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Regulation of Volatile Fatty Acids and Total Alkalinity in Anaerobic Digesters

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Abstract: This paper deals with the simultaneous regulation of volatile fatty acids (VFA's) and total alkalinity (TA) in anaerobic digesters. The control scheme is conformed by an output feedback control and an extended Luenberger observer used to estimate the uncertainties associated to the controlled states (*i.e.,* kinetics terms and inlet composition). The inlet flow rate is used to regulate the VFA's concentration, whereas an alkali solution is added directly to the digester for the regulation of the TA concentration. The control scheme is evaluated via numerical simulations under different operating conditions. Results show that the control law is capable to regulate the VFA's and TA despite of load disturbances, uncertainties in the kinetics terms, noisy measurements and control inputs restrictions. *Copyright © 2008 IFAC*

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1. INTRODUCTION

The anaerobic digestion (AD) process is an interesting alternative for the treatment of effluents with high organic loads. This process is carried out under the absence of molecular oxygen by an heterogeneous bacterial community. The AD process can be described by two main steps: acidogenesis and methanization [Malina and Pohland, 1992]. In the acidogenic step, the organic matter expressed as chemical oxygen demand (COD) is consumed by the acidogenic microorganisms and converted to volatile fatty acids (VFA's) and CO_2. Later, in the methanogenic phase, the VFA's are consumed by the methanogens and transformed to methane and CO_2. Several advantages are recognized when the AD process is used for wastewater treatment: a) low sludge production; b) great capacity to degrade complex substrates at high concentrations of organic matter; c) low energy requirements and d) the possibility of recovering energy through the methane production. However, sudden changes in temperature, hydraulic or organic overloading, and the presence of inhibitory substances may alter the digester stability [Chen et al., 2007]. Under these conditions, the digester becomes unstable due to the accumulation of VFA's, which induces an overflow of protons that decompose the bicarbonates in the liquid phase to produce CO_2, increasing the CO_2 fraction in the gas phase and decreasing drastically the digester pH [Ripley et al., 1985]. If the perturbation causing the digester instability is not corrected in an early stage, the global irreversible digester failure is expected [Rozzi, 1991].

One way to overcome the aforementioned difficulties is by implementing advanced control and monitoring schemes that use reliable information of the key variables in addition to the variables that are traditionally monitored in AD processes: pH and biogas production. The biogas production is monitored in almost all the AD processes but, unfortunately, this variable does not yield accurate information about the digester stability. pH is readily available but only indicates the process stability in wastewater with low buffering capacity *i.e.,* low bicarbonates, because high bicarbonates concentrations may compensate the pH changes due to the VFA's accumulation. In this case, the pH drop will occur only when the process has been severely unbalanced. Unlike pH, alkalinity allows the detection of changes in the buffer capacity of an AD process [Rozzi, 1991, Malina and Pohland, 1992]. Thus, this variable is a better alternative to monitor the digester stability when the wastewater has a high buffer capacity [Rozzi, 1991]. However, since the alkalinity depends of the VFA's concentration, it must be monitored together with VFA's in order to have an accurate overview of the digester stability [Ahring and Angelidaki, 1997]. Therefore, both VFA's and alkalinity readings offer, accurate information about the digester stability. Some criterions based on relations between VFA's and alkalinity have been proposed to evaluate the digester stability. Zickefoose and Hayes [1976] for instance suggested that the ratio VFA's/TA should be maintained in the range of 0.1-0.35$\frac{mmol/l}{mmol/l}$ in order to improve the digester stability, where TA is the total alkalinity. Ripley et al. [1985] found that a successful digestion occurs when the ratio IA/PA is less than $0.3\frac{mmol/l}{mmol/l}$ and TA higher than 60mmol/l, where PA approximates the alkalinity due to bicarbonates, whereas IA = TA - PA is mainly composed by VFA's. Recently, Bernard et al. [2001b] found that the ratio IA/TA must be less or equal to $0.3\frac{mmol/l}{mmol/l}$ in order to avoid the digester instability.

