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A Non Linear Observer for a Fishery Model

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Abstract: The aim of this paper is to apply some tools of observability theory to an age-structured model of a harvested fish population in order to estimate the stock state. We construct an observer that uses the data of caught fish and gives a dynamical estimation of the number of fish by stage.

Keywords: Nonlinear systems, Observers, State estimation, High-gain, Population dynamics, Management of natural resources, Natural and environmental systems, Parameter and state estimation.

1. INTRODUCTION

The natural stock management is a problem which received great attention during the last decades. The development of management policies in the exploitation of renewable resource stocks needs to have a good estimate of the available resource. Nowadays, mathematical models together with computer simulations are useful to describe the evolution of complex systems. One of the important problems in control theory is to reconcile the available data with the used mathematical model. This problem is known as the observability problem and it is related to the construction of "observers" (called some times software sensors) for dynamical systems. In this paper, we show how to apply this theory in order to address the stock estimation problem for an exploited fish population.

It is often not possible to measure all the state variables. So it is necessary to have an algorithm to estimate the unmeasured state variables.

In this paper we construct an observer for a nonlinear system which model the dynamical evolution of a harvested fish population Touzeau [1997].

This system is of the form

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)), \\ y(t) = h(x(t), u(t)), \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, $y \in \mathbf{R}^p$, The maps f and h are assumed to be smooth.

Our aim is to construct an estimation $\hat{x}(t)$ of the state $x(t)$ by supposing that the input $u(t)$ and the output $y(t)$ are well known.

1.1 Problem Statement

We consider the following mathematical model describing the dynamical evolution of a fish population submitted to fishing. The population is structured by age class (see for instance Touzeau et al. [1998], Touzeau [1997], Ouahbi et al. [2003]):

$$\begin{cases} \dot{X}_0(t) &=& -\alpha_0 X_0(t) + \sum_{i=1}^{2} f_i l_i X_i(t) \\ & & -\sum_{i=1}^{2} p_i X_i(t) X_0(t) - p_0 X_0^2(t) \\ \dot{X}_1(t) &=& \alpha X_0(t) - (\alpha_1 + q_1 E) X_1(t) \\ \dot{X}_2(t) &=& \alpha X_1(t) - (\alpha_2 + q_2 E) X_2(t). \end{cases} \tag{2}$$

where :

X_i: the number of fish in the stage i.

α : linear aging coefficient (in time^{-1})

m_i : natural mortality rate (in time^{-1})

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$\alpha_i = m_i + \alpha$ (in time^{-1})

p_0: juvenile competition parameter (in time^{-1}.number^{-1})

f_i: fecundity rate of class (no dimension)

l_i: reproduction efficiency of class i (in time^{-1})

p_i: predation rate of class i on class 0 (time^{-1}.num^{-1})

q_i: capturability coefficient of class i (in unit effort^{-1})

E: instantaneous fishing effort. (in unit effort×time^{-1})

The fishing effort applied on a stock of a fish population can be seen as the whole means of production used by the fisherman. Fishing effort can be considered as the sum , over all units of the product of the fishing power on each unit, and its fishing time, or a number of unit operations Gulland [1983] (page 38-39).

To the system (2) we associate the output

$$Y(t) = q_2 E X_2(t)$$

$Y(t)$ represents the total catch that we assume can be measured. This means that we suppose that only the last class is submitted to fishing, so $q_1 = 0$. Then, we obtain the following system

$$\begin{cases} \dot{X}_0(t) &= -\alpha_0 X_0(t) + \displaystyle\sum_{i=1}^{2} f_i l_i X_i(t) \\ & \quad - \displaystyle\sum_{i=1}^{2} p_i X_i(t) X_0(t) - p_0 X_0^2(t) \\ \dot{X}_1(t) &= \alpha X_0(t) - \alpha_1 X_1(t) \\ \dot{X}_2(t) &= \alpha X_1(t) - (\alpha_2 + q_2 E) X_2(t) \\ Y(t) &= q_2 E X_2(t). \end{cases} \tag{3}$$

