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Decentralized Output Feedback Control of Interconnected Systems Using Low Gain-High Gain Feedback Domination ⋆

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Abstract: This paper examines a decentralized control technique which makes use of a gain domination method that implements both *low and high gain* to negate the effects of high order nonlinearities found in a series of interconnected systems that are coupled by both measurable and unmeasurable states. We develop a linear controller and observer design technique that applies this low gain-high gain feedback domination technique and by doing so we construct a method that allows for the global stabilization of a general class of nonlinear system. The low gain-high gain feedback domination method is applied to an example to illustrate its performance.

1. INTRODUCTION

The decentralized control of large-scale interconnected systems has been an area of considerable research due to its obvious practical application to current problems in the field of controls. Large-scale systems have very complex dynamic models due to the uncertain environment, the varying system parameters, and the decentralized structure of the system. Also it is inevitable that nonlinearities are prevalent throughout the dynamics of the interconnected systems. All these make the stabilization of such large-scale systems a difficult control problem. Though quite challenging, the research of large-scale systems are relevant to such areas as communication networks, a system of satellites, and formation flying of autonomous vehicles, and hence are important in control practice.

In this paper, we investigate the decentralized output feedback stabilization problem for the following class of interconnected systems that is comprised of linear integrators in combination with nonlinear terms represented by $\phi(x,y)$ and $\varphi(x,y)$,

$$\begin{aligned}
\dot{x}_1 &= x_2 + \phi_1(x,y)\\
\dot{x}_2 &= x_3 + \phi_2(x,y)\\
&\vdots\\
\dot{x}_n &= u + \phi_n(x,y)\\
\\
\dot{y}_1 &= y_2 + \varphi_1(x,y)\\
\dot{y}_2 &= y_3 + \varphi_2(x,y)\\
&\vdots\\
\dot{y}_{n-2} &= y_{n-1} + \varphi_{n-2}(x,y)\\
\dot{y}_n &= \varphi_n(x,y).
\end{aligned} \tag{1}$$

The research of large-scale nonlinear systems began in the late 1960's and early 1970's. One of the earliest investigations into the nonlinear issues of large-scale systems centered around time-varying stabilization [1]. The early research in [2] demonstrated a method of using high-gain state feedback to stabilize the nonlinearities of the large-scale systems. The research in the early 1980's focused on the use of state feedback to globally stabilize large-scale nonlinear systems. Adaptive control was applied in [3] to stabilize a class of large-scale nonlinear systems with success. Output feedback had also be applied to linear large-scale system in such papers as [4], [5], and [6] during the same time. The use of output feedback has certain apparent advantages because of the fact that not all of the state variables of a large-scale system can be measured. Note that most of the existing decentralized output feedback results are developed for large-scale systems interconnected only by the outputs. There are very few results dealing with large-scale systems interconnected by the unmeasurable states.

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The papers [7] and [8] dealt with the problem of decentralized control by output feedback control by applying a gain domination technique originally developed in [9]. In [7], lower triangular interconnected systems were shown to be globally stabilizable by employing a high gain domination output feedback controller. Under a linear growth condition imposed on the uncertain nonlinear vector fields, a linear controller was designed for each subsystem only using its own output. As shown in [9], this output feedback controller needs no information of the uncertain nonlinearities. The new structure of the observer and controller allowed for the ability to overcome the difficulty in dealing with the output feedback control problem in the presence of unmeasurable states in each subsystem. A combination of the observers and controllers constructed for subsystems globally stabilized the whole large-scale system.

In [8], upper-triangular nonlinear systems whose subsystems are interconnected by unmeasurable states were examined. In [8], a low gain design, rather than the high gain in [7], was employed to stabilize a class of upper-triangular large-scale systems by implementing an output feedback stabilization method. It developed a design procedure where a linear observer in parallel with a state feedback controller for each individual subsystem was constructed. Furthermore, assuming that the upper-triangular system meets the linear growth condition, no prior information about the nonlinearities was necessary.

This paper generalizes the results from [7] and [8] and examines the problem of interconnected systems coupled with unmeasurable states where the structure can be both lower and upper triangular. By taking advantage of the structure of the problem, we show that a *low gain-high gain domination technique* can be developed in a step by step process which can overcome the high order functions present in the systems.