Clearly, meeting such ratios have required the implementation of number of control strategies; however, these strategies have addressed the AD stability problem by using single-input single-output (SISO) control schemes focused in the regulation of either VFA's or alkalinity but no both [Wilcox et al., 1995, Marsilli-Libelli and Beni, 1996, Guwy et al., 1997, Bernard et al., 2001b, Steyer et al., 2006]. Thus, this paper address the digester stability problem from a multiple-input multiple-output (MIMO) point of view, by the simultaneous regulation of both the VFA's and TA concentrations, where the wastewater dilution rate is used to regulate the VFA's concentration, whereas an alkali solution is added directly to the digester in order to maintain TA at a given set point. The paper is organized as follows. First, the considered AD model is briefly described and TA is defined in terms of the model. Later, the conditions that must be satisfied in order to assure the digester stability are established in terms of the model. Then, the geometric properties of the AD model are analyzed and the control approach is developed. Thereafter, the control scheme is evaluated via numerical simulations under different operating conditions including load disturbances, uncertain kinetics, noisy measurements and restrictions in the control inputs. Finally, some conclusions are drawn.

2. THE CONSIDERED MODEL

Several models dealing with the AD process can be found in the current literature. However, most of these models describe in detail particular aspects of the process resulting difficult to use for monitoring and control purposes [Bastin and Dochain, 1990]. Recently, a model useful in the monitoring and control of AD processes has been proposed and validated by Bernard et al. [2001a]. This model was developed under the following assumptions: i) the AD process is operated in the pH range $6 \leq pH \leq 8$, ii) the VFA's are totally dissociated in the liquid phase and they are mainly composed of acetic acid, iii) the AD process is carried out under isothermal conditions. Such a model was validated in a great range of operating conditions including changes in both the loading rates and the retention time. Nevertheless, this model does not considers the alkali addition to digester. Then, the model proposed by Bernard et al. [2001a] is modified in this work to take into account this fact, by introducing the following assumption: iv) the wastewater dilution rate (D_1) is much greater than the alkali dilution rate (D_2) (*i.e.*, $D_1 \gg D_2$). This means that the total dilution rate ($D = D_1 + D_2$) can be approximated by $D \approx D_1$. Then, from assumptions i-iv the considered model is given by

$$\begin{aligned}
\dot{X}_1 &= (\mu_1(.) - \alpha D_1)X_1 \\
\dot{X}_2 &= (\mu_2(.) - \alpha D_1)X_2 \\
\dot{S}_1 &= (S_{1,in} - S_1)D_1 - k_1\mu_1(.)X_1 \\
\dot{S}_2 &= (S_{2,in} - S_2)D_1 + k_2\mu_1(.)X_1 - k_3\mu_2(.)X_2 \\
\dot{Z} &= (Z_{in} - Z)D_1 + (Z'_{in} - Z)D_2
\end{aligned} \quad (1)$$

where X_1, X_2, S_1 and S_2 are respectively the concentrations of acidogenic bacteria (g/l), methanogenic bacteria (g/l), primary substrate measured as COD (g/l) and VFA's (mmol/l). The strong ions concentration Z (mmol/l) can be defined in the considered pH range as: $Z = S_2 + B$, where B is the bicarbonates concentration. The subscript in is used to identify the concentration of each component in the wastewater inlet flow. Z'_{in}(mmol/l) represents the concentration of strong ions in the alkali flow, which is a constant and known value. D_1 and D_2 are the dilution rates related to the wastewater and alkali flow rates, respectively (*i.e.*, $D_i = Q_i/V$). The parameter α ($0 \leq \alpha \leq 1$) reflects the process heterogeneity: $\alpha = 0$ corresponds to an ideal fixed-bed reactor, whereas $\alpha = 1$ represents an ideal continuous stirred tank reactor (CSTR). $\mu_1(.)$ and $\mu_2(.)$ are the growth rates associated to the acidogenic and methanogenic microorganism, and are expressed in the model by a Monod-type and Haldane-type kinetics, respectively. Readers interested in a more detailed description are referenced to [Bernard et al., 2001a].

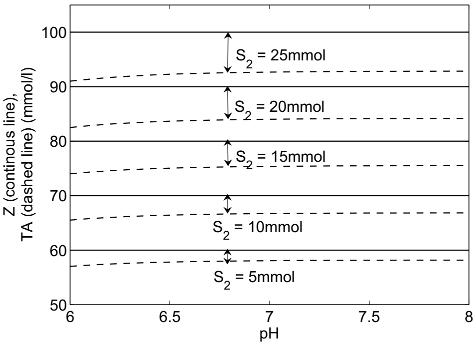

Fig. 1. Z and TA concentrations for the VFA's and pH ranges under NOC.

