(x_1) = \tau, \quad (6)$$

which approximates the rigid manipulator model. If the parasitic elasticity parameter K^{-1} is not very small, also the 'flexible' terms in the equations of motion have to be compensated for in the robust control, instead of approximating only the rigid manipulator system with $K^{-1} \to 0$.

Considering the flexible robot model (4) and (5), by substituting the elastic forces $h(t)$ of (4) into (5) we get

$$K^{-1}J\ddot{h} + K^{-1}D\dot{h} + [M + J]\ddot{x}_1 + [C + D]\dot{x}_1 + G = \tau. \quad (7)$$

Therefore we can design a controller using this transformed dynamics. The designed controller has simple structure, more easy tuning factor, and control forms having direct relation with control performance, because it is designed using a system dynamics directly instead of a back-stepping method.

3. ROBUST CONTROLLER

The link motion of the robot cannot be directly controlled by the driving torque because of elastic interconnecting mechanism. So usually it is assumed that there is a fictitious control to be used in the position of the motor angle as virtual input for robust stabilization of the link dynamics. And because the fictitious control is not real control, the real control is recursively designed to make the overall system robustly stable. These control method is called back-stepping based robust control. However the finally saturation-type control input of the back-stepping based robust control becomes the function of angular velocity error and bound function denoted the preceding inequality [9,10,12,13]. Thus, the designer must chose between robust range and tracking accuracy. In this paper, we proposed actual robust control satisfying those two main properties.

To design the robust control, we use the transformed dynamics (7) directly. The design procedure consists of two phases. A model based computed torque control part, and robust control part to maintain the tracking performance against model uncertainties. The control input, τ is proposed as

$$\tau = \tau_{ct} + \tau_{ro} \tag{8}$$

where τ_{ct} is a model based computed torque control input, and τ_{ro} is a robust control input to designed with H_∞ theory.

A model based computed torque control input is designed, which controls the nominal model, as

$$\begin{aligned}\tau_{ct} &= [\hat{M}+\hat{J}](\ddot{x}_1 d + \Lambda_1\dot{e}_1 + \Lambda_2 e_1) \\ &+ [\hat{C}+\hat{D}](\dot{x}_1 d + \Lambda_1 e_1 + \Lambda_2 \int e_1) + \hat{G} \\ &+ \hat{K}^{-1}\hat{J}(\ddot{h}_d + K_d\dot{e}_r) + \hat{K}^{-1}\hat{B}(\dot{h}_d + K_p\,\mathrm{e}_r)\end{aligned} \tag{9}$$

where $\hat{M}$, $\hat{J}$, $\hat{C}$, $\hat{D}$, $\hat{G}$, and $\hat{K}$ are the matrixes with nominal parameter values. For simple derivation.

To use the H_∞ theory, the new state s, which is the modified error for motor side joint tracking, is defined as

$$s = -\dot{e}_1 - \Lambda_1 e_1 - \Lambda_2 \int e_1 \tag{10}$$

where e_1 is link side angular errors, $e_1 = x_{1d} - x_1$, and x_{1d} is a desired position. If the elements approach zero at $t \to \infty$, the tracking errors of joints approach zero.

Then, the transformed dynamics (7) is re-expressed by the state s such as

$$\dot{s} = As + Bw + B\tau_r, \tag{11}$$

where $A = -[\hat{M}+\hat{J}]^{-1}[\hat{C}+\hat{D}]$, $B = [\hat{M}+\hat{J}]^{-1}$, and $w = \{[\hat{M}+\hat{J}] - [M+J]\}\ddot{x}_1 + ([\hat{C}+\hat{D}] - [C+D])\dot{x}_1 + [\hat{G}-G] + \hat{K}^{-1}\hat{J}(\ddot{e}_r + K_d\dot{e}_r) + \hat{K}^{-1}\hat{B}(\dot{e}_r + K_p e_r)$ which is a disturbance vector caused by model uncertainties.

The performance index matrix, z is designed such as

$$z = Hs + R\tau_{ro}, \quad H^T R = 0, \quad R^T R > 0 \tag{12}$$

where H and R are the constant matrices of suitable dimensions.