Our aim is to construct an observer (estimator) for system (3). This observer is an auxiliary dynamical system that can be written

$$\frac{d\hat{X}(t)}{dt} = g(\hat{X}(t), E(t), Y(t)), \tag{4}$$

and whose state $\hat{X}(t)$ gives a "good" asymptotic estimation of the state$X(t)$ of system (3); that is, the solutions of (3-4) satisfy $\lim\limits_{t\to+\infty} \|\hat{X}(t) - X(t)\| = 0$, for all initial conditions $X(0)$ and $\hat{X}(0)$. The observer we built will be in fact an exponential observer; the estimation error will converges to zero with exponential speed, i.e.,

$$\|\hat{X}(t) - X(t)\| \leq \exp(-\lambda t)\|\hat{X}(0) - X(0)\|.$$

2. OBSERVABILITY AND OBSERVER DESIGN

2.1 Definitions (see for instance, Bernard et al. [1998], Iggidr [2004], or Sontag [1998], chap. 6)

Observability: We use the notation $x(x_0, u(.), t)$ to denote the solution of the differential equation (1) corresponding to the admissible control $u(.)$ and with initial condition x_0, and The corresponding output is denoted $y(x_0, u(.), t) = h\left(x(x_0, u(.), t), u(t)\right)$. The system (1) is said to be **observable** if for any pair of different initial states (x_0, x_1) $x_0 \neq x_1$, there exists an admissible control $u(.)$ such that the outputs corresponding to those initial conditions are not identically equals; that is, there exists a time $t \geq 0$ such that :

$$y(x_0, u(.), t) \neq y(x_1, u(.), t)$$

Observability for any input: The system (1) is said to be **uniformly input observable** if for any input $u(.)$ and for any (x_0, x_1), $x_0 \neq x_1$, there exists a time $t \geq 0$ such that :

$$y(x_0, u(.), t) \neq y(x_1, u(.), t)$$

2.2 Observability of system (3)

For the observer design, we will use the High Gain observer (Gauthier et al. [1992], Gauthier et al. [1994]) and the Kalman like observer (Deza et al. [1992]) to design an observer for system (3).

Let $Y(t) = h(X(t)) = q_2 E X_2(t)$ and the function Φ : $\mathbf{R}^3 \to \mathbf{R}^3$, defined by :

$$\Phi(X) = \begin{pmatrix} h(X) \\ L_f h(X) \\ L_f^2 h(X) \end{pmatrix} = Z \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} \text{ and}$$

$$f(X) = \begin{pmatrix} -\alpha_0 X_0(t) + \displaystyle\sum_{i=1}^{2} f_i l_i X_i(t) - \sum_{i=1}^{2} p_i X_i(t) X_0(t) \\ -p_0 X_0^2(t) \\ \alpha X_0(t) - \alpha_1 X_1(t) \\ \alpha X_1(t) - (\alpha_2 + q_2 E) X_2(t) \end{pmatrix}.$$

where L denote the Lie derivative operator with respect to the vector field f, and h is the output function.

Φ is a diffeomorphism of the state space $\mathbf{R}^3$ to its image $\Phi(\mathbf{R}^3)$. In fact :

$$\frac{d\Phi}{dx} = \begin{pmatrix} 0 & 0 & q_2 E \\ 0 & \alpha q_2 E & -q_2 E(\alpha_2 + q_2 E) \\ \alpha^2 q_2 E & -\alpha_1 q_2 E & q_2 E(\alpha_2 + q_2 E)^2 \\ & -q_2 E(\alpha_2 + q_2 E) & \end{pmatrix},$$

$$\left[\frac{d\Phi}{dx}\right]^{-1} = \begin{pmatrix} \dfrac{\alpha_1 q_2 E + \alpha_1 \alpha_2}{\alpha^2 q_2 E} & \dfrac{q_2 E + \alpha_1 + \alpha_2}{\alpha^2 q_2 E} & \dfrac{1}{\alpha^2 q_2 E} \\ \dfrac{q_2 E + \alpha_2}{\alpha q_2 E} & \dfrac{1}{\alpha q_2 E} & 0 \\ \dfrac{1}{q_2 E} & 0 & 0 \end{pmatrix}$$