This paper is organized as follows, Section 2 *Global Decentralized Control of Interconnected System* presents the problem statement, where we will present our assumption which utilizes a growth condition for bounding the nonlinearities of systems. Also we will present our main results and demonstrate that a linear observer coupled with its output feedback controller can globally stabilize an interconnected system. A stability analysis is performed that proves that by employing a low gain and high gain feedback domination, global stability is guaranteed. Section 3, *An Example*, implements a high gain L and low gain ℓ output feedback controller on an two interconnected system with three states. We summary and conclude our results in Section 4.

2. GLOBAL DECENTRALIZED CONTROL OF INTERCONNECTED SYSTEM

In this paper, we examine a class of large-scale uncertain nonlinear systems comprised of 2-interconnected systems that are coupled not only by their measurable states but also by their unmeasurable states. Due to this interconnection, an output feedback controller will be developed to stabilize this interconnected system. Consider the following interconnected systems,

$$\dot{x} = Ax + Bu + \begin{bmatrix} \phi_1(x,y) \\ \vdots \\ \phi_{n-1}(x,y) \\ \phi_n(x,y) \end{bmatrix},$$

$$\dot{y} = Ay + Bu + \begin{bmatrix} \varphi_1(x,y) \\ \vdots \\ \varphi_{n-1}(x,y) \\ \varphi_n(x,y) \end{bmatrix},$$

with the following measurable states,

$$x_{out} = x_1 \qquad y_{out} = y_1,$$

and

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

The functions $\phi(x,y)$ and $\varphi(x,y)$ found in System (2) are nonlinear coupled functions that are interconnected between the two systems. We make use of the following assumption,

Assumption 2.1. For any constant $L > 1$, the following holds

$$\left|\frac{\phi(x,y)}{L^{i-1}}\right| = \left|\frac{\phi(\xi_1, L\xi_2, \cdots, L^{n-1}\xi_n, \eta y_1, \frac{1}{L}\eta y_2, \cdots, \frac{1}{L^{n-1}}\eta y_n)}{L^{i-1}}\right|$$
$$\leq C(L^{1-a}\|\xi\| + \frac{1}{L^b}\|\eta y\|)$$
$$L^{i-1}|\varphi(x,y)| = \left|L^{i-1}\varphi(\xi_1, L\xi_2, \cdots, L^{n-1}\xi_n, \eta y_1, \frac{1}{L}\eta y_2, \cdots, \frac{1}{L^{n-1}}\eta y_n)\right|$$
$$\leq C(\frac{1}{L^c}\|\xi\| + (\frac{1}{L})^{1+d}\|\eta y\|),$$

where a, b, c, and d are positive constants and $C > 0$ is a constant.

The objective of this paper is to show that under Assumption 2.1, there exist dynamic observers and controllers such that the closed-loop system is globally asymptotically stable at the equilibrium.

We will now develop a step-by-step design which implements a linear controller and observer to globally stabilize system (2) by making use of Assumption 2.1. The main result of this section is the following Theorem,

Theorem 2.2. Under Assumption 2.1, there exists a linear output feedback controller that renders the large-scale interconnected system (2) globally asymptotically stable.

Proof:
We prove Theorem 2.2 by first designing a linear controller and then an observer. The nonlinearities in System (2) are negated by performing a change of coordinates (diffeomorphism) on the interconnected systems. By making use of scaling gains which were involved in the diffeomorphism, low gain-high gain feedback domination is achievable.

2.1 Construction and Design of the Linear Controller

Perform a change of coordinates on System (2) where

$$\xi_i = \frac{x_i}{L^{i-1}}, \quad i = 1, \cdots, n$$
$$\eta_i = L^{i-1} y_i, \quad i = 1, \cdots, n$$

noting that the scaling gains involved in the change of coordinates are related as follows, $\ell = \frac{1}{L}$. After the necessary substitutions, we arrive at