Now, let us define TA in terms of (1). Recall that TA is defined as the equivalent sum of all the bases that can be titrated with a strong acid to the first equivalence point of the system (*i.e.*, pH = 4.3) [Ripley et al., 1985]. Since the model considers only the presence of bicarbonates and VFA's in the digester, then TA can be expressed as [Alcaraz-Gonzalez, 2001]:

$$TA = f_{Tc}[HCO_3^-] + f_{Ta}[S_2^-] \quad (2)$$

where S_2^- and HCO_3^- represent respectively the concentrations of dissociated VFA's and bicarbonate, whereas f_{Tc} and f_{Ta} are given by:

$$f_{Tc} = \left(1 - \frac{10^{-pH} + K_c}{10^{-4.3} + K_c}\right); \; f_{Ta} = \left(1 - \frac{10^{-pH} + K_{ac}}{10^{-4.3} + K_{ac}}\right)$$

where K_c and K_{ac}(mmol/l) are the affinity constants for the HCO_3^-/CO_2 and S_2^-/S_2 equilibriums, respectively. Figure 1 depicts Z calculated from (1) and TA obtained from (2) as function of both pH and VFA's. Note that there exist a difference approximately constant between Z and TA in the pH range considered, thus, TA (2) can be approached by

$$TA \approx Z - \beta \quad (3)$$

where β represents the non titrated fraction of both, bicarbonates and VFA's.

2.1 Normal Operating Conditions (NOC)

Now, let us define the conditions that must be satisfied in order to assure the digester stability in terms of (1), which are called normal operating conditions (NOC). It is said that (1) is under NOC if the following conditions holds.

- The biomass remains active, which implies that a fraction of the substrates entering to the digester is consumed by the bacterial culture. This means in terms of the model that $X_j > 0$ and $(S_{j,in} - S_j) > 0$ $\forall t \geq 0$ and $j = 1, 2$.

- The dilution rate D_1 is constrained to avoid the digester washout (*i.e.*, the bacterial culture is dragged out of the digester), whereas D_2 is constrained to avoid the digester alkalinization (*i.e.*, $D_j(t) \in [D_j^-, D_j^+]$ $\forall t \geq 0$ and $j = 1, 2$).
- The system TA is greater than 60mmol/l.

3. NONLINEAR CONTROL APPROACH

3.1 Control Problem Statement

As it was already stated, the control of VFA's and TA is of paramount importance in AD processes since these variables are directly related to the process stability. Hence, in this paper the control problem can be stated as follows. The proposal of a MIMO control scheme based on differential geometry capable to achieve the regulation of both VFA's and TA concentrations in AD processes in the face of load disturbances, restrictions in the control inputs and uncertainties in the kinetics terms.

3.2 The Geometric Properties

In this section, key geometric properties of (1) are analyzed and then used in the controller design. First, let us rewrite (1) in the affine form for MIMO nonlinear systems [Isidori, 1995]

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i$$
$$y_1 = h_1(x), \ldots, y_m = h_m(x)$$

where m is the number of state variables to be regulated, $f(x)$, $g_i(x)'s$ are smooth vector fields, $h_i(x)'s$ are smooth functions defined on $U \subset \mathbb{R}_+^5$ and U the set of NOC. y_i represents the output functions whereas the control inputs are denoted by u_i. Particularly, the input and output vectors are given by

$$y = [S_2, TA]; \qquad u = [D_1, D_2]$$

Proposition 1. Let $y = [S_2, TA]$ and $u = [D_1, D_2]$ the output and input vectors of (1), respectively. Then, (1) has a relative degree vector $r = [1, 1]$ for any $x(0) = x(t = 0) \in U$, where x is the state vector.