There exists a non-negative function $V(s) = s^T P s \geq 0$. The time-derivative of the non-negative energy storage function is

$$\begin{aligned}\dot{V} &= 2s^T P^T \dot{s} \\ &= 2s^T P^T (As + Bw + B\tau_r) \\ &= s^T(P^T A + A^T P)s + 2s^T P^T (Bw + B\tau_r).\end{aligned} \tag{13}$$

Introducing $\gamma^2\|w\|^2 - \|z\|^2$ into the upper equation,

$$\begin{aligned}\dot{V} &= \gamma^2\|w\|^2 - \|z\|^2 - \gamma^2\|w - \frac{1}{\gamma^2}B^T Ps\|^2 \\ &+ \|Ru_r + R^{-T}B^T Ps\|^2 + s^T\{P^T A + A^T P \\ &+ \frac{1}{\gamma^2}P^T BB^T P - P^T B[R^T R]^{-1}B^T P + H^T H\}s \\ &+ 2s^T H^T R\tau_{ro}.\end{aligned} \tag{14}$$

If there exists a matrix P satisfying the following HJ inequality such as

$$\begin{aligned}&P^T A + A^T P + \frac{1}{\gamma^2}P^T BB^T P \\ &-P^T B[R^T R]^{-1}B^T P + H^T H \leq 0\end{aligned} \tag{15}$$

and control input is designed such as

$$\tau_{ro} = -[R^T R]^{-1}B^T Ps. \tag{16}$$

Then the derivative of the storage function satisfies

$$\dot{V} \leq \gamma^2\|w\|^2 - \|z\|^2, \tag{17}$$

which achieves $L_2 - gain$ property.

To derive the HJ inequality for the robust control input, each matrix term of (11) is substituted into (15), then

$$\begin{aligned}&-[\hat{C}+\hat{D}]Q[\hat{M}+\hat{J}]^T - [\hat{M}+\hat{J}]Q^T[\hat{C}+\hat{D}]^T + \frac{1}{\gamma^2}I \\ &-[R^T R]^{-1} + [\hat{M}+\hat{J}]Q^T H^T HQ[\hat{M}+\hat{J}]^T \leq 0.\end{aligned} \tag{18}$$

where $Q = P^{-1}$.

Using the Schur complement, (18) can be described as a NLMI

$$\begin{bmatrix} W & [\hat{M}+\hat{J}]Q^T H^T \\ HQ[\hat{M}+\hat{J}]^T & -I \end{bmatrix} \leq 0 \tag{19}$$

where $W = -[\hat{C}+\hat{D}]Q[\hat{M}+\hat{J}]^T - [\hat{M}+\hat{J}]Q^T[\hat{C}+\hat{D}]^T + \frac{1}{\gamma^2}I - [R^T R]^{-1}$. The matrices $\hat{M}$ and $\hat{C}$ are the nonlinear function. However, those matrices include trigonometric functions and can be bounded when each joint velocity range between two empirically determined external values. Using this fact, we suppose that the matrices forming above NMLI vary in some bounded sets of the space of matrices.

$$[M, J, C, D, K] \in Co\{[M_i, J_i, C_i, D_i, K_i]|_{i\in\{1,2,\cdots,L\}}\}$$

where Co represents the convex hull and L is the number of vertices of bounded space. Therefore, if there exists a solution Q to (20), then it is also a solution to (19) [9].

$$\begin{bmatrix} W_i & [\hat{M}_i + \hat{J}_i]Q^T H^T \\ HQ[\hat{M}_i + \hat{J}_i]^T & -I \end{bmatrix} \leq 0 \qquad (20)$$

where $W_i = -[\hat{C}_i + \hat{D}_i]Q[\hat{M}_i + \hat{J}_i]^T - [\hat{M}_i + \hat{J}_i]Q^T[\hat{C}_i + \hat{D}_i]^T + \frac{1}{\gamma^2}I - [R^T R]^{-1}$. This approach provides a tractable method for obtaining a constant solution to NLMI, which can be used to design the robust control input. However, this approach generally leads to conservative results if the prescribed bound is large.

