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Distributed Control with Integral Quadratic Constraints

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Abstract: In this paper, stability conditions for distributed systems with general Integral Quadratic Constraints (IQC) on the interconnections are derived. These results take the form of coupled Linear Matrix Inequalities (LMIs), where the multipliers are shaped by the underlying IQCs. It is further shown how these results can be exploited to design distributed controllers in a way similar to the gain-scheduling controller design in Linear Parameter Varying (LPV) systems.

Keywords: Distributed Control, IQC, Controller Synthesis.

1. INTRODUCTION

Over the past few years, there has been renewed research interest in the distributed control of large scale systems; see for example, Langbort et al. [2004], Dullerud and D' Andrea [2004], Langbort et al. [2006], Ugrinovskii et al. [2000], Scorletti and Due [2001], Chen and Lall [2003]. Many of these systems are formed by interconnecting multiple homogeneous or heterogeneous subsystems. These systems typically exhibit overall complex dynamical behavior dictated by their distributed nature and the dynamical interactions between the subsystems.

In physically distributed systems, the observations are highly distributed and this has motivated the development of new research directions in control theory, namely control under communication constraints. In particular, researchers have considered control problems with non-ideal communication links such as limited bandwidth (Tatikonda [2000], Nair and Evens [2000]), delay, and packet dropout between sensors, actuator of these subsystems. See the special issues Antsaklis and Baillieul [2004], Antsaklis and Baillieul [2007]. Standard control design techniques for these systems often fail because of the high dimension of the system and the high communication and computation costs to implement centralized control algorithms. Decentralized control schemes have been deployed for large-scale applications in special cases. Some synthesis methods have also been proposed for decentralized controllers that guarantee performance, however, these decentralized controllers are generally conservative.

Recently, a distributed control theory for spatially-invariant distributed systems has been developed by Langbort et al. [2004]. It was shown that the controllers have 'identical' structure as the underlying subsystems. A Linear Matrix Inequality (LMI) based control synthesis algorithm for this class of interconnected systems was developed in D' Andrea and Dullerud [2003], Dullerud and D' Andrea [2004] using a multidimensional system theory. These results were further extended in Langbort et al. [2004], Langbort et al. [2006] and Di et al. [2006] to distributed system over an arbitrary graph under various connections. Specifically, the results take the form of a set of coupled linear matrix inequalities. The design variables for the LMIs are shaped by the interconnections.

The distributed control approach described above can also be derived using gain-scheduling techniques for Linear Parameter Varying (LPV) systems (Scorletti and Ghaoui [1998], Packard [1994]). This point of view will be taken in this paper. The distributed stability results follow from an application of the S-procedure developed in Yakubovich [1977], Megretski and Treil [1993], Yakubovich [1992] where the interconnections are parameterized as a family of IQCs. Furthermore, the stability conditions under perfect communication can be proved via the block S-procedure if a set of proper quadratic separators is chosen (Scherer [2001], Iwasaki and Shibata [2001]). As it is shown below, all the stability results can be interpreted from a graph separation point view (Safonov [1980], Moylan and Hill [1978], Iwasaki and Hara [1998]) following a similar proof as in Scorletti and Ghaoui [1998]. While the sufficiency of the distributed stability results can be easily derived via a graph separation argument, the necessity part, which can be derived only for special interconnections, follows from the lossless (D, G) scaling theorem for LPV uncertainties (Meinsma et al. [2000]). As mentioned in Langbort et al. [2004], these stability results can be explained in the general framework of dissipative theory (Willems [1972]). They are well connected to the integral quadratic constraints analysis methods since the interconnections are generally modeled by IQCs (Megretski and Rantzer [1997]). For stability, in the present paper, we explore this connection so that we can unify all these stability results and treat systems with more general interconnections in this framework. As for synthesis, based on a recently extended elimination lemma in Helmersson [1999], the synthesis inequalities turn out to be convex in all variables, including the scalings and controller parameters (Scherer [2001]). However, these techniques can only be applied under certain inertia hypothesis on the multipliers. We further point out that in Langbort et al. [2004] the synthesis condition for the ideal interconnection case $n_{ij}^K = 3n_{ij}$ was derived with an inertia restriction on the scaling matrix. it can be shown that there exist distributed controllers to guarantee the global control performance without that inertial assumption and under milder restrictions on the dimension, namely $n_{ij}^K = n_{ij}$ (Hui and Antsaklis [2007]).

The remainder of the paper is organized as follows. The interconnected system is introduced in Section 2, where each of the individual Linear Time Invariant (LTI) subsystems is represented in state space form, and operators are introduced to model the interconnections. The main result, the performance and stability analysis theorem for the global systems with a general IQC for the interconnections is presented in Section 3. As an application of this result, stability conditions for several interconnections are also derived. In Section 4, based on the stability results for the specific interconnections and the elimination lemma, we present results for the synthesis of distributed controller.

Notation The notation is standard. Real and negative reals are denoted by $\mathbb{R}$, $\mathbb{R}^+$. $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. The transpose (complex conjugate transpose) of matrix M is denoted by $M^T(M^*)$. Let $U \in \mathbb{R}^{r\times n}$ with $r < n$. $U_\perp$ denotes an orthogonal complement of U, i.e $UU_\perp = 0$ and $[U^T U_\perp]$ is of full rank. We use $\mathbb{R}_S^n$ to denote $n \times n$ real symmetric matrices. If $M \in \mathbb{R}_S^n$, then $M > 0 (M \geq 0)$ indicate M is positive definite (positive semidefinite) matrix, and $M < 0 (M \leq)$ denotes negative definite (negative semidefinite) matrix. For any matrix P, $\ker(P)$ stands for the null space of P. The inertia of a symmetric matrix A is the ordered triple $\text{in}(A) = (i_+(A), i_0(A), i_-(A))$ where $i_+(A), i_-(A), i_0(A)$ are the numbers of positive, negative and zero eigenvalues of A, all counting multiplicity.