$$\begin{aligned}
\dot{\xi}_1 &= x_2 + \phi_1(x,y) & \dot{\eta}_1 &= y_2 + \varphi_1(x,y)\\
\dot{\xi}_2 &= \frac{x_3}{L} + \frac{\phi_2(x,y)}{L} & \dot{\eta}_2 &= \frac{y_3}{\ell} + \frac{\varphi_2(x,y)}{\ell}\\
\dot{\xi}_3 &= \frac{x_4}{L^2} + \frac{\phi_3(x,y)}{L^2} & \dot{\eta}_3 &= \frac{y_4}{\ell^2} + \frac{\varphi_3(x,y)}{\ell^2}\\
&\vdots & &\vdots\\
\dot{\xi}_{n-1} &= \frac{x_n}{L^{n-2}} + \frac{\phi_{n-1}(x,y)}{L^{n-2}} & \dot{\eta}_{n-1} &= \frac{y_n}{\ell^{n-2}} + \frac{\varphi_{n-1}(x,y)}{\ell^{n-2}}\\
\dot{\xi}_n &= \frac{u}{L^{n-1}} + \frac{\phi_n(x,y)}{L^{n-1}} & \dot{\eta}_n &= \frac{v}{\ell^{n-1}} + \frac{\varphi_n(x,y)}{\ell^{n-1}}.
\end{aligned}$$

Recognizing that $x_i = \xi_i L^{i-1}$ and $y_i = \eta_i \ell^{i-1}$ we have

$$\begin{aligned}
\dot{\xi}_1 &= L\xi_2 + \phi_1(x,y) & \dot{\eta}_1 &= \ell\eta_2 + \varphi_1(x,y)\\
\dot{\xi}_2 &= L\xi_3 + \frac{\phi_2(x,y)}{L} & \dot{\eta}_2 &= \ell\eta_3 + \frac{\varphi_2(x,y)}{\ell}\\
&\vdots & &\vdots\\
\dot{\xi}_{n-1} &= L\xi_n + \frac{\phi_{n-1}(x,y)}{L^{n-2}} & \dot{\eta}_{n-1} &= \ell\eta_n + \frac{\varphi_{n-1}(x,y)}{\ell^{n-2}}\\
\dot{\xi}_n &= \frac{u}{L^{n-1}} + \frac{\phi_n(x,y)}{L^{n-1}} & \dot{\eta}_n &= \frac{v}{\ell^{n-1}} + \frac{\varphi_n(x,y)}{\ell^{n-1}}.
\end{aligned} \tag{2}$$

We construct the virtual controllers, $u^* = -L^n[k_1\xi_1 + k_2\xi_2 + \cdots + k_n\xi_n]$ and $v^* = -\ell^n[k_1\eta_1 + k_2\eta_2 + \cdots + k_n\eta_n]$, where $k_1, \cdots, k_n$ are the coefficients of the Hurwitz polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1 = 0$. System (2) can be written compactly as

$$\dot{\xi} = L\bar{A}\xi + \begin{bmatrix} \phi_1(x,y) \\ \frac{\phi_2(x,y)}{L} \\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{(u-\hat{u}^*)}{L^{n-1}} \end{bmatrix}$$

$$\dot{\bar{\eta}} = \ell\bar{A}\bar{\eta} + \begin{bmatrix} \varphi_1(x,y) \\ \frac{\varphi_2(x,y)}{\ell} \\ \vdots \\ \frac{\varphi_n(x,y)}{\ell^{n-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{\ell(v-\hat{v}^*)}{l^{n-1}} \end{bmatrix}$$

where

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix}, \qquad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix}, \quad \text{and} \tag{3}$$

$$\bar{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & \cdots & -k_n \end{bmatrix}. \tag{4}$$

2.2 Construction and Design of the Linear Observer

Next we design the following linear observer for System (2),

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + La_1(x_1 - \hat{x}_1)\\
&\vdots\\
\dot{\hat{x}}_{(n-1)} &= \hat{x}_n + L^{(n-1)}a_{n-1}(x_1 - \hat{x}_1)\\
\dot{\hat{x}}_n &= u + L^n a_n(x_1 - \hat{x}_1),\\
\dot{\hat{y}}_1 &= \hat{y}_2 + \frac{1}{L}a_1(y_1 - \hat{y}_1)\\
&\vdots\\
\dot{\hat{y}}_{(n-1)} &= \hat{y}_n + \frac{1}{L^{n-1}}a_{n-1}(y_1 - \hat{y}_1)\\
\dot{\hat{y}}_n &= v + \frac{1}{L^n}a_n(y_1 - \hat{y}_1)
\end{aligned} \tag{5}$$