Therefore, the stabilizing robust control becomes

$$\begin{aligned} \tau &= \tau_{ct} + \tau_{ro} \qquad (21) \\ &= [\hat{M} + \hat{J}](\ddot{x}_1 d + \Lambda_1 \dot{e}_1 + \Lambda_2 e_1) \\ &+ [\hat{C} + \hat{D}](\dot{x}_1 d + \Lambda_1 e_1 + \Lambda_2 \int e_1) + \hat{G} \\ &+ \hat{K}^{-1}\hat{J}(\ddot{h}_d + K_d \dot{e}_r) + \hat{K}^{-1}\hat{B}(\dot{h}_d + K_p e_r) \\ &- [R^T R]^{-1}[\hat{M} + \hat{J}]^{-T} Ps. \end{aligned}$$

4. SIMULATIONS

The target model of the simulation is a heavy payload industrial robot which handles 165kg load. We used MATLAB/SimMechanics toolbox to design a flexible joint mechanical system. We assumed that this robot has model uncertainties about it inertia and joint stiffness.

The robust performance of the proposed robust control for the 6-DOF robot manipulators is verified through simulation against inertia and stiffness uncertainties. For estimating the performance of a proposed controller, we use a rectangular trajectory in the 3 dimensional spaces.

The matrices $\hat{M}$ and $\hat{C}$ include trigonometric functions and can be bounded when each joint velocity range between two empirically determined external values. Using this fact, we suppose that the matrics forming above NMLI vary in some bounded sets of the space of matrices. Therefore the NLMI is solved off-line. The performance level can be determined by parameter γ and weighting matrix H, and the control input energy can be adjusted by using matrix D.

The end position errors are shown in Fig. 3. It has very small size error excepting four corners, starting and stoping times. In these phases, acceleration and deceleration are so high, so this situations are occupied. Figure 4 show the joint angle errors. In starting stage, the noisy signal is caused by initial angular velocity errors.

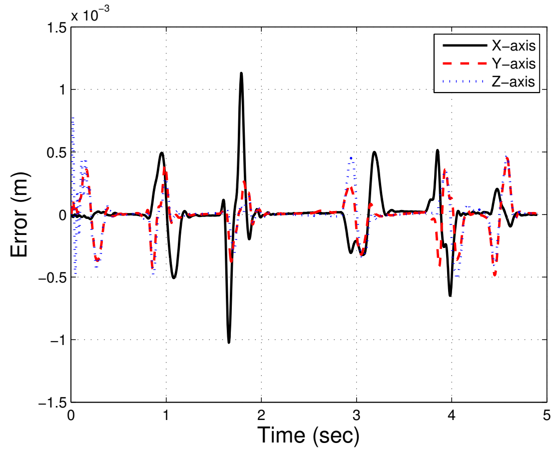

Fig. 3. Position errors of robot end point.

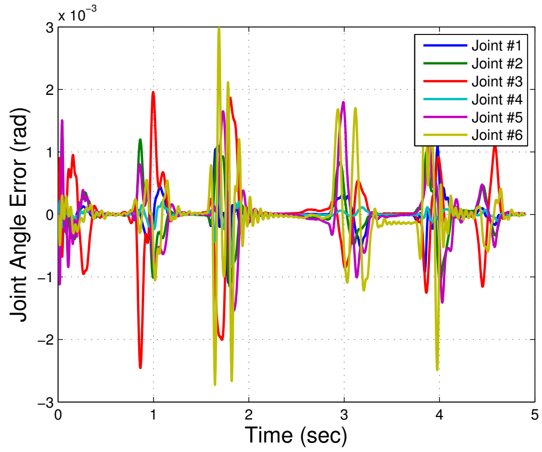

Fig. 4. Joint angle errors.

Figures 5 and 6 show the end position of 6-DOF robot manipulator with model uncertainties of 30%. The legend 'DT' means a desired trajectory. The proposed robust controller has small size error changes, it means that the proposed control has a robustness to the parameter uncertainties.

Though the most of robust controllers has a fine performance to a inertia uncertainty, it dose not to a stiffness uncertainty. However the proposed robust control has a good robustness to a stiffness uncertainty especially. Figures 7 and 8 show the end position errors. In case of the stiffness uncertainty, there is a little change of position errors.