Proof. From (3) and (1), the derivative of TA with respect to time is given by

$$\dot{TA} = (TA_{in} - TA)D_1 + (TA'_{in} - TA)D_2$$

where $TA_{in} = Z_{in} - \beta$ and $TA'_{in} = Z'_{in} - \beta$. Now, by computing the Lie derivative of the output vector y along the vector fields $f(x)$ and $g_i(x)$, one gets $L_{g_1} L_f^0 h_1(x) = (S_{2,in} - S_2)$ and $L_{g_2} L_f^0 h_2(x) = (TA'_{in} - TA)$. Then, the relative degree matrix

$$\mathbb{A}(x) = \begin{pmatrix} (S_{2,in} - S_2) & 0 \\ (TA_{in} - TA) & (TA'_{in} - TA) \end{pmatrix} \tag{4}$$

is nonsingular under NOC since $(S_{2,in} - S_2) \neq 0$, $(TA'_{in} - TA) \neq 0$; which means that (1) has a well-defined relative degree vector $r = [1, 1]$ for all $x(0) \in U$. $\diamond$

Proposition 1 implies the existence of an invertible map $z = \Phi(x)$. Then, since the relative degree vector (r) is strictly less than the system order (n) (i.e., $r < n$), (1) is locally partially input-output linearizable. Thus, $n - r$ complementary functions $\phi_i(x)$ must be proposed in order to complete the map $\Phi(x)$, where the complementary functions are solutions of the partial differential equation $L_{g_j} \phi_i(x) = 0$, for $j = 1, 2$ and $i = 3, 4, 5$.

Proposition 2. Let

$$z = \begin{pmatrix} S_2 \\ TA \\ X_1/(S_{2,in} - S_2)^\alpha \\ X_2/(S_{2,in} - S_2)^\alpha \\ X_1/[(S_{1,in} - S_1) + k_1/k_2(S_{2,in} - S_2)]^\alpha \end{pmatrix} \tag{5}$$

a dipheomorphism of (1). Then, (5) qualifies as a local coordinates transformation in a neighborhood of $x(0)$.

Proof. Let, $[S_{T,in} - S_T] = [(S_{1,in} - S_1) + k_1/k_2(S_{2,in} - S_2)]$. Then, by computing the determinant of the Jacobian matrix of (5), it is obtained that

$$det(\mathbb{J}(z)) = -\frac{\alpha X_1}{(S_{2,in} - S_2)^{2\alpha}(S_{T,in} - S_T)^{\alpha+1}} \tag{6}$$

Since $X_1 > 0$, $\alpha > 0$ (non ideal digester) and $(S_{j,in} - S_j) \neq 0$ for $j = 1, 2, T$, the Jacobian matrix $\mathbb{J}(z)$ is nonsingular for all $x \subset U$ guaranteeing the existence of an inverse $x = \Phi(z)^{-1}$. $\diamond$

From map (5), model (1) can be rewritten in the normal form as follows:

$$\dot{z}_1 = (S_{2,in} - z_1)D + (k_2\mu_1(.)z_2 - k_3\mu_2(.)z_3)(S_{2,in} - z_1)^\alpha$$
$$\dot{z}_2 = (TA_{in} - TA)D_1 + (TA'_{in} - TA)D_2 \tag{7a}$$

$$\dot{z}_3 = z_3\left(\mu_1(.) - \alpha\frac{k_3\mu_2(.)z_4 - k_2\mu_1(.)z_3}{(S_{2,in} - z_1)^{1-\alpha}}\right)$$
$$\dot{z}_4 = z_4\left(\mu_2(.) - \alpha\frac{k_3\mu_2(.)z_4 - k_2\mu_1(.)z_3}{(S_{2,in} - z_1)^{1-\alpha}}\right) \tag{7b}$$
$$\dot{z}_5 = z_5\left[\mu_1(.) - \frac{\alpha k_1 k_3 \mu_2(.)z_4}{k_2(S_{2,in} - z_1)^{1-\alpha}}\right]$$

where (7a) represents the linearizable part of (1), whereas (7b) denotes its internal dynamics.

Lemma 3. Under NOC, the internal dynamics of (1) is asymptotically stable.

Proof. Let

$$\mathbb{V} = \frac{X_T}{(S_{T,in} - S_T)^\alpha} \tag{8}$$

a candidate Lyapunov function (CLF), where X_T represents the total concentration of biomass into the digester (*i.e.*, $X_1 + X_2$). Under NOC, it is easy to verify that the CLF is positive defined for all $t \geq 0$. Now, by taking the time derivative of the CLF we get

$$\dot{\mathbb{V}} = \frac{X_T\mu_T(.)}{(S_{T,in} - S_T)^\alpha}\left[1 - \frac{\alpha K_T X_T}{(S_{T,in} - S_T)}\right] \tag{9}$$

where K_T is the global yield coefficient and $\mu_T(.)$ is the global grow rate. Under NOC, the following inequality is fulfilled

$$X_T \geq \frac{(S_{T,in} - S_T)}{\alpha K_T} \qquad \forall t \geq 0 \tag{10}$$

which means that in order to maintain the digester stability, the active biomass must be capable to degrade a fraction of the total substrate within the digester. Then, $\dot{\mathbb{V}} \leq 0$ for all $t \geq 0$ and thus the asymptotic stability of the internal dynamics is guaranteed. ⋄