where the linear controllers u and v are defined as

$$u = -L^n[k_1\hat{\xi x}_1 + k_2\hat{\xi x}_2 + \cdots + k_n\hat{\xi x}_n]$$
$$v = -\ell^n[k_1\hat{\eta}_1 + k_2\hat{\eta}_2 + \cdots + k_n\hat{\eta}_n],$$

where $\hat{\xi}_i = \hat{x}_i L^{i-1}$ and $\hat{\eta}_i = \hat{y}_i \ell^{i-1}$. Next, we introduce a change of coordinates on (5) and define the following error terms

$$e_i = \frac{x_i - \hat{x}_i}{L^{i-1}} = \xi_i - \hat{\xi}_i, \quad i = 1, \cdots, n$$
$$\varepsilon_i = \frac{y_i - \hat{y}_i}{\ell^{i-1}} = \eta_i - \hat{\eta}_i, \quad i = 1, \cdots, n.$$

A simple calculation yields the following error dynamics:

$$\begin{aligned}
\dot{e}_1^x &= e_2^x - La_1e_1^x + \phi_1(x,y)\\
&\vdots\\
\dot{e}_{(n-1)}^x &= e_n^x - L^{n-1}a_{n-1}e_1^x + \phi_{n-1}(x,y)\\
\dot{e}_n^x &= -L^n a_n e_1^x + \phi_n(x,y),\\
\dot{e}_1^y &= e_2^y - \ell a_1 e_1^y + \varphi_1(x,y)\\
&\vdots\\
\dot{e}_{(n-1)}^y &= e_n^y - \ell^{n-1}a_{n-1}e_1^y + \varphi_{n-1}(x,y)\\
dote_n^y &= -\ell^n a_n e_1^y + \varphi_n(x,y)
\end{aligned}$$

Performing the change of coordinates on (5) yields the following error dynamics,

$$\dot{e} = L\hat{A}e + \begin{bmatrix} \phi_1(x,y) \\ \frac{\phi_2(x,y)}{L} \\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}} \end{bmatrix}, \; \dot{\varepsilon} = L\hat{A}\varepsilon + \begin{bmatrix} \varphi_1(x,y) \\ \frac{\varphi_2(x,y)}{\ell} \\ \vdots \\ \frac{\varphi_n(x,y)}{\ell^{n-1}} \end{bmatrix} \tag{6}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, \qquad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}.$$

2.3 Stability Analysis

This section will perform a Lyapunov stability analysis on (3) and (6). As noted previously in the introduction, we apply a low gain-high gain feedback domination method which negates the effects of nonlinearities in the interconnected system. The Lyapunov inequalities for both (3) and (6) will be shown to be made negative definite by appropriate choices in the scaling gains L and ℓ. As will be clearly seen, this gain domination technique plays a vital role in the stability analysis.

Noting that $\bar{A}$ from (4) is a Hurwitz matrix from (3), there is a positive definite matrix $P = P^T > 0$ such that $\bar{A}^T P + P\bar{A} = -I$. Consider the following Lyapunov functions $V_\xi = \xi^T P\xi$ and $V_{\bar{y}} = \eta^T P\eta$ for (3). The derivative of V_ξ along (3) is found to be,

$$\begin{aligned}\dot{V}_\xi &= L\xi^T(\bar{A}^T P + P\bar{A})\xi + 2\xi^T P \\ &\quad \times \left(\begin{bmatrix} \phi_1(x,y) \\ \frac{\phi_2(x,y)}{L} \\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{(u-\hat{u}^*)}{L^{n-1}} \end{bmatrix} \right), \\ &\le -L\|\xi\|^2 + 2\|P\|\|\xi\| \left\| \begin{matrix} \phi_1(x,y) \\ \frac{\phi_2(x,y)}{L} \\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}} \end{matrix} \right\| \\ &\quad + 2\|P\|\|\xi\| \frac{|u-\hat{u}^*|}{L^{n-1}}, \\ &\quad \text{where} \quad \frac{(u-\hat{u}^*)}{L^{n-1}} \le CL\|e\|, \\ &\le -\frac{L}{2}\|\xi\|^2 + LM\|e\|^2 \\ &\quad + 2\|P\|\|\xi\| \left\| \begin{matrix} \phi_1(x,y) \\ \frac{\phi_2(x,y)}{L} \\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}} \end{matrix} \right\|, \end{aligned} \tag{7}$$