A block diagonal matrix with $X_k, \ldots, X_l$ is denoted $\mathbf{diag}_{k\leq i\leq l} X_i = \mathbf{diag}\{X_k, \ldots, X_l\}$ on the diagonal, likewise, if $e1, \ldots, e_L$ are elements of sets $E_1, \ldots, E_L$, $\mathbf{cat}_{k\leq i\leq l} e_i$ will designate the elements $(e_k, \ldots, e_l) \in E_k \times \ldots E_l$ when $1 \leq k \leq l \leq L$. We will sometimes write $\mathbf{diag}_i$ and $\mathbf{cat}_i$ instead of $\mathbf{diag}_{1\leq i\leq L}$ and $\mathbf{cat}_{1\leq i\leq L}$.

The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $\|x\| = (x^T x)^{1/2}$. The space of square integrable n-dimensional functions $f : (0, \infty) \to \mathbb{R}^n$ is denote by $\mathcal{L}_2^n$; this is abbreviated as $\mathcal{L}_2$ when n is clear from context or not relevant. The inner product between two signals in $\mathcal{L}_2$ is denoted by $< \cdot, \cdot >$. The Fourier transform of a $\mathcal{L}_2$ function f is denoted as $\hat{f}(j\omega)$. The norm of an $\mathcal{L}_2$ signal and the induced norm of an operator on $\mathcal{L}_2$ is denoted by $\|\cdot\|$, so for an operator $F : \mathcal{L}_2 \to \mathcal{L}_2$, $\|F\| = \sup_{u\in\mathcal{L}_2} \frac{\|Fu\|}{\|u\|}$. An operator $\Delta : \mathcal{L}_2^n \to \mathcal{L}_2^n$ is said to be contractive if $\|\Delta v\| < \|v\|, \forall v \in \mathcal{L}_2^n$. Lower case δ's always denote operators from $\mathcal{L}_2^1$ to $\mathcal{L}_2^1$. Then for $u, v \in \mathcal{L}_2^n$, the expression $v = \delta I_n u$ is defined to mean that u_k of u and v_k of v satisfy $u_k = \delta v_k$. An operator $\delta : \mathcal{L}_2 \to \mathcal{L}_2$ is called self-adjoint if $< u, \delta v > = < \delta u, v >, \forall u, v \in \mathcal{L}_2$. Note that all real-valued static Linear Time Varying (LTV) operators are self-adjoint.

2. PROBLEM FORMULATION

2.1 Problem Formulation

The global system consists of an assembly of L subsystem $G_i, i = 1, \ldots, L$, connected arbitrarily.

Each subsystem G_i is described by the following state-space equation:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ z_i(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^i & B_{Td}^i \\ A_{ST}^i & A_{SS}^i & B_{Sd}^i \\ C_{Tz}^i & C_{Sz}^i & D_{zd}^i \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \end{bmatrix} \tag{2.1}$$

$$x_i(0) = x_i^0 \tag{2.2}$$

where $x_i(t) \in \mathbb{R}^{m_i}, d_i(t) \in \mathbb{R}^{p_i}, z_i(t) \in \mathbb{R}^{q_i}, v_i(t), w_i(t) \in \mathbb{R}^{n_i}$ for all $t \geq 0$. In (2.1), d_i is the disturbance and z_i is the performance associated with G_i, while v_i and w_i are the overall interconnection signals used by G_i. For each given i, v_i and w_i are further partitioned into v_{ij}, w_{ij} respectively, i.e., the n_{ij}-dimension signal that is shared by G_i and G_j. We model the interconnection via an operator Δ_{ij}, such that,

$$v_{ij} = \Delta_{ji} w_{ji}, \quad \forall i, j, 1 \leq i, j \leq L \tag{2.3}$$

For example, a simple case would be, $w_{ij} = v_{ji}$ which is called ideal/perfect interconnection. However, we generally model the interconnection signal subspace as $\mathcal{W}(\Delta_{ij})$, such that

$$\mathcal{W}(\Delta_{ji}) = \left\{ \begin{bmatrix} v_{ij} \\ w_{ji} \end{bmatrix} \in \mathcal{L}_2^{2n_{ji}} : v_{ij} = \Delta_{ji} w_{ji} \right\} \tag{2.4}$$

We denote $v = \mathbf{cat}_i v_i$, where each v_i can be further partioned as $v_i = \mathbf{cat}_j v_{ij}$. Note that the dimension of v_{ij}, v_i and v are n_{ij}, n_i and $\mathcal{N}$ where $n_i = \sum_{j=1}^L n_{ij}$, $\mathcal{N} = \sum_{i=1}^L n_i$. The global system signals, $x = \mathbf{cat}_i x_i, w = \mathbf{cat}_i w_i, z = \mathbf{cat}_i z_i, d = \mathbf{cat}_i d_i$ are similarly defined.

Based on the representations of G_i in (2.1), the state space representation of the global system can be described as

$$\begin{bmatrix} \dot{x}(t) \\ w(t) \\ z(t) \end{bmatrix} = \left[\begin{array}{c|cc} A_{TT} & B_{TS} & B_{Td} \\ \hline A_{ST} & A_{SS} & B_{Sd} \\ C_{Tz} & C_{Sz} & D_{zd} \end{array}\right] \begin{bmatrix} x(t) \\ v(t) \\ d(t) \end{bmatrix} \tag{2.5}$$

$$v(t) = \Delta P_r w(t) \tag{2.6}$$

where Δ is a (causal) operator from $\mathcal{L}_2^{\mathcal{N}}$ to $\mathcal{L}_2^{\mathcal{N}}$ generated via Δ_{ij},

$$\Delta = \mathbf{diag}_i \mathbf{diag}_j \Delta_{ji} \tag{2.7}$$

and the permutation matrix P_r is chosen such that

$$\bar{w} = \mathbf{cat}_i \mathbf{cat}_j w_{ji} = P_r w = P_r \mathbf{cat}_i \mathbf{cat}_j w_{ij} \tag{2.8}$$

$A_{TT} = \mathbf{diag}_i A_{TT}^i$. All other matrices in (2.5) are similarly defined. The signals $w(t)$ and $v(t)$ are $\mathbb{R}^{\mathcal{N}}$-valued internal signals. The signal space for for v, w can be described as

$$\mathcal{W}(\Delta) = \left\{ \begin{bmatrix} v \\ w \end{bmatrix} \in \mathcal{L}_2^{2\mathcal{N}} : \begin{bmatrix} v_{ij} \\ w_{ji} \end{bmatrix} \in \mathcal{L}_2^{2n_{ji}}, v_{ij} = \Delta_{ji} w_{ji} \right\} \tag{2.9}$$

From (2.5), we obtain the transfer function

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{2.10}$$

which has been partitioned to conform with the vector (v, d). In this paper, the interconnected system is called well-posed and stable if the system (2.5) is internally stable regardless of the uncertainty of the interconnection operator Δ_{ij} defined by 2.9.