3.3 Robust Approach

In this section, a MIMO output feedback approach is designed for the regulation of VFA's and TA. In order to take into account real operating conditions, the controller design is carried out under the following assumptions:

A1 The digester outputs (S_2, TA) are available from on-line measurements.
A2 From a biological point of view, it is not restrictive to assume that the growth rates $\mu_1(.)$ and $\mu_2(.)$ are bounded and positive uncertain functions. $S_{j,in}$ for $j = 1, 2$ and Z_{in} are smooth, bounded but uncertain functions.
A3 The composition of the alkali flow (TA'_{in}) is constant but the fraction of alkali ionized is unknown.
A4 The digester inputs D_1, D_2 are constrained in order to avoid the washout and the alkalinization of the digester by the saturation function

$$sat(D_i) = \left\{ \begin{array}{ll} D_i^+, \; if & D_i \geq D_i^+ \\ D_i, \; if & D_i^- < D_i < D_i^+ \\ D_i^-, \; if & D_i \leq D_i^- \end{array} \right\} \tag{11}$$

where $D_i \in \mathbb{R}^+$ and $D_{sat,i} = sat(D_i)$.

Theorem 4. Since the relative degree vector is well defined (see Proposition 1) and (7b) is asymptotically stable under NOC, then the following output feedback control assures the exponential convergence of the output vector $y = [z_1, z_2]$ toward its set-point values $y^* = [S_2^*, TA^*]$

$$[D_1, D_2]' = \mathbb{A}^{-1}(z) \begin{pmatrix} -L_f h_1(z) - v_1(z) \\ -L_f h_2(z) - v_2(z) \end{pmatrix} \tag{12}$$

where K_1, K_2 are the control gains and $v_1(z) = K_1(z_1 - S_2^*)$, $v_2(z) = K_2(z_2 - TA^*)$ are such that the polynomials $P_1(s) = s + K_1 = 0$ and $P_2(s) = s + K_2 = 0$ are Hurwitz.

Proof. From (12) and (7), it is easy to see that the error closed-loop dynamics is given by

$$\begin{aligned} \dot{e}_1 &= -K_1 e_1 \\ \dot{e}_2 &= -K_2 e_2 \\ \dot{z}_i &= \Upsilon(z) \qquad for \;\; i = 3, 4, 5 \end{aligned} \tag{13}$$

where $e_1 = z_1 - S_2^*$, $e_1 = z_2 - TA^*$ and $\Upsilon(z)$ represents the internal dynamics (7b) which is asymptotically stable under NOC (see Theorem 3). Notice that the dynamics of the error vector $e = [e_1, e_2]' \to 0$ as $t \to \infty$ completing the proof. ⋄

Unfortunately, the output feedback control law (12) cannot be directly implemented in practice, since the knowledge of both the inlet composition and the kinetic terms are required, which is a condition difficult to satisfy under real operating conditions (see assumptions A1-A4). Therefore, in order to overcome these limitations, $S_{2,in}$ and TA_{in} are rewritten as follows: $S_{2,in} = \tilde{S}_{2,in} + \Delta_{S_2}$, $TA_{in} = \tilde{TA}_{in} + \Delta_{TA}$, where Δ_{S_2} and Δ_{TA} are uncertain and bounded functions associated to the variation in the influent composition around a well-known nominal values $\tilde{S}_{2,in}$ and $\tilde{TA}_{in}$. These values can be determined by a single off-line measurement of the wastewater to be treated. TA'_{in} is rewritten as $TA'_{in} = \tilde{TA}'_{in} + \Delta'_{TA}$, where $\tilde{TA}'_{in}$ is a nominal value of the alkalinity concentration in the alkali flow and Δ'_{TA} is an uncertain function associated to the variation of the non titrated fraction. Then, by defining the uncertain functions $\eta_1 = k_2\mu_1(.)X_1 - k_3\mu_2(.)X_2 + \Delta_{S_2}D_1$ and $\eta_2 = \Delta_{TA}D_1 + \Delta'_{TA}D_2$, model (1) can be rewritten in the following extended state-space representation