where $M > 0$ is a constant. Applying Assumption 2.1 on the functions $\phi(\xi,\eta)$, the inequality (7) becomes

$$\begin{aligned}\dot{V}_\xi &\le -\frac{L}{2}\|\xi\|^2 + LM\|e\|^2 + C\|\xi\| \left[L^{1-a}\|\xi| + \frac{1}{L^b}\|\eta|\right] \\ &\le -\frac{L}{2}\|\xi\|^2 + LM\|e\|^2 + CL^{1-a}\|\xi\|^2 + C\|\xi\|\frac{1}{L^b}\|\eta\|,\end{aligned}$$

where $c > 0$. Completing the square, it is easy to calculate,

$$\dot{V}_\xi \le -\frac{L}{2}\|\xi\|^2 + LM\|e\|^2 + CL^{1-a}\|\xi\|^2 + CL^{1-b}\|\xi\|^2 + \frac{C}{L^{1+b}}\|\eta\|^2. \tag{8}$$

Remark 2.3. Note that the choice of L, a, and b can be any positive constant which dominates the dynamics of the nonlinear functions $\phi(\bar{x},\bar{y})$ and $\varphi(\xi,\eta)$. For example in the term from (8),

$$CL^{1-a}\|\xi\|^2,$$

an appropriately large enough L will dominate the states from System (1). Conversely, in the term from (8)

$$\frac{1}{L^{1+b}}\|\eta\|^2,$$

an appropriately large enough L can be made small enough to negate the states from System 2, hence the effect of low gain-high feedback domination can be clearly seen.

Using a similar argument, the derivative of V_η along (3) is,

$$\begin{aligned}\dot{V}_\eta &= \ell\eta^T(\bar{A}^T P + P\bar{A})\eta + 2\eta^T P \\ &\quad \times \left(\begin{bmatrix} \varphi_1(x,y) \\ \frac{\varphi_2(x,y)}{\ell} \\ \vdots \\ \frac{\varphi_n(x,y)}{\ell^{n-1}} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{v-\hat{v}^*}{\ell^{n-1}} \end{bmatrix} \right) \\ &\le -\ell\|\eta\|^2 + 2\|P\|\|\eta\| \left\| \begin{matrix} \varphi_1(x,y) \\ \frac{\varphi_2(x,y)}{L} \\ \vdots \\ \frac{\varphi_n(x,y)}{L^{n-1}} \end{matrix} \right\| \\ &\quad + 2\|P\|\|\eta\|\frac{|v-\hat{v}^*|}{\ell^{n-1}}, \\ &\quad \text{where} \quad \frac{(v-\hat{v}^*)}{\ell^{n-1}} \le C\ell\|\varepsilon\|, \\ &\le -\frac{\ell}{2}\|\eta\|^2 + \ell M\|\varepsilon|^2 + 2\eta^T P \left\| \begin{matrix} \varphi_1(x,y) \\ \frac{\varphi_2(x,y)}{L} \\ \vdots \\ \frac{\varphi_n(x,y)}{L^{n-1}} \end{matrix} \right\|, \end{aligned} \tag{9}$$

where $M > 0$ is a constant. Using Assumption 2.1 for $\varphi(x,y)$, the inequality (9) becomes

$$\begin{aligned}\dot{V}_\eta &\le -\frac{\ell}{2}\|\eta\|^2 + \ell M\|\varepsilon\|^2 + C\|\eta\| \\ &\quad \times \left[\frac{1}{L^c}\|\xi| + \left(\frac{1}{L}\right)^{1+d}\|\eta\| \right] \\ &\le -\frac{\ell}{2}\|\eta\|^2 + \ell M\|\varepsilon\|^2 + C\frac{1}{L^c}\|\eta\|\|\xi\| \\ &\quad + C\left(\frac{1}{L}\right)^{1+d}\|\eta\|^2,\end{aligned}$$

where $C > 0$. Completing the square and noting that $\ell = \frac{1}{L}$,

$$\begin{aligned}\dot{V}_\eta &\le -\frac{1}{2}\ell\|\eta\|^2 + \ell M\|\varepsilon|^2 + C\left(\frac{1}{L}\right)^{1+d}\|\eta\|^2 \\ &\quad + C\left(\frac{1}{L}\right)^{c+1}\|\eta\|^2 + L^{1-c}\|\xi\|^2.\end{aligned} \tag{10}$$