Definition 2.1. The interconnected system consisting of subsystems (2.5) and the interconnection constraints (2.9) is said to be well-posed and stable if the map $(I-\Delta PG_{11})$ has a bounded inverse on $\mathcal{L}_2$, for any Δ in a prescribed uncertainty set.

Finally, we say that a system (2.5) is contractive if it is stable and $\|z\| < \|d\|$ for all $d \in \mathcal{L}_2$ and all interconnection uncertainty modelled by Δ_{ij} in 2.9.

3. STABILITY ANALYSIS VIA IQC

The main idea here is to first use integral quadratic constraints to model the interconnection operator Δ_{ij}. The performance under the integral quadratic constraints (IQC) for the internal signal v, w can then be casted as an unconstrained quadratic optimization problem via the S-procedure (Megretski and Rantzer [1997]). For the LTI system, the stability results admit a LMI formulation. For this purpose, we need the following definitions of IQC and of a dissipative operator.

Definition 3.1. A class of signal $\mathcal{W}, \mathcal{W} \subset \{w : w \in \mathcal{L}_2^n\}$ is said to satisfy the IQC defined by $\Pi(\omega)$ if $\sigma(w, \Pi(\omega)) \geq 0, \forall w \in \mathcal{W}$, where σ is of the form

$$\sigma(w, \Pi(\omega)) = \int_{-\infty}^{\infty} \hat{w}(j\omega)^* \Pi(\omega) \hat{w}(j\omega) d\omega \qquad (3.11)$$

$\hat{w}(j\omega)$ is the Fourier transform of w, and $\Pi(j\omega) = \Pi^*(j\omega)$ is a matrix function referred to as the multiplier of σ and assumed to be bounded on the imaginary axis. In the sequel, we will refer to condition $\sigma(w, \Pi(\omega)) \geq 0$ (3.11) as an IQC with multiplier $\Pi(\omega)$.

Definition 3.2. Let $\mathcal{H} : \mathcal{L}_{2e} \to \mathcal{L}_{2e}$ be an operator. $\mathcal{H}$ is $\{X, Y, Z\}$-dissipative if there exist real matrices X, Y, Z such that

$$\Phi = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$$

is a full rank matrix and with $p(t) = \mathcal{H}(q(t)), p, q \in \mathcal{L}_{2e}$

$$\int_0^{\infty} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix}^T \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix} dt \geq 0 \qquad (3.12)$$

Note that, condition (3.12) can be easily represented in the frequency domain as an IQC of the form (3.11). If $\mathcal{H}$ is stable and time-invariant, $\Pi(\omega)$ is restricted to be a constant matrix. We often call (3.12) an IQC in the time-domain form.

Many important interconnections used in stability analysis can be characterized by IQC's with proper multiplier $\Pi(\omega)$. A collection of commonly used IQC's has been summarized in Megretski and Rantzer [1997]. Based on results on (D, G)-scaling, the following linear time varying (LTV) operators of fixed block and scalar operators can be equivalently represented by IQCs with proper constant multiplier (Π Iwasaki and Hara [1998]).

Lemma 3.1.
- Suppose $\widetilde{\delta} : \mathcal{L}_n^2 \to \mathcal{L}_n^2$, if the LTV operator $\widetilde{\delta}$ is self-adjoint and contractive, then for any $D \in \mathbb{R}_S^{n\times n}, D \geq 0$ and $G = -G^T$, $\widetilde{\delta} I_n$ is $(-D, G, D)$-dissipative.
- Suppose $\delta : \mathcal{L}_n^2 \to \mathcal{L}_n^2$, if the LTV operator is contractive , then for any $D \in \mathbb{R}_S^{n\times n}, D \geq 0$, δI_n is $(-D, 0, D)$-dissipative.
- There is a contractive LTV operator, $\Delta : \mathcal{L}_n^2 \to \mathcal{L}_n^2$ such that $p = \Delta q$ if and only if Δ is $(-I, 0, I)$-dissipative.

Definition 3.3. A quadratic performance criterior is a quadratic functional $\sigma_p(z, d)$ defined as

$$\sigma_p(z, d) = \int_0^{\infty} \begin{bmatrix} d(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{p1} & \Pi_{p2} \\ \Pi_{p2}^T & \Pi_{p3} \end{bmatrix} \begin{bmatrix} d(t) \\ z(t) \end{bmatrix} dt \qquad (3.13)$$

A system satisfies the σ_p-performance criterion over a set of disturbances $\mathcal{W}$ if the system is well-posed, internally stable and its performance measurement z satisfies $\sigma_z(z, d) < 0$.

The following proposition gives a sufficient condition for the system that satisfies the performance criterion $\sigma_p < 0$ over a class of signals $\mathcal{W}$ which can be characterized by IQCs. If the operator Δ used to model the interconnection $v = \Delta w$ can be characterized by several IQCs, $\sigma_{w1}, \sigma_{w2}, \ldots, \sigma_{wn}$, then the performance can be formulated as a convex feasibility problem over the set of IQCs via the S-procedures,

$$\sigma_p(z, d) + \sum_{i=1}^{n} \lambda_i \sigma_{wi}(w) < 0, \ \forall w \in \mathcal{L}_2. \qquad (3.14)$$

The following proposition proved in Scorletti and Ghaoui [1998] is a direct application of the S-procedure to the interconnected system (2.5) and (2.6).