$$\begin{aligned} \dot{z}_1 &= \eta_1 + (\tilde{S}_{2,in} - z_1)D_1 \\ \dot{z}_2 &= \eta_2 + (\tilde{TA}_{in} - z_2)D_1 + (\tilde{TA}'_{in} - z_2)D_2 \\ \dot{\eta} &= \Xi(z); \qquad \eta = [\eta_1, \eta_2]' \\ \dot{z}_i &= \Upsilon(z); \qquad for \;\; i = 3, 4, 5 \end{aligned} \tag{14}$$

where the augmented state vector η can be reconstructed from on-line measurements of the output and input variables [Femat et al., 1999]. In the present work, an extended Luenberger observer is used to estimate the uncertain states (η_1, η_2). Thus, by coupling the extended Luenberger observer to (12), the following robust control approach is obtained

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{\eta}_1 + (\tilde{S}_{2,in} - \hat{z}_1)D_{1,sat} + \Gamma_1 g_{11}(z_1 - \hat{z}_1) \\ \dot{\hat{z}}_2 &= \hat{\eta}_2 + (\tilde{TA}_{in} - \hat{z}_2)D_{1,sat} + (\tilde{TA}'_{in} - \hat{z}_2)D_{2,sat} \\ &\quad + \Gamma_2 g_{21}(z_2 - \hat{z}_2) \\ \dot{\hat{\eta}}_1 &= \Gamma_1^2 g_{12}(z_1 - \hat{z}_1) \\ \dot{\hat{\eta}}_2 &= \Gamma_2^2 g_{22}(z_2 - \hat{z}_2) \end{aligned} \tag{15a}$$

$$D_1 = -\frac{\hat{\eta}_1 + K_1(\hat{z}_1 - S_2^*)}{(\tilde{S}_{2,in} - \hat{z}_1)} \tag{15b}$$

$$D_2 = -\frac{(\tilde{TA}_{in} - \hat{z}_2)D_1 + \hat{\eta}_2 + K_2(\hat{z}_2 - TA^*)}{(\tilde{TA}'_{in} - \hat{z}_2)} \tag{15c}$$

where (15a) allows the estimation of the uncertain states η_1 and η_2, whereas (15b) and (15c) induce a desired behavior on the VFA's and TA concentrations, respectively. g_{11}, g_{12}, g_{21} and g_{22} are chosen such that the characteristic polynomial of the linear part of the estimation error ($e_i = z_i - \hat{z}_i$ for $i = 1, 2$) is Hurwitz, whereas Γ_i and K_i for $i = 1, 2$ are the estimation and control gains (tuning parameters), respectively. In order to avoid undesired effects in the controller performance due to the restrictions in the control inputs such as the windup phenomena, the restricted values of the dilution rates given by the saturation function (11) are feeding back to the observer allowing an observer-based antiwindup structure [Méndez-Acosta et al., 2004]. Thereby, (15) considers assumptions A1-A4. In addition, by analyzing the closed-loop behavior of (15), the following low-pass filter structures are obtained