5. CONCLUSIONS

A practical robust control was proposed for flexible joint manipulators. To design the robust control, we use the

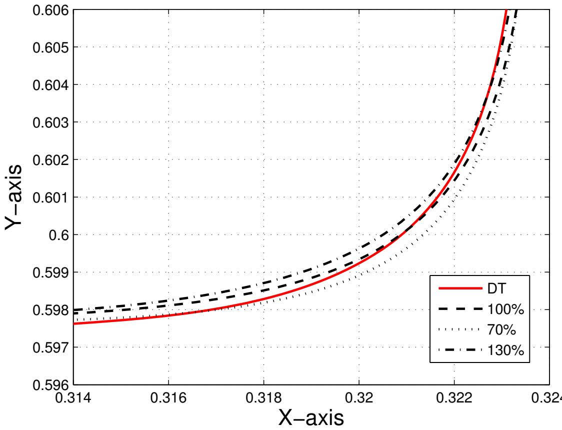

Fig. 5. Position under mass uncertainties.

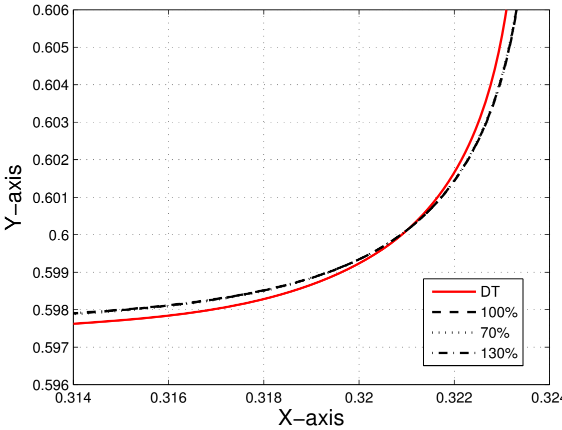

Fig. 6. Position under stiffness uncertainties.

transformed dynamics directly. And the designed controller consists of two phases, a model based computed torque control part, and robust control part to maintain the tracking performance against model uncertainties using H-infinity theory. In case of the saturation-type control input of a recursive robust control is very difficult to tuning gain values. However the proposed robust control has simple structure, more easy tuning factor, and control forms having direct relation with control performance. As a result of simulations, the proposed robust controller has high accuracy performance and robustness against the disturbances and model uncertainties. Especially it has a nice robustness to a stiffness uncertainty.

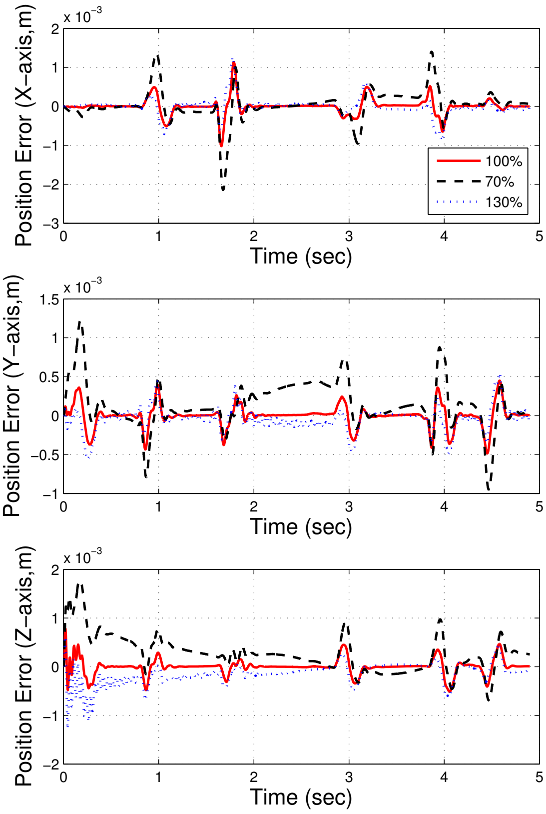

Fig. 7. Position errors under mass uncertainties.

REFERENCES

[1] I. Kanellakppoulos, P. V. Kokotovic, and A. S. Morse, "Systematic Design of Adaptive Controllers for Feedback Lilearizable Systems," IEEE Trans. Automat. Contr., Vol. 36, pp. 1241-1253, 1997.