Proposition 1. Suppose the operator ΔP in (2.6) is $\{X, Y, Z\}$-dissipative, then the interconnected systems (2.5), (2.6) satisfies $\sigma_p(z, d)$ performance (3.13), if there exists symmetric matrix $X_T \in \mathbb{R}_S^{m\times m}, X_T > 0$, such that the following LMI holds true.

$$M^T P M < 0 \qquad (3.15)$$

where

$$M = \left[\begin{array}{ccc} I & 0 & 0 \\ A_{TT} & B_{TS} & B_{Td} \\ \hline 0 & I & 0 \\ A_{ST} & A_{SS} & B_{Sd} \\ \hline 0 & 0 & I \\ C_{Tz} & C_{Sz} & D_{zd} \end{array}\right]$$

$$P = \left[\begin{array}{cc|cc|cc} 0 & X_T & 0 & 0 & 0 & 0 \\ X_T & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & X & Y & 0 & 0 \\ 0 & 0 & Y^T & Z & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \Pi_{p1} & \Pi_{p2} \\ 0 & 0 & 0 & 0 & \Pi_{p2}^T & \Pi_{p3} \end{array}\right]$$

Note here, we are only use the sufficient part of the S-procedure to derive the above sufficient performance conditions. From the lossless (D, G) scaling theorem for linear time invariant(LTI) systems with LPV uncertainties, we know that for the contractive operators ($\widetilde{\delta}, \delta$ and Δ) considered in lemma 3.1, the above results are both necessary and sufficient (Meinsma et al. [2000]) with proper multipliers X, Y, Z, and they are referred to as (D, G)-scalings for such LTV operators. Generally speaking, the sufficient part of Proposition 1 can be easily proved via a separation of graph argument. The inner matrix in equation (3.15) can be interpreted as a hyperplane that separates the graph of the linear time invariant system and the operators that model the time-varying interconnections.

The necessary part follows the idea proposed in Shamma [1994] for the full block uncertainty LTV Δ to construct a causal destabilizing operator when strict separation of the two graph is violated; the scalar case $\delta, \widetilde{\delta}$ has been proved in Megretski and Treil [1993], Meinsma et al. [2000] respectively. For the contractive operator list in lemma 3.1, the above proposition is a LMI reformulation of the necessary and sufficient conditions presented in Meinsma et al. [2000] via an application of the KYP lemma to the LTI system (2.5) with scaling matrices X, Y, Z.

IQC for the interconnections We introduce the following IQC to model the global interconnection, $v = \Delta P w$. For each $i = 1, \ldots, L$, let us introduce the quadratic form on $\mathbb{R}^{n_i} \times \mathbb{R}^{n_i}$, such that

$$\mathcal{P}_{ij}(v_{ij}, w_{ij}) = \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T X_{ij} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} \tag{3.16}$$

The scaling matrix X_{ij} is further partitioned into four $n_{ij} \times n_{ij}$ blocks as

$$X_{ij} = \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ (X_{ij}^{12})^T & X_{ij}^{22} \end{bmatrix} \tag{3.17}$$

We are now able to state our first analysis conditions. The proof of Theorem 2 follows from Proposition 1 by utilizing the diagonal structure of the global system (2.5).

Theorem 2. The interconnected system (2.5), (2.6) is well-posed, stable and contractive if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $X_{ij} \in \mathbb{R}_S^{2n_{ij} \times 2n_{ij}}$, $X_T^i > 0$ such that

$$M_i^T P_i M_i < 0 \tag{3.18}$$

for all $i = 1, \ldots, L$. where

$$M_i = \left[\begin{array}{ccc} I & 0 & 0 \\ A_{TT}^i & A_{TS}^i & B_{Td}^i \\ \hline 0 & I & 0 \\ A_{ST}^i & A_{SS}^i & B_{Sd}^i \\ \hline 0 & 0 & I \\ C_{Tz}^i & C_{Sz}^i & D_{zd}^i \end{array}\right] \tag{3.19}$$

$$P_i = \left[\begin{array}{cc|cc|cc} 0 & X_T^i & 0 & 0 & 0 & 0 \\ X_T^i & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & P_i^{11} & P_i^{12} & 0 & 0 \\ 0 & 0 & (P_i^{12})^* & P_i^{22} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{array}\right] \tag{3.20}$$

$$P_i^{11} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{11} \tag{3.21}$$

$$P_i^{22} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{22} \tag{3.22}$$

$$P_i^{12} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{12} \tag{3.23}$$

and

$$\begin{aligned} \sigma(P_X) &= \int_0^\infty \begin{bmatrix} v \\ w \end{bmatrix}^T P_X \begin{bmatrix} v \\ w \end{bmatrix} dt \\ &= \sum_{1 \le i,j \le L} \int_0^\infty \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ (X_{ij}^{12})^T & X_{ij}^{22} \end{bmatrix} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} dt \\ &\ge 0 \end{aligned} \tag{3.24}$$

As applications of Theorem 2, it is of interest to use the above stability results to model different interconnections.

Ideal Interconnections Here we assume that $\Delta_{i,j=I_{n_{ij}}}$, $\forall i, j = 1, \ldots, L$, i.e., at anytime t

$$v_{ij}(t) = w_{ji}(t), \quad \forall i, j, t \ge 0 \tag{3.25}$$

In this case, suppose we choose for all $1 \le i, j \le L$

$$X_{ij}^{11} + X_{ji}^{22} = 0$$
$$X_{ij}^{12} + (X_{ji}^{12})^T = 0$$

then

$$\begin{aligned} \sigma(P_{X_{ideal}}) &= \sum_{1 \le i,j \le L} \int_0^\infty \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T X_{ij} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} dt \\ &= 0 \end{aligned}$$

The family of multiplier X_{ideal} can thus be characterized by the two sets of matrices,

$$\left\{ X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}, i, j = 1, \ldots, L \right\}$$

and

$$\left\{ X_{ij}^{12} \in \mathbb{R}_S^{n_{ij} \times n_{ij}} : X_{ii}^{12} \text{skew-symmetric}, 1 \le j \le i \le L \right\}$$

Once we have identified the IQC for such interconnection $\Delta_{ij} = I_{n_{ij}}$, we have the following proposition, as an application of Theorem 2.