and we have

$$\text{Det}\left(\frac{d\Phi}{dx}\right) = \alpha^3 q_2^3 E^3.$$

So Φ is a diffeomorphism provided that $E \neq 0$ and then the system (3) is uniformly observable (Gauthier et al. [1992]). Then, in a new coordinate system $(\mathbf{R}^3, \Phi(\mathbf{R}^3))$, with $Z = \Phi(X) = (h(X), L_f h(X), L_f^2(X))^t$, our system can be written in the canonical form:

$$\begin{cases} \dot{Z} &= AZ + \psi(Z) \\ Y &= CZ \end{cases} \tag{5}$$

where : $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $C = (1,0,0)$ and

$$\psi(Z) = \begin{pmatrix} 0 \\ 0 \\ L_f^3 h(\Phi^{-1}(Z)) \end{pmatrix}$$

We denote by φ and φ_1 the maps defined by

$$\begin{aligned}
\varphi_1(Z) &= L_f^3 h(\Phi^{-1}(Z)) \\
&= \Big(-\alpha_0\alpha q_2 E - \alpha^2 q_2 E\alpha_1 - \alpha^2 q_2 E(\alpha_2 + q_2 E)\Big)X_0 \\
&+ \Big(\alpha q_2 E f_1 l_1 + \alpha\alpha_1^2 q_2 E + \alpha\alpha_1 q_2 E(\alpha_2 + q_2 E) \\
&+ \alpha q_2 E(\alpha_2 + q_2 E)^2\Big)X_1 \\
&+ \Big(\alpha q_2 E f_2 l_2 - \alpha_1 q_2 E(\alpha_2 + q_2 E)\Big)X_2 \\
&- \alpha q_2 p_0 E {X_0}^2 - \alpha q_2 p_1 E X_0 X_1 - \alpha q_2 p_2 E X_0 X_2. \\
&= \varphi(X)
\end{aligned}$$

It has been proved in Touzeau [1997] that there is a positively invariant compact set for system (3). This set is of the form $D = [a_0, b_0] \times [a_1, b_1] \times [a_2, b_2]$, where The numbers a_i can be chosen as small as we need and the numbers b_i are function of the parameters f_i, l_i and p_i. More precisely:

$$\begin{aligned}
&b_i = (1+\nu_i)\pi_i\mu \\
&\text{with } 0 = \nu_0 < \nu_1 < \nu_2 < 1,\ \pi_i = \frac{\alpha^i}{\prod_{j=1}^{i}(\alpha_j + q_j E)}, \\
&\text{and } \mu = \min_{f_i l_i p_i \neq 0}\{\frac{f_i l_i}{p_i}\}
\end{aligned} \tag{6}$$

The function φ is smooth on the compact set D. Hence, it is globally Lipschitz on D. Therefore it can be extended by $\tilde{\varphi}$, a Lipschitz function on $\mathbf{R}^3$ which satisfies $\tilde{\varphi}(X) = \varphi(X)$, for all $X \in D$. In the same way we define $\tilde{\psi}$ the Lipschitz prolongation of the vector function ψ.

$\tilde{\psi}(Z) = \begin{pmatrix} 0 \\ 0 \\ \tilde{\varphi}_1(Z) \end{pmatrix}$; where $\tilde{\varphi}_1$ is the prolongation of φ_1 to $\mathbf{R}^3$, i.e., $\tilde{\varphi}_1(Z) = \varphi_1(Z) = L_f^3 h(\Phi^{-1}(Z))$ for all $Z \in D$.

So now we are going to work with the following system (7) defined in the whole space $\mathbf{R}^3$. Notice that its restriction to the domain D is the system (5).

$$\begin{cases} \dot{Z} &= AZ + \tilde{\psi}(Z) \\ Y &= CZ \end{cases} \tag{7}$$

where : $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $C = (1,0,0)$ and

$$\tilde{\psi}(Z) = \begin{pmatrix} 0 \\ 0 \\ \tilde{\varphi}_1(Z) \end{pmatrix}$$

Remarque: For the simulations we extend the vector field f that defines the system (3) by continuity in order to make it globally lipschitz on $\mathbf{R}^3$ in the following way: We denote $\tilde{f}$ the prolongation of f to $\mathbf{R}^3$ and the function π the projection on the domain D and we construct $\tilde{f} = f \circ \pi$. The extended function $\tilde{f}$ has the same lipschitz coefficient as f. The projection π is defined as follows $\pi(X) = \bar{X}$, where $X \in \mathbf{R}^3$ and $\bar{X} \in D$ such that $\text{Dist}(X, D) = \|X - \bar{X}\|$, i.e. $\bar{X}$ satisfies $\|X - \bar{X}\| = \min_{Y \in D} \|Y - X\|$.