[2] S. Gutman, "Uncertain Dynamical Systems-A Lyapunov Min-Max Approach," IEEE Trans. Automat. Contr., Vol. 24, pp. 737-463, 1999.

[3] M.J. Corless and G. Leitmann, "Continuout State Feedback Guaranteeing Uniform Boundedness for Uncertain Dynamic System," IEEE Trans. Automat. Contr,. Vol. 26, pp. 1739-2144, 1981.

[4] A.J. van der Schaft, "L_2-Gain Analysis of Nonlinear Systems and Nonlinear State Feedback," IEEE Trans. Automat. Contr., Vol. 37, No. 6, pp. 770-784, 1992.

[5] A. Isidori and A. Astolfi, "Disturbance Attenuation and H_∞-Control via Measurement Feedback in Nonlinear Systems," IEEE Trans. Automat. Contr., Vol. 37, No. 3, pp. 1282-1293, 1992.

[6] A. Astnlfi and L. Lanari, "H_∞ Control of Rigid Robots," IFAC Robot Control, pp. 271-176, 1994.

[7] S.-S. Hu and B.-C. Chang, "Design of a Nonlinear H_∞ Controller for the Inverted Pendulum System," Proc. Of IEEE International Conference on Control Application, pp. 699-707, 1998.

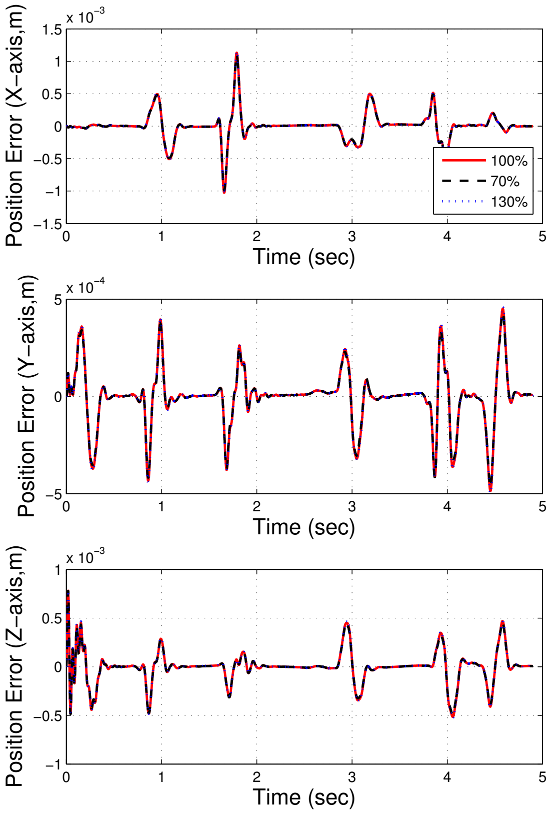

Fig. 8. Position errors under stiffness uncertainties.

[8] W. M. Lu and J. C. Doyle, "H_∞-Control of Nonlinear Systems: a Convex Characterization," Caltech CDS Tech. Memo., No. CTI-CDS-93-020, 1993.

[9] J. H. Park and J. Yim, "Nonlinear H_∞ Control of Robotic Manipulator," Proc. Of IEEE Conference on Systems, Man and Cybernetics, pp. 866-871, 1999.

[10] J. H. Park and J. Yim, "Robust Control of Robot Manipulator with Actuators," Proc. Of KSME International Journal, Vol. 15, pp. 320-326, 2001.

[11] Z. Qu and D. M. Dawson, Robust Tracking Control of Robot Mnipulators, IEEE Press, 1996.

[12] S. Lee and J. Hur, "A PID-Type Robust Controller Design for Industrial Robots with Flexible Joints," Proc. Of SICE-ICASE International Joint Conference, pp. 5905-5910, 2006.

[13] J. Yim, J. S. Yeon and J. H. Park, "Robust Control using Recursive Design Method for Flexible Joint Robot Manipulator," Proc. Of IEEE International Conference on Robotics and Automation, pp. 3806-3811, 2007.