We now examine the Lyapunov inequalities for the error dynamics of (6). For Hurwitz matrix $\hat{A}$ from (7), there is a positive definite matrix $Q = Q^T > 0$ such that $\hat{A}^T Q + Q\hat{A} = -I$. Consider the following Lyapunov functions $U_e = (M+1)e^T Qe$ and $U_\varepsilon = (M+1)\varepsilon^T Q\varepsilon$. The derivative of U_e along (6) is calculated to be

$$\dot{U}_e \leq -L\|e\|^2(M+1)+2(M+1)e^TQ\begin{bmatrix}\phi_1(x,y)\\ \vdots \\ \frac{\phi_n(x,y)}{L^{n-1}}\end{bmatrix}$$
$$\leq -L\|e\|^2(M+1)+C\|e\|\left[L^{1-a}\|\xi\|+\frac{1}{L^b}\|\eta\|\right]$$
$$\leq -L\|e\|^2(M+1)+C\|e\|L^{1-a}\|\xi\|+C\|e\|\frac{1}{L^b}\|\eta\|$$
$$\leq -L\|e\|^2(M+1)+CL^{1-a}\left(\|e\|^2+\|\xi\|^2\right)$$
$$+CL^{1-b}\|e\|^2+\left(\frac{1}{L}\right)^{b+1}\|\eta\|^2, \qquad (11)$$

where $M > 0$ and $C > 0$.

The derivative of U_ε along (6) is calculated to be

$$\dot{U}_\varepsilon \leq -\ell\|\varepsilon\|^2(M+1)+2(M+1)\varepsilon^TQ\begin{bmatrix}\varphi_1(x,y)\\ \vdots \\ \frac{\varphi_n(x,y)}{\ell^{n-1}}\end{bmatrix}$$
$$\leq -\ell\|\varepsilon\|^2(M+1)+c\|\varepsilon\|\left[\ell^c\|\varepsilon\|+\ell^{1+d}\|\eta\|\right]$$
$$\leq -\ell\|\varepsilon\|^2(M+1)+C\ell^{1+d}\|\varepsilon\|\|\eta\|+C\ell^c\|\varepsilon\|\|\xi\|$$
$$\leq -\ell\|\varepsilon\|^2(M+1)+C\ell^{1+d}\left(\|\varepsilon\|^2+\|\eta\|^2\right)$$
$$+CL^{1-c}\|\xi\|^2+C\|\varepsilon\|^2\ell^{1+c}, \qquad (12)$$

where $M > 0$ and $C > 0$ are constants.

Therefore we can show that, $\dot{W} = \dot{V}_\xi + \dot{V}_\eta + \dot{U}_e + \dot{U}_\varepsilon$ can be made negative definite by an appropriate choice of L. Hence, (2) can be made globally asymptotically stable. □

Remark 2.4. For convenience, this proof assumed only two subsystems. However, this method can be easily extended to include m interconnected subsystems.

Remark 2.5. This low gain-high gain feedback domination technique can be extended to include homogeneous feedback domination based on the technique in [10]. In this case, the nonlinearities are not necessarily required to satisfy the linear growth condition. In fact with the help of the low gain-high gain domination technique, the nonlinearities can include a variety of high order terms while the decentralized output feedback stabilization problem is solvable. The interested reader can be referred to [10].

3. AN EXAMPLE

We apply Theorem 2.2 on the following systems that are interconnected by unmeasurable states,

$$\dot{x}_1 = x_2 + x_1 + x_3^{\frac{1}{3}}\log(1+y_3^2)$$
$$\dot{x}_2 = x_3 + y_2$$
$$\dot{x}_3 = u + x_1 + x_2 + y_3$$
$$x_{out} = x_1$$

$$\dot{y}_1 = y_2 + x_1^{\frac{1}{3}}y_3^{\frac{2}{3}}$$
$$\dot{y}_2 = y_3 + v$$
$$\dot{y}_3 = v,$$
$$y_{out} = y_1. \qquad (13)$$