2.3 High Gain observer design (Fixed gain)

Proposition 1. (Gauthier et al. [1992]) Consider the following system :

$$\dot{Z} = AZ + \psi(Z) + S^{-1}(\theta)(y - CZ). \tag{8}$$

where A is the anti-shift operator and $S(\theta)$ is the solution of

$$0 = -\theta S(\theta) - A^t S(\theta) - S(\theta)A^t + C^t C.$$

Here, $S(\theta) = \begin{pmatrix} \theta^{-1} & -\theta^{-2} & \theta^{-3} \\ -\theta^{-2} & 2\,\theta^{-3} & -3\,\theta^{-4} \\ \theta^{-3} & -3\,\theta^{-4} & 6\,\theta^{-5} \end{pmatrix}$.

For θ large enough, system (8) is an exponential observer for the system (7).

Precisely $\theta \geq 2ncK\sqrt{S}$, where K is the lipschitz coefficient of the function ψ, n is the dimension of the space, and $S = sup_{i,j}|S(1)_{i,j}|$ See Gauthier et al. [1992] for the proof.

Going back to the our original system (3) via the transformation Φ^{-1}, we have

$$\dot{\hat{X}} = \tilde{f}(\hat{X}) + [\frac{d\Phi}{dx}]^{-1}_{X=\hat{X}} S(\theta)^{-1} C^t (y - h(\hat{X}))$$

such that the restriction to D is the following system

$$\begin{cases}
\dot{\hat{X}}_0 = -\alpha_0 \hat{X}_0 + \sum_{i=1}^{2} f_i l_i \hat{X}_i - \sum_{i=1}^{2} p_i \hat{X}_i \hat{X}_0 - p_0 \hat{X}_0^2 + \\
\dfrac{3\theta\alpha_1(q_2E + \alpha_2) + 3\theta^2(q_2E + \alpha_1 + \alpha_2) + \theta^3}{\alpha^2}(X_2 - \hat{X}_2) \\
\dot{\hat{X}}_1 = \alpha\hat{X}_0 - \alpha_1\hat{X}_1 + \dfrac{3\theta(q_2E + \alpha_2) + 3\theta^2}{\alpha}(X_2 - \hat{X}_2) \\
\dot{\hat{X}}_2 = \alpha\hat{X}_1 - (\alpha_2 + q_2E)\hat{X}_2 + 3\theta(X_2 - \hat{X}_2)
\end{cases}$$

which is the observer for the fishery model (3). This observer is particularly simple since it is only a copy of (2), together with a corrective term depending on θ.

2.4 Kalman-like observer design (Variable gain)

Proposition 2. (Deza et al. [1992]) Assume that

H_1 : Φ is a diffeomorphism from $\overset{o}{D}$ to $\Phi(\overset{o}{D})$. ($\overset{o}{D}$ is the interior of D).

H_2 : φ can be extended from D to $\mathbf{R}^3$ by a C^∞ function, globally Lipschitz on $\mathbf{R}^3$.

Then for θ large enough, the following differential system (9) is an exponential observer for system(7).

$$\left\{\begin{array}{lcl}\dot{\hat{X}} & = & \tilde{f}(\hat{X}) - \frac{1}{r}\left[\frac{d\Phi}{dX}\right]^{-1}_{X=\hat{X}} \times S^{-1}C^t(h(\hat{X}) - Y) \\ \dot{S} & = & -SQ_\theta S - A^{*t}(\hat{X})S - SA^{*(\hat{X})} + \frac{1}{r}C^tC.\end{array}\right. \quad (9)$$

with $r > 0$, Q_θ is defined from Q a symmetric positive definite matrix, by taking $Q_\theta = \Delta_\theta Q \Delta_\theta$ ($\Delta_\theta = diag(\theta, \theta^2, \theta^3)$).