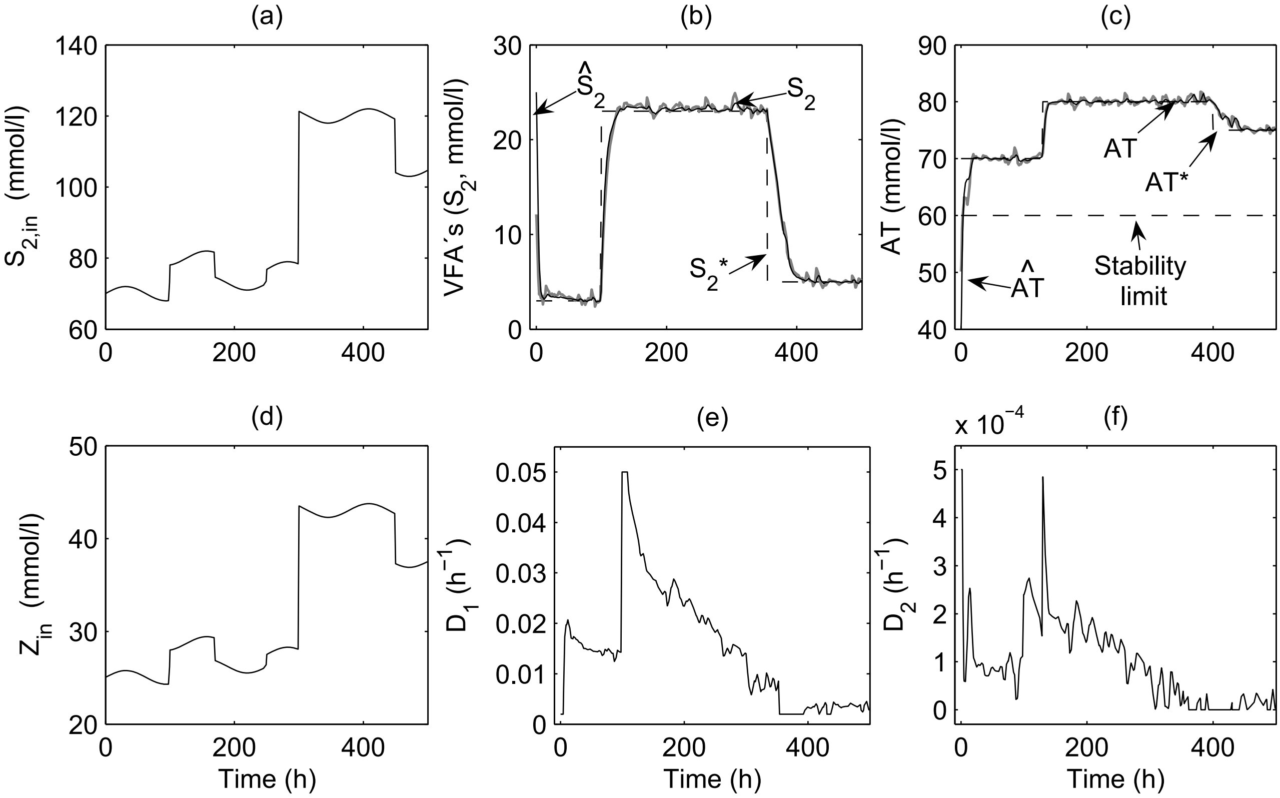

Fig. 2. a) VFA's concentration in the inlet flow ($S_{2,in}$). b) VFA's concentration in the AD process (S_2). c) Response of the digester TA during the simulation. d) Strong ions concentration in the inlet flow (Z_{in}). e) Wastewater dilution rate (D_1). f) Alkali dilution rate (D_2).

$$\frac{\hat{z}_1}{z_1} = \frac{\Gamma_1 g_{11}}{s + \Gamma_1 g_{11} + K_1} \qquad D_{i,sat} = D_i$$
$$\frac{\hat{z}_2}{z_2} = \frac{\Gamma_2 g_{21}}{s + \Gamma_2 g_{21} + K_2}$$

$$\frac{\hat{z}_1}{z_1} = \frac{\Gamma_1 g_{11} s + \Gamma_1^2 g_{22}}{s^2 + (\Gamma_1 g_{11} + D_1^{+,-})s + \Gamma_1^2 g_{12}} \qquad D_{i,sat} = D_i^{+,-}$$
$$\frac{\hat{z}_2}{z_2} = \frac{\Gamma_2 g_{21} s + \Gamma_2^2 g_{22}}{s^2 + (\Gamma_2 g_{21} + D_1^{+,-} + D_2^{+,-})s + \Gamma_2^2 g_{22}}$$

which means that the effect of noisy measurements can be handled by the suitable selection of the estimation and control gains. Notice that if $D_{i,sat} = D_i$ we have a first order low-pass filter which depends of both the controller and the observer gains, whereas if $D_{i,sat} = D_i^{+,-}$ the structure is a second order low pass filter which depends only of the observer gain and the saturation values.