Proposition 3. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all $\Delta_{ij} = I_{n_{ij}}$ if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $X_{ij}^{12} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i \ge j$ with X_{ii}^{12} skew-symmetric, such that $X_T^i > 0$ and the LMIs(3.18) hold true for all $i = 1, \ldots, L$ with

$$P_i^{11} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{11}$$
$$P_i^{22} = \mathbf{diag}_{1 \le j \le L} - X_{ji}^{11}$$
$$P_i^{12} = \mathbf{diag}\left(\mathbf{diag}_{1 \le j \le i} X_{ij}^{12}, \mathbf{diag}_{i \le j \le L} - (X_{ji}^{12})^T\right)$$

Directed interconnection with $\Delta_{ij} = \delta I_{n_{ij}}$, $\|\delta\| \le 1$. Let us now consider the new class of interconnected systems with $\Delta_{ij} = \delta_{ij} I_{n_{ij}}$ and $\|\delta_{ij}\| \le 1$. We are seeking a new IQC to model such interconnections.

Following similar derivation, we can parameterize the multipliers X_{ij} by the following sets of matrices

$$\left\{ X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}} : X_{ij}^{11} < 0, i, j = 1, \ldots, L \right\}$$

Besides, we require $X_{ji}^{22} = X_{ij}^{11}$ and $X_{ij}^{12} = 0$ for all $i, j = 1 \ldots, L$.

In this case, it is easy to verify

$$\begin{aligned} \sigma(P_{X_\delta}) &= \frac{1}{2} \sum_{1 \le i,j \le L} \left\langle \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}, \begin{bmatrix} X_{ij}^{11} & 0 \\ 0 & -X_{ji}^{11} \end{bmatrix} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} \right\rangle \\ &\quad + \left\langle \begin{bmatrix} v_{ji} \\ w_{ji} \end{bmatrix}, \begin{bmatrix} X_{ji}^{11} & 0 \\ 0 & -X_{ij}^{11} \end{bmatrix} \begin{bmatrix} v_{ji} \\ w_{ji} \end{bmatrix} \right\rangle \\ &= \sum_{1 \le j \le i \le L} < v_{ij}, X_{ij}^{11} v_{ij} > - < w_{ji}, X_{ij}^{11} w_{ji} > \\ &\ge 0 \end{aligned}$$

Once we have found the *IQC* to model the interconnection δ_{ij}, it's straightforward to apply Theorem 2 to get the sufficient part of the propositions that follow below. The necessary part follows from the lossless-(D, G)-scaling theorem for LTV uncertainties; the details are omitted here.

Proposition 4. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all $\Delta_{ij} = I_{n_{ij}}\delta, \|\delta\| \leq 1$ if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $d_{ij} \in \mathbb{R}$ for all $i, j = 1, \ldots, L$, such that $X_T^i > 0, d_{ij} < 0, X_{ij}^{11} = d_{ij}I_{n_{ij}}$ and LMI (3.18) are satisfied for all $i = 1 \ldots, L$, with $P_i^{11} = \mathbf{diag}_j(X_{ij}^{11}), P_i^{22} = \mathbf{diag}_j(-X_{ji}^{11})$ and $P_i^{12} = 0$.

Following similar argument, we have the following Proposition 5 below. The sufficient part can be similarly proved with proper chosen multipliers X_{ij}, while the necessary part follows from the lossless-(D, G)-scaling theorem for these LTV interconnection operators (Meinsma et al. [2000]).

Proposition 5. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all LTV $\Delta_{ij}, \|\Delta_{ij}\| \leq 1$, if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$, , $X_T^i > 0$ and for all $i, j = 1, \ldots, L$, $d_{ij} < 0$, $X_{ij}^{11} = d_{ij}I_{n_{ij}}$ and the LMIs (3.18) are satisfied for all $i = 1 \ldots, L$, with $P_i^{11} = \mathbf{diag}_j(X_{ij}^{11}), P_i^{22} = \mathbf{diag}_j(-X_{ji}^{11})$ and $P_i^{12} = 0$.

The necessity part of the following proposition has been proved in Langbort et al. [2004] as an extension of the standard S-procedure, and the sufficient part can be similarly derived via Theorem 2.

Proposition 6. The interconnected system (2.5) , (2.6) is well-posed, stable and contractive for all LTV unitary operator $\delta_{ij}, 1 \leq j \leq i \leq L$ with $\Delta_{ij} = I_{n_{ij}}\delta_{ij}$ and $\delta_{ji} = \delta_{ij}^{-1}$ for $i \geq j$ if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $X_{ij}^{12} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i \geq j$ with X_{ii}^{12} skew-symmetric, such that $X_T^i > 0$ and the LMIs(3.18) hold true for all $i = 1, \ldots, L$.

Before we apply the stability analysis results to controller synthesis, the following remark is in order.

Remark 3.1. Theorem 2 unifies stability results for different interconnections which can be modeled as integral quadratic constraints. This theorem renders the performance specification based on the interconnected implicit uncertain systems to an explicit expression through S-procedure with multipliers X_{ij}, which are shaped by the structure and properties of the interconnection operator Δ_{ij}. Generally speaking, Theorem 2 reflects the simple idea of topological separation of the graph generated via the LTI plant and the LTV uncertainty. Although sufficient stability conditions can be easily derived in this framework, the necessity part is challenging; it has been shown only in special cases (Langbort et al. [2004], Megretski and Treil [1993], Meinsma et al. [2000] and Shamma [1994]).