The matrix A^* can be analytically computed from the diffeomorphism Φ.

$$A^*(\hat{X}) = A + \left[\frac{d\psi}{dZ}\right]_{Z=\Phi(\hat{X})}$$

See Deza et al. [1992] for the proofs.

For our system, we have:

$$\tilde{\varphi}(X) = \varphi(\Phi(X)), \text{ so } \frac{d\varphi}{dX} = \left(\frac{d\tilde{\varphi}}{dX}\right)\left(\frac{d\Phi}{dX}\right)^{-1}$$

we compute now A^* as follow:

$$A^*(X) = A + \left(\frac{d\tilde{\varphi}}{dX}\right)\left(\frac{d\Phi}{dX}\right)^{-1}.$$

With $\frac{d\tilde{\varphi}}{dX} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{31} & \beta_{32} & \beta_{33}\end{pmatrix}$ and

$\beta_{31} = -\alpha_0\alpha q_2 E - \alpha^2\alpha_1 q_2 E - \alpha^2 q_2 E(\alpha_2 + q_2 E) - 2\alpha q_2 p_0 E X_0 - \alpha q_2 p_1 E X_1 - \alpha q_2 p_2 E X_2$

$\beta_{32} = \alpha q_2 E f_1 l_1 + \alpha q_2 E(\alpha_1 + q_1 E)^2 + \alpha\alpha_1 q_2 E(\alpha_2 + q_2 E) + \alpha q_2 E(\alpha_2 + q_2 E)^2 - \alpha q_2 p_1 E X_0$

$\beta_{33} = \alpha q_2 E f_2 l_2 - \alpha_1 q_2 E(\alpha_2 + q_2 E) - \alpha q_2 p_2 E X_0.$

Finally we have:

$$A^*(X) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \gamma_{31} & \gamma_{32} & \gamma_{33}\end{pmatrix},$$

with: $\gamma_{31} = \beta_{31}(\frac{\alpha_1 q_2 E + \alpha_1\alpha_2}{\alpha^2 q_2 E}) + \beta_{31}(\frac{\alpha_2 + q_2 E}{\alpha q_2 E}) + \frac{\beta_{33}}{q_2 E}$

$\gamma_{32} = \beta_{31}(\frac{\alpha_1 + \alpha_2 + q_2 E}{\alpha^2 q_2 E}) + \frac{\beta_{32}}{\alpha q_2 E}$

$\gamma_{33} = \frac{\beta_{31}}{\alpha^2 q_2 E}.$

3. SIMULATION RESULTS

We present in this section some simulation results showing the performance of the constructed observers for system (3). We use the following fishery parameters (Ouahbi et al. [2003], Touzeau [1997]):

$\alpha_0 = 1.3$; $\alpha_1 = 0.9$; $\alpha_2 = 0.85$; $p_0 = 0.2$; $p_1 = 0.1$; $p_2 = 0.1$; $q_1 = 0$; $q_2 = 0.15$; $f_1 = 0.5$; $f_2 = 0.5$; $l_1 = 5$; $l_2 = 10$; $E = 1$; $\alpha = 0.8$.

With these parameters, we compute the coordinates of the higher corner B of the parallelepiped D by using the formulas (6). This gives B= $(25; 22,444; 18,1333)$ i.e. $0 < X_i \leq b_i$.

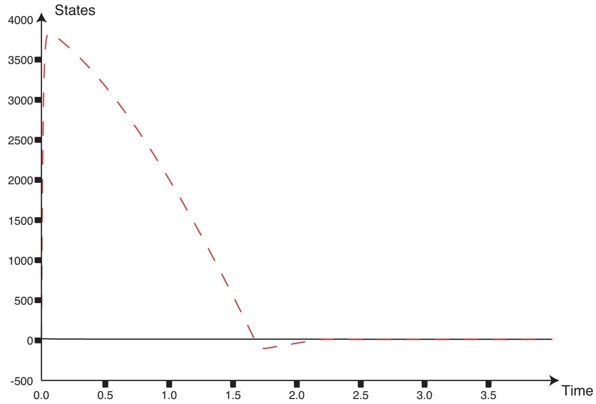

Fig. 1. X_0 (solid line) and its estimate $\hat{X}_0$ (dashed line) when φ is extended