4. NUMERICAL IMPLEMENTATION

Here, the performance of (15) is evaluated via numerical simulations under different operating conditions. The implementation was carried out using the Matlab-Simulink® software. The model parameters used along the simulation are those proposed by Bernard et al. [2001a], whereas the initial conditions were the following: $X_1(0) = 0.5$g/l, $X_2(0) = 0.7$g/l, $TA(0) = 50$mmol/l, $S_1(0) = 2.0$g/l, $S_2(0) = 12$mmol/l, $\hat{z}_1(0) = 25$mmol/l, $\hat{z}_2(0) = 40$mmol/l, $\hat{\eta}_1(0) = 0$ mmol/l-h and $\hat{\eta}_2(0) = 0$ mmol/l-h. The nominal values used are $\tilde{S}_{2,in} = 80$mmol/l, $\tilde{TA}_{in} = 40$mmol/l and $\tilde{TA}_{in}^{'} = 8000$mmol/l. The value of $\tilde{TA}_{in}^{'}$ is closer to the concentration of the industrial soda. In order to consider the stability criterions reported by Ripley et al. [1985] and Hill et al. [1987], the set-point values are chosen from the following restrictions

$$0 < S_2^* \leq 25\text{mmol/l}$$
$$TA^* \geq 60\text{mmol/l}$$

The upper and lower bounds used in (11) to constrain the dilution rates D_1, D_2 are $D_1 = [0.002, 0.05]h^{-1}$ and $D_2 = [0, 0.0005]h^{-1}$. Notice that the operation range for D_1 is greater than D_2, fulfilling the assumption iv (see Section 2). The control parameters are listed in Table 1, where the control and estimation gains are selected in order to have a specific cutoff-frequency for the low pass filter. For testing the controller under the influence of noisy measurements, white noise was added to the simulated measurements (*i.e.,* z_1, z_2). The sample time for the noise was 5h for both states, whereas the amplitude was 0.5 for S_2 and 1.0 for TA. Since the estimated $\hat{z}_1$, $\hat{z}_2$ are used in the control scheme, it is observed in Figures 2c), d) that the proposed control scheme was not sensitive to noisy measurements as it was able to trade the set point in a fast and smooth way.

Table 1. Parameters used in the numerical implementation of the control law (15)

g_{11}	g_{12}	g_{21}	g_{22}
2.0	1.0	2.0	1.0
Γ_1 (h^{-1})	Γ_2 (h^{-1})	K_1 (h^{-1})	K_2 (h^{-1})
0.2	0.3	0.3	0.3

Figures 2a) and d) depict the behavior of VFA's and Z inlet composition during simulation. As seen at time $t = 300$h, the wastewater VFA's and Z inlet concentration increased in order to evaluate the performance of (15) under the influence of load disturbances. The response of the VFA's concentration (S_2) is depicted in Figure 2b). Three set-point changes between 3 to 23 mmol/l were induced along the simulation. It can be observed in Figure 2b) that (15) yields a good set-point tracking performance in the VFA's regulation. In addition, notice that (15) is capable to attenuate the noisy measurements as well as the load disturbance induced at t = 300h. Figure 2e) shows the dynamic response of the wastewater dilution rate (D_1). It reached its upper saturation value when the reference value S_2^* is changed from 3 to 23mmol/l at t = 200h, nevertheless, the performance of (15) is not deteriorated during the saturation due to its antiwindup structure. When S_2^* is switched from 23 to 5mmol/l D_1 reached its lower saturation value again, despite this, the performance of the input control D_1 was acceptable. Figure 2c) shows that the performance of (15) in the regulation of TA for both cases the set-point tracking and the disturbance rejection is acceptable. The response of the alkali dilution rate D_2 is depicted in Figure 2f). Observe that D_2 has a smooth response to the set-points changes induced to the TA concentrations, which implies that the wearing down of the feeding pump is minimum. Observe that even when the alkali dilution rate D_2 reached the lower saturation value a t = 400h, the performance of (15) did not deteriorate.

5. CONCLUSIONS

In this work, a robust scheme was proposed to regulate the concentrations of VFA's and TA in anaerobic digesters. The model proposed by Bernard et al. [2001a] was modified in order to take into account: a) the addition of an alkali flow to the digester and b) the definition of TA as a state variable in terms of the model. The proposed scheme was conformed by an output feedback control and an extended Luenberger observer used to estimate the uncertain terms associated to the controlled states. The wastewater dilution rate was used as manipulated variable to control the VFA's concentration, whereas the alkali dilution rate was used to regulate TA. The control scheme was evaluated via numerical simulations showing a good performance in the regulation of VFA's and TA in spite of load disturbances, noisy measurements, control input restrictions and uncertainties in the kinetics terms. In addition, it is shown that the proposed scheme allows to fulfill the stability criterion defined for AD processes. Finally, the experimental implementation of the control scheme will be reported in future work.

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