4. SYNTHESIS VIA THE ELIMINATION LEMMA

The synthesis part of this paper follows the same line of the derivation presented in Langbort et al. [2004], which is based on the extended elimination lemma (Helmersson [1999]). We want to point out that for the synthesis condition in Theorem 2, we need $n_{ij}^K = n_{ij}$; this is enough since the inertia constraints are automatically satisfied if the associated LMIs are feasible and the multipliers are nonsingular. Note that in Langbort et al. [2004], the stricter requirement on the controllers dimension, namely $n_{ij}^K = 3n_{ij}$, is used.

Now let us consider each of subsystem G_i with control input u_i and a measured output y_i, in addition to the signals given in (2.1), such that

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ z_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^i & B_{Td}^i & B_{Tu}^i \\ A_{ST}^i & A_{SS}^i & B_S^i & B_{Su}^i \\ C_T^i & C_S^i & D^i & D_{zu}^i \\ C_{Ty}^i & C_{Sy}^i & D_{yd}^i & D_{yu}^i \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \\ u_i(t) \end{bmatrix}$$
$$v_{ij} = \Delta_{ji} w_{ji}$$

for all $t \geq 0$ and $i = 1, \ldots, L$, here Δ_{ji} is an operator used to model the interconnection. In the rest of this paper, without loss of generality, we assume that $D_{yu}^i = 0$ for all i. Similarly to the controllers considered in the LPV literature, we are seeking controllers with similar structure as the plant: another interconnected system K with subsystems $K_i, i = 1, \ldots, L$ given by

$$\begin{bmatrix} \dot{x}_i^K(t) \\ w_i^K(t) \\ u_i(t) \end{bmatrix} \begin{bmatrix} (A_{TT}^i)_K & (A_{TS}^i)_K & (B_T^i)_K \\ (A_{ST}^i)_K & (A_{SS}^i)_K & (B_S^i)_K \\ (C_T^i)_K & (C_S^i)_K & D_K^i \end{bmatrix} \begin{bmatrix} x_i^K(t) \\ v_i^K(t) \\ y_i(t) \end{bmatrix} \tag{4.26}$$

such that the closed loop system is well-posed, stable and contractive. In addition, we require $n_{ij}^K = 0$ whenever $n_{ij} = 0$, which means if there is no interaction between G_i and G_j, the controllers K_i and K_j will not communicate with each other either.

Here superscripts K, C are introduced to denote the controller signals and closed-loop signals respectively. The state variable for the subsystem x_i^C has dimension $m_i^C = m_i + m_i^K$,

$$x_i^C = \begin{bmatrix} x_i \\ x_i^K \end{bmatrix}.$$

The interconnection signal w_{ij}^C, v_{ij}^C has dimension $n_{ij}^C = n_{ij} + n_{ij}^K$,

$$w_{ij}^C = \begin{bmatrix} w_{ij} \\ w_{ij}^K \end{bmatrix} \tag{4.27}$$

$$v_{ij}^C = \begin{bmatrix} v_{ij} \\ v_{ij}^K \end{bmatrix} \tag{4.28}$$

We further require

$$w_{ij}^C = \Delta_{ji} v_{ji}^C \tag{4.29}$$

. since the controller K and the plant G share the same interconnection operator Δ_{ij} between each subsystem.

We are now ready to apply the analysis result to the close-loop systems.

Proposition 7. The closed-loop system is well-posed, stable and contractive if there exist symmetric matrices $(X_T^i)^C \in R_S^{m_i^C \times m_i^C}$ and $X_{ij}^{11} \in R_S^{n_{ij}^C \times n_{ij}^C}$ for all $i, j = 1, \ldots, L$, and $(X_{ij}^{12})_C \in R^{n_{ij}^C \times n_{ij}^C}$ for all $i \geq j$, with $(X_{ii}^{12})_C$ skew symmetric , such that $(X_T^i)_C > 0$ and

$$(M_i^C)^T P_i^C M_i^C < 0 \tag{4.30}$$

with

$$M_i^C = \left[\begin{array}{ccc} I & 0 & 0 \\ (A_{TT}^i)_C & (A_{TS}^i)_C & (B_T^i)_C \\ \hline 0 & I & 0 \\ (A_{ST}^i)_C & (A_{SS}^i)_C & (B_S^i)_C \\ \hline 0 & 0 & I \\ (C_T^i)_C & (C_S^i)_C & (D^i)_C \end{array}\right] \quad (4.31)$$

$$P_i^C = \left[\begin{array}{cc|cc|cc} 0 & (X_T^i)_C & 0 & 0 & 0 & 0 \\ (X_T^i)_C & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (Z_i^{11})_C & (Z_i^{12})_C & 0 & 0 \\ 0 & 0 & (Z_i^{12})_C^* & (Z_i^{22})_C & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right] \quad (4.32)$$

for all $i = 1, \ldots, L$. and

$$\begin{aligned} (Z_i^{11})_C &= \mathbf{diag}_j (X_{ij}^{11})_C \\ (Z_i^{22})_C &= \mathbf{diag}_j - (X_{ji}^{11})_C \\ (Z_i^{12})_C &= \mathbf{diag}(\mathbf{diag}_{1\le j\le i}(X_{ij^{12}})_C, \mathbf{diag}_{i<j\le L}((X_{ji}^{12})^T)_C) \end{aligned}$$

The following synthesis result can be derived if we use the elimination lemma Helmersson [1999] to eliminate controller parameters from the above closed-loop performance conditions.