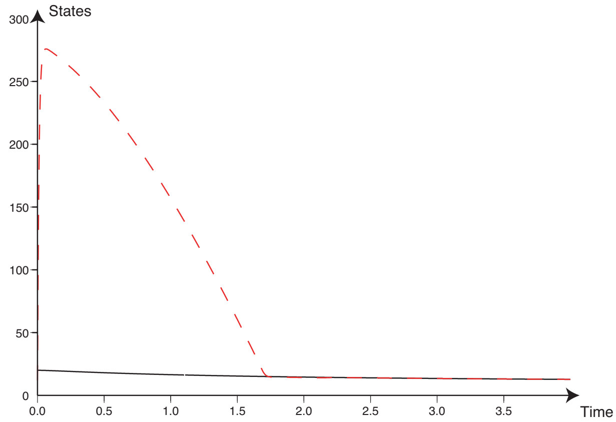

Fig. 2. X_1 (solid line) and its estimate $\hat{X}_1$ (dashed line) when φ is extended

The equilibrium point is $X^* = (12,4382; 11,0562; 8,8449)$.

For the High gain observer, we take $\theta = 30$, $X(0) = [21; 20; 15]$ and $\hat{X}(0) = [35; 12; 8]$.

For the kalman-like observer, we take $\theta = 5$, $X(0) = [25; 20; 15]$ and $\hat{X}(0) = [30; 14; 10]$. The positive definite symmetric matrix Q has been chosen $Q = I$, and $r = 0.2$, the matrix S is initialized as $S_0 = 10^{-10}I$, with $I = diag(1, 1, 1)$.

Using the same parameter values as above, when we do not use the Lipschitz prolongation of the function φ to the whole $\mathbf{R}^3$, the estimation $\hat{X}(t)$ computed by the observer tends to infinity for finite time. This actually happens in the beginning of the integration process as it can be seen in Figures 4, 5, and 6.

Kalman-like observer simulations : We observe the same phenomena. In particular the function f must be extended in a globally Lipschitz function on $\mathbf{R}^3$ otherwise the Kalman observer does not work with the chosen parameters values: this fact is illustrated by Figures 10, 11, and 12; but when we use the prolongation of f to built the observer then the convergence of the estimates toward

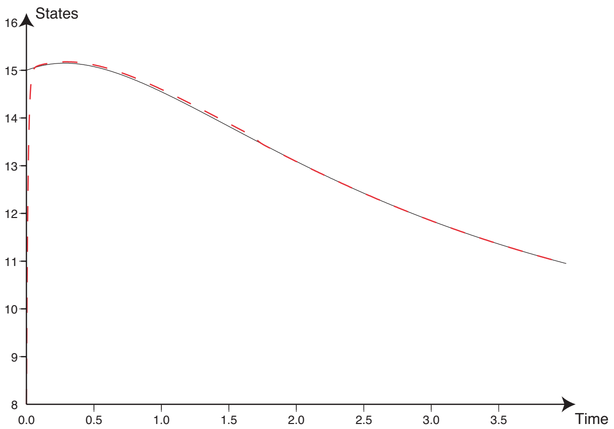

Fig. 3. X_2 (solid line) and its estimate $\hat{X}_2$ (dashed line) when φ is extended

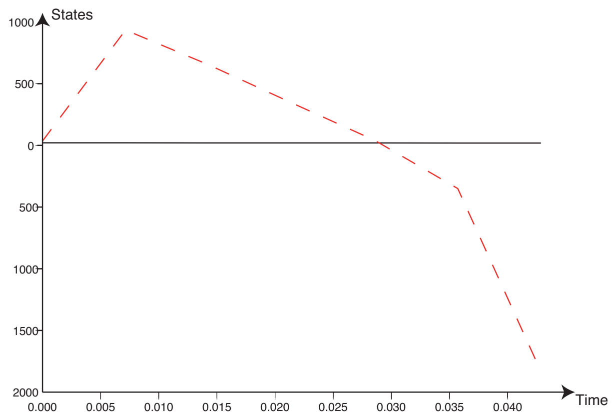

Fig. 4. X_0 (solid line) and its estimate $\hat{X}_0$ (dashed line) when φ is not extended