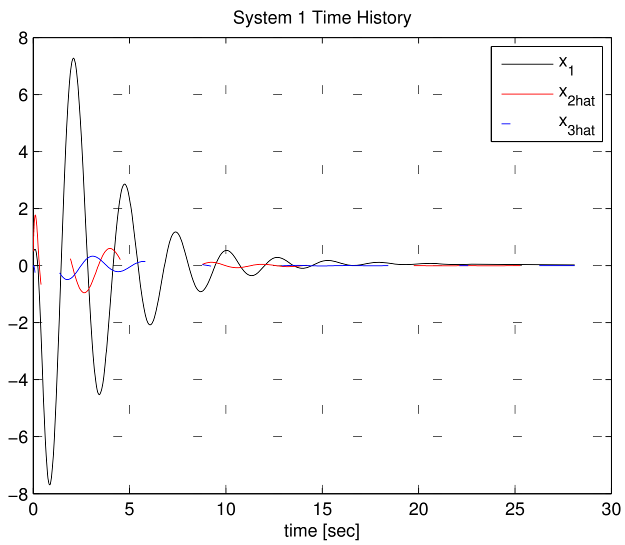

Fig. 1. System One Time History using High Gain L.

We develop the following control laws for System 1 and System 2 from (13)

$$u = -L^3\left(k_1\hat{x}_1 + k_2\frac{\hat{x}_2}{L} + k_3\frac{\hat{x}_3}{L^2}\right)$$
$$v = -\ell^3\left(k_1\hat{y}_1 + k_2\frac{\hat{y}_2}{\ell} + k_3\frac{\hat{y}_3}{\ell^2}\right) \qquad (14)$$

where the scaling gains are found to be $L =$**4** and $\ell = \frac{1}{L}$.

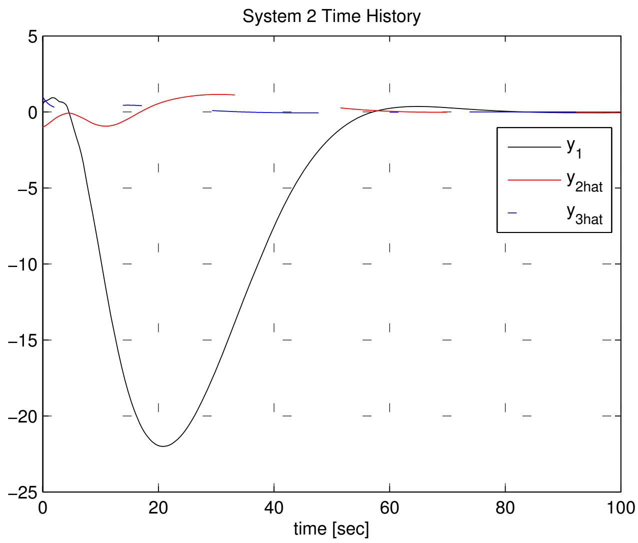

Fig. 2. System Two Time History using Low Gain ℓ.

Finally the observers used were designed as follows

$$\dot{\hat{x}}_1 = \hat{x}_2 + a_1 L (x_1 - \hat{x}_1)$$
$$\dot{\hat{x}}_2 = \hat{x}_3 + a_2 L^2 (x_1 - \hat{x}_1)$$
$$\dot{\hat{x}}_3 = u + a_3 L^3 (x_1 - \hat{x}_1)$$

$$\dot{\hat{y}}_1 = \hat{y}_2 + a_1 \ell (x_1 - \hat{y}_1)$$
$$\dot{\hat{y}}_2 = \hat{y}_3 + a_2 \ell^2 (y_1 - \hat{y}_1)$$
$$\dot{\hat{y}}_3 = v + a_3 \ell^3 (y_1 - \hat{y}_1).$$

The Hurwitz polynomial $s^3 + 3s^2 + 3s + 1$ was used for both the controller gains k_1, k_2, k_3 and observer gains a_1, a_2, a_3. The interconnected system from (13) was combined with the control and observer dynamics (14) and (15) and simulated. Figures 1 and 2 are the closed loop time history responses applying Theorem 2.2.

4. CONCLUSION

We have developed a method of using output feedback to globally stabilize two interconnected system whose subsystems are coupled by both upper-triangular and lower-triangular nonlinearities. Under the linear growth condition, we explicitly construct a set of linear observers and controllers only using the output feedback information of each system. It is shown that global output feedback stabilization is achieved for the closed-loop system by applying a *low gain-high gain domination* method. This method can be easily extended to m subsystems.

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