Proposition 8. There exist distributed controllers with state representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij} = I$ such that such the closed-loop system conditions (4.30) are satisfied if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i \times m_i}$ and $(X_{ij}^{11})_G^T, (Y_{ij}^{11})_G^T \in \mathbb{R}_S^{n_{ij}\times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $(X_{ij}^{12})_G, (Y_{ij}^{12})_G \in \mathbb{R}^{n_{ij}\times n_{ij}}$ for $i \ge j$, with $(X_{ii}^{12})_G, (Y_{ii}^{12})_G$ skew-symmetric such that $(X_T^i)_G > 0, (Y_T^i)_G > 0$ and (4.36),(4.37),(4.38) are satisfied, where Ψ^i, Φ^i, M_i, N_i are defined as (4.33), (4.34), (3.19), (4.35), respectively.

$$\Psi^i = \ker \left[\, C_{Ty}^i \;\; C_{Sy}^i \;\; D_{yd}^i \, \right] \quad (4.33)$$

$$\Phi^i = \ker \left[\, (B_{Tu}^i)^T \;\; (B_{Su}^i)^T \;\; (D_{zu}^i)^T \, \right] \quad (4.34)$$

and

$$\begin{aligned} (Z_i^{11}) &= \mathbf{diag}_{1\le j\le L}(X_{ij}^{11})_G \\ (Z_i^{22}) &= -\mathbf{diag}_{1\le j\le L}(X_{ji}^{11})_G \\ (Z_i^{12}) &= \mathbf{diag}\left\{\mathbf{diag}_{1\le j\le i}(X_{ij}^{12})_G, -\mathbf{diag}_{i<j\le L}(X_{ji}^{12})_G^*\right\} \\ (\widetilde{Z}_i^{11}) &= \mathbf{diag}_{1\le j\le L}(Y_{ij}^{11})_G \\ (\widetilde{Z}_i^{22}) &= -\mathbf{diag}_{1\le j\le L}(Y_{ji}^{11})_G \\ (\widetilde{Z}_i^{12}) &= \mathbf{diag}\left\{\mathbf{diag}_{1\le j\le i}(Y_{ij}^{12})_G, -\mathbf{diag}_{i<j\le L}(Y_{ji}^{12})_G^*\right\} \end{aligned}$$

$$Z_i = \left[\begin{array}{cc|cc|cc} 0 & (X_T^i)_G & 0 & 0 & 0 & 0 \\ (X_T^i)_G & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (Z_i^{11})_G & (Z_i^{12})_G & 0 & 0 \\ 0 & 0 & (Z_i^{12})_G^* & (Z_i^{22})_G & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right]$$

$$\widetilde{Z}_i = \left[\begin{array}{cc|cc|cc} 0 & (\widetilde{X}_T^i)_G & 0 & 0 & 0 & 0 \\ (\widetilde{X}_T^i)_G & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (\widetilde{Z}_i^{11})_G & (\widetilde{Z}_i^{12})_G & 0 & 0 \\ 0 & 0 & (\widetilde{Z}_i^{12})_G^* & (\widetilde{Z}_i^{22})_G & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right]$$

$$N_i = \left[\begin{array}{ccc} -(A_{TT}^i)^T & -(A_{ST}^i)^T & -(C_{Tz}^i)^T \\ I & 0 & 0 \\ \hline -(A_{TS}^i)^T & -(A_{SS}^i)^T & -(C_{Sz}^i)^T \\ 0 & I & 0 \\ \hline -(B_{Td}^i)^T & -(B_{Sd}^i)^T & -(D_{zd}^i)^T \\ 0 & 0 & I \end{array}\right] \quad (4.35)$$

$$(\Psi^i)^* M_i^* Z_i M_i \Psi_i < 0 \quad (4.36)$$

$$(\Phi^i)^* N_i^* \widetilde{Z}_i N_i \Phi_i > 0 \quad (4.37)$$

$$\begin{bmatrix} (X_T^i)_G & I \\ I & (Y_T^i)_G \end{bmatrix} > 0 \quad (4.38)$$

Proof 4.1. Notice that, the closed loop system for the individual subsystem with the controller described by (4.26) is linear in the controller's parameter Θ_i with

$$\Theta_i = \begin{bmatrix} (A_{TT}^i)_K & (A_{TS}^i)_K & (B_T^i)_K \\ (A_{ST}^i)_K & (A_{SS}^i)_K & (B_S^i)_K \\ (C_T^i)_K & (C_S^i)_K & D_K^i \end{bmatrix} \quad (4.39)$$

Apply now the elimination lemma from Helmersson [1999] to each individual stability LMI(3.18) condition derived in Proposition 3 for the closed-loop systems The necessity part follows instantly. The sufficience part follows from similar techniques as in Scherer [2001] to construct the extended multiplier for the overall interconnection w_{ij}^C, v_{ji}^C. Conditions (4.36), (4.37), (4.38) are sufficient to construct the extended multipliers and controller parameters. □

The following synthesis conditions in Proposition 9 and Proposition 10 correspond to condtions in Proposition 4 and Proposition 5 respectively, they can be proved similarly to Proposition 8 via the elimination lemma.

Proposition 9. There exist distributed controllers with state space representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij} = \delta I, \|\delta\| \le 1$ such that the closed-loop system is well-posed, stable and contractive if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i\times m_i}$ and matrices $(X_{ij}^{11})_G, (Y_{ij}^{11})_G \in \mathbb{R}_S^{n_{ij}\times n_{ij}}, (X_{ij}^{11})_G < 0, (Y_{ij}^{11})_G < 0$ for all $i, j = 1, \ldots, L$ such that $(X_T^i)_G > 0, (Y_T^i)_G > 0$ and (4.36),(4.36),(4.36) are satisfied for all i with $(Z_{12}^i)_G = (\widetilde{Z}_{12}^i)_G = 0$, and

$$\begin{bmatrix} (X_{ij}^{11})_G & -I \\ -I & (Y_{ij}^{11})_G \end{bmatrix} \le 0 \quad \text{for all } i, j \quad (4.40)$$

Proposition 10. There exist distributed controllers with state space representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij}, \|\Delta_{ij}\| \le 1$ such that the closed-loop system is well-posed, stable and contractive if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i\times m_i}$, and $x_{ij}, y_{ij} \in \mathbb{R}$ such that $x_{ij} < 0, y_{ij} < 0, (X_{ij}^{11})_G = x_{ij} I_{n_{ij}}, (Y_{ij}^{11})_G = y_{ij} I_{n_{ij}}$ for all $i, j = 1, \ldots, L$ such that $(X_T^i)_G > 0, (Y_T^i)_G > 0$ and (4.36),(4.37),(4.38) (4.40) are satisfied for all i with $(Z_{12}^i)_G = (\widetilde{Z}_{12}^i)_G = 0$.