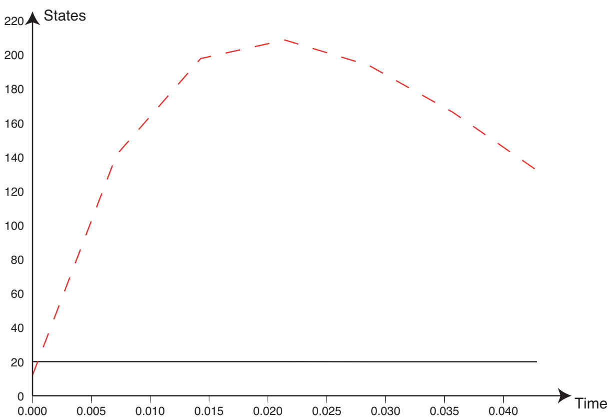

Fig. 5. X_1 (solid line) and its estimate $\hat{X}_1$ (dashed line) when φ is not extended

the real states is quite good and fast as it can be seen in Figures 7, 8, and 9.

4. CONCLUSION

Nonlinear control techniques are useful to validate biological models which are generally build on empiric observations. Indeed the construction of observers allows to have an estimate of unmeasured states. In this work, we construct an observer for a harvested fish model by using high gain and Kalman-like methods. We also show that it is necessary to extend the vector field f (that defines the dynamical evolution of the system) outside the invariant domain D by a globally Lipschitz function on $\mathbf{R}^3$. If the prolongation of f is not done then the observer does not converge.

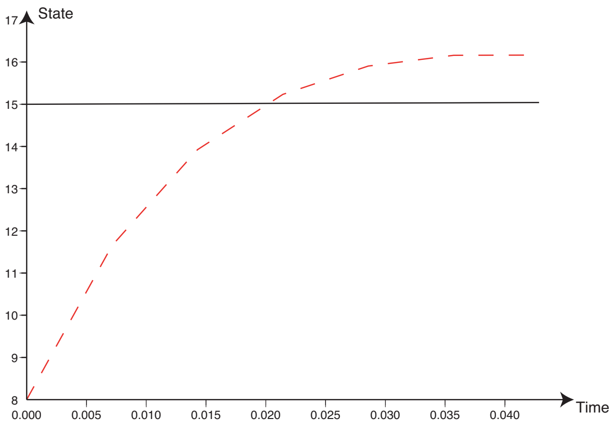

Fig. 6. X_2 (solid line) and its estimate $\hat{X}_2$ (dashed line) when φ is not extended

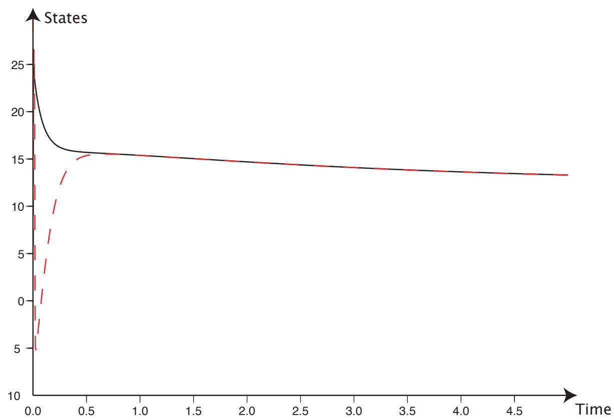

Fig. 7. X_0 (solid line) and its estimate $\hat{X}_0$ (dashed line) when φ is extended

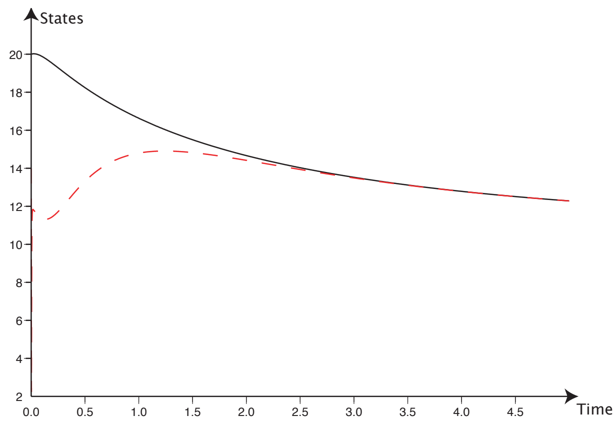

Fig. 8. X_1 (solid line) and its estimate $\hat{X}_1$ (dashed line) when φ is extended