5. CONCLUSION

In this paper, we derived stability conditions for distributed systems with various IQC constraints on the interconnections. Technically, the stability results follow from an application of the S-procedure and can be proved via a graph separation argument. Our stability theorem

(Theorem 2) expresses the global performance with implicit uncertainty interconnections in terms of a set of explicit conditions with design multipliers parameterized by the uncertainty. Specifically, our results generalize the stability results presented in Langbort et al. [2004]. They are applicable to systems with more general interconnections. The approach used to derive our results relies on the S-procedure and the connections to the gain-scheduling techniques in linear parameter varying systems. These connnections have been made explicit.

REFERENCES

Panos. J. Antsaklis and J. Baillieul. Guest editorial: Special issue on networked control systems. *IEEE Transaction on Automatic Control, Special Issue on Networked Control Systems*, 49(9):1241–1243, 2004.

Panos. J. Antsaklis and John Baillieul. Special Issue on Technology of Networked Control Systems. *Proceedings of the IEEE*, 95(1):5–8, January 2007.

B.D. Chen and S. Lall. Dissipation inequlaities for distributed systems on graph. In *Proceedings of the IEEE Conference on Decision and Control*, pages 3084–3090, 2003.

R D' Andrea and G.E Dullerud. Distributed control design for spatially interconnected systems. *IEEE Transaction on Automatic Control*, 48(9):1478–1495, September 2003.

Zhe Di, Mazen Farhood, and Geir E. Dullerud. Control of distributed systems over graphs. In *Proceedings of the 45th IEEE Conference on Decision and Control*, pages 3486–3491, San Diego, CA, USA, Dec 2006.

G.E Dullerud and R D' Andrea. Distributed control of heterogeneous systems. *IEEE Transaction on Automatic Control*, 49(12):2113–2128, December 2004.

Anders Helmersson. Iqc synthesis based on inertial constraints. In *Proceedings of the 14th IFAC World Congress*, pages 163–168, Beijing, China, 1999.

Fang Hui and Panos. J. Antsaklis. Distributed Control with Integral Quadratic Constraints. Tech Report of ISIS (Interdisciplinary Studies of Intelligent Systems) Group ISIS-2007-001, University of Notre Dame, March 2007.

T. Iwasaki and S. Hara. Well-poseness of feedback systems: insights into exact robustness analysis and approximate computations. *IEEE Transaction on Automatic Control*, 43(5):619–630, May 1998.

T. Iwasaki and G. Shibata. LPV system analysis via quadratic separator for uncertain implicit systems. *IEEE Transactions on Automatic Control*, 46(8):1195–1208, August 2001.

Cedric Langbort, Ramu S. Chandra, and R D' Andrea. Distributed control design for systems interconnected over an arbitray graph. *IEEE Transaction on Automatic Control, Special Issue on Networked Control Systems*, 49 (9):1502–1519, September 2004.

Cedric Langbort, Vijak Gupta, and Richard Murray. Distributed control over failing channels. In Panos J. Antsaklis and Paulo Tabuada, editors, *Lecture Notes in Control and Information Sciences series*, pages 325–342. Workshop on Networked Embedded Sensing and Control, University of Notre Dame, Springer, 2006.

A. Megretski and A. Rantzer. System analysis via intergral qudratic constraints. *IEEE Transaction on Automatic control*, 42(6):819–830, June 1997.

A. Megretski and S. Treil. Power distribution inequalities in optimization and robustness of uncertain systems. *Journal of Mathematical Systems, Estimation, and Control*, 3(3):301–319, March 1993.

G. Meinsma, T. Iwasaki, and M. Fu. When is (D, G)-scaling both necessary and sufficient. *IEEE Transaction on Automatic Control*, 45(9):1755–1759, September 2000.

P.J. Moylan and D.J. Hill. Stability criteria for large-scale systems. *IEEE Transaction on Automatic Control*, 23 (2):143–149, April 1978.

G.N. Nair and R.J. Evens. Stabilization with data-rate limited feedback: Tightest attainable bounds. *System and Control Letters*, 41(1):49–56, September 2000.

A Packard. Gain scheduling via linear fractional transformations. *System and Control letters*, 22(2):79–92, February 1994.

M.G. Safonov. *Stability and Robustness of Multivariable Feedback Systems*. MIT Press, Cambridge, MA, 1980.

C.W. Scherer. LPV control and full block multipliers. *Automatica*, 37(3):361–375, March 2001.

G. Scorletti and G. Due. An lmi approach to decentralized control. *International Journal of Control*, 74(3):211–224, Feb 2001.

G. Scorletti and L. El Ghaoui. Improved linear matrix inequlity conditions for gain scheduling. *International Journal of Robust and Nonlinear Control*, 8(10):3626–3631, Aug 1998.

J.S. Shamma. Robust stability with time-varying strctured uncertainty. *IEEE Transaction on Automatic Control*, 39(4):714–724, April 1994.

S. Tatikonda. *Control under Communcation Constraints*. PhD thesis, M.I.T, Cambridge, MA, 2000.

Valery A. Ugrinovskii, I.R. Peterson, A.V. Savkin, and E.Y. Ugrinovskaya. Decentralized state-feedback stabilization and robust control of uncertain large-scale systems with integrally constrained interconnections. *System and Control Letters*, 40(2):107–119, Jun 2000.

J.C. Willems. Dissipative dynamical systems i & ii. *Arch. Rational Mech. Anal*, 45:321–393, 1972.

V.A. Yakubovich. Nonconvex optimization problem: The infinite-horizon linear-qudratic control problem with qudratic constraints. *Systems and Control Letters*, 19 (1):13–22, Jul 1992.

V.A. Yakubovich. The s-procedure in nonlinear control theory. *Vestnik Leningrad University*, 4:73–93, 1977.