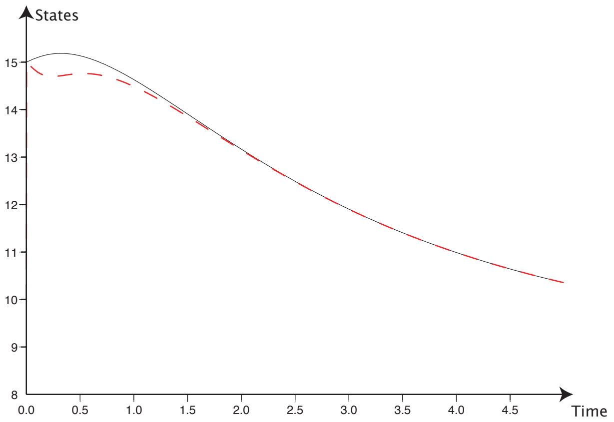

Fig. 9. X_2 (solid line) and its estimate $\hat{X}_2$ (dashed line) when φ is extended

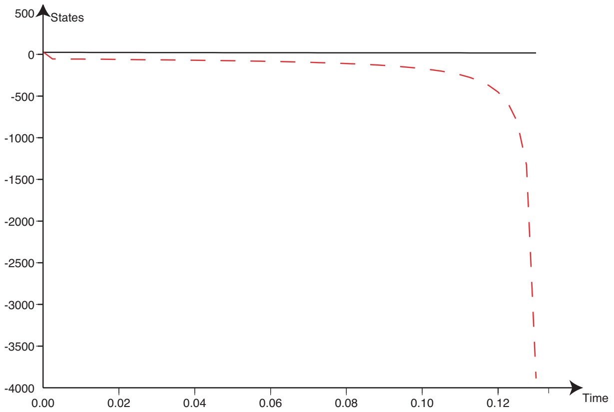

Fig. 10. X_0 (solid line) and its estimate $\hat{X}_0$ (dashed line) when φ is not extended

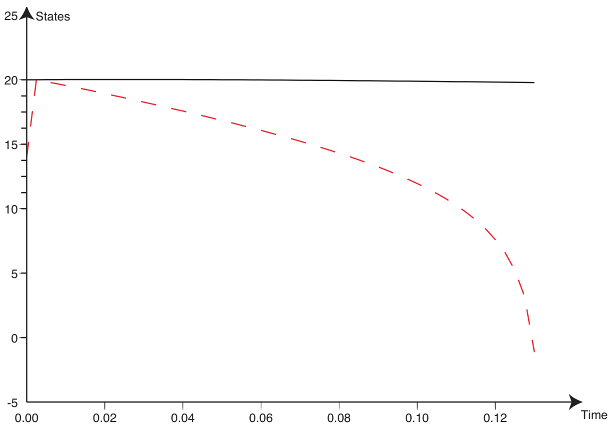

Fig. 11. X_1 (solid line) and its estimate $\hat{X}_1$ (dashed line) when φ is not extended

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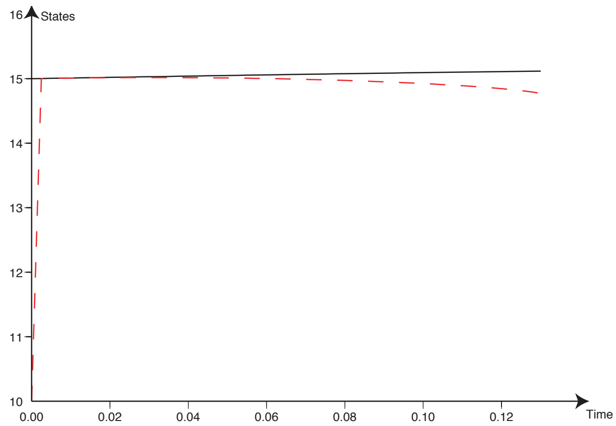

Fig. 12. X_2 (solid line) and its estimate $\hat{X}_2$ (dashed line) when φ is not extended

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