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Stabilization for a Class of Nonlinear Systems: A Fuzzy Logic Approach ⋆

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Abstract: In this paper, the problem of stabilization for the class of continuous time nonlinear systems which are exactly discretized is addressed. By using the Takagi–Sugeno model approach, a discrete controller capable of stabilizing the discrete TS model and the continuous model as well, is obtained. This scheme allows the use of a digital controller for stabilizing an analog plant.

1. INTRODUCTION

The problem of stabilizing dynamical systems is an interesting problem, and many approaches dealing with it for different classes of systems – linear, nonlinear, continuous, discrete – have been developed. In general, the control law for each type of system has the same nature of the original system, namely, for continuous systems, continuous controllers are designed to guarantee the stabilizability of the closed–loop system.

The use of faster digital computers has motivated the design of sampled data controllers for continuous time plants. However, when applied to the continuous system, the performance of the digital controller may not be necessarily satisfactory. A solution consists of discretizing the continuous system and designing a suitable controller on the basis of the discrete system obtained through this procedure.

Several methods for the approximate discretization can be found in the literature, but obviously the performance of a controller designed on the basis of the approximate discretization depends on the degree of approximation. For example, when using the simple Euler method, it is possible that the controller does not guarantee the stability of the closed loop system (see Monaco and Normand-Cyrot [2007] for an overview on this issue). Obviously, a way to avoid this is to design the controller on the basis of the exact discretization, although this is not always possible. Recently, however, several methods for the exact discretization have appeared in the literature. Moreover, some results suggest that it is possible to induce the "exact discretizability property" by a suitable feedback (Di Giamberardino et al. [2000]). An example of such systems are the systems completely linearizable by feedback or the class of systems transformable to a polynomial triangular form.

On the other hand, recent results on the fuzzy modeling, in particular, the Takagi–Sugeno (TS) model, have allowed the use of this modeling approach to deal with the problem of stabilizing nonlinear systems. Indeed, the design of the controller is made on the basis of the linear subsystems that describe locally the aggregate nonlinear TS model. A relative advantage of using this approach is that it is possible to stabilize nonlinear systems by means of the design based on the linear subsystems.

In this paper, we propose the use of the TS model approach for the discrete model obtained from an exact discretization. This discretized model will be used to calculate a discrete controller which stabilizes the exact discretized model. We show that under some conditions, this discrete controller, when applied to the continuous time system via a zero order holder, it stabilizes this continuous system as well.

The paper is organized as follows. In Section 2 some facts about the discretization of dynamical systems are recalled. In Section 3 the Takagi–Sugeno Fuzzy Model is introduced, while in Section 4 the discrete fuzzy stabilization problem is solved. An example is presented in Section 5, and some comments conclude the paper.

2. SOME FACTS ABOUT THE DISCRETIZATION OF DYNAMICAL SYSTEMS

Consider a linear time invariant system described by

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$$\dot{x} = A_c x + B_c u \quad (1)$$
$$y = C_c x \quad (2)$$

where $x \in \mathbb{R}^n$, $u, y \in \mathbb{R}^m$. It is well known that the discretization of this linear system with a sampling interval δ is given by

$$x_{k+1} = A_d x_k + B_d u_k$$
$$y_k = C_d x_k$$

where

$$x_{k+1} = x(k\delta + \delta), \quad x_k = x(k\delta), \quad u_k = u(k\delta),$$
$$A_d = e^{A_c \delta}, \quad B_d = \int_0^\delta e^{A_c s}\, ds\, B_c, \quad C_d = C_c.$$

For nonlinear systems, however, finding the exact solution of the differential equations is in most cases difficult, if not impossible. Hence, various authors consider approximate discretizations. As a result, at the sampling instants the solutions of the differential and approximate discretized systems do not coincide, and poor accuracy may result. Also, relatively large sampling period may cause instability or undesired behavior.

However, some cases for exact discretization have been studied in the literature (Di Giamberardino et al. [1996], Monaco et al. [1996]; see also Monaco and Normand-Cyrot [2007] and references therein). These schemes allow expressing the discretization process as a power Lie series, and exact discretization can be obtained if some conditions on the residuals hold. To precise the ideas, let us consider the nonlinear system

$$\dot{x} = f(x, u).$$

Expanding its solutions $x(t)$ around $t = 0$ we get

$$\begin{aligned} x(t) &= \left[\frac{x(t)}{0!}\right]_{t=0} + \left[\frac{\dot{x}(t)}{1!}\right]_{t=0} t + \left[\frac{\ddot{x}(t)}{2!}\right]_{t=0} t^2 + \cdots \\ &= x(0) + f(x(0), u(0))t + \frac{1}{2!}\left[\dot{f}(x, u)\right]_{t=0} t^2 + \cdots \quad (3) \\ &= x(0) + \sum_{i=1}^{\infty} \frac{t^i}{i!}\left[f^{(i)}(x, u, \dot{u}, \cdots, u^{(i-1)})\right]_{t=0} \end{aligned}$$

where the operator $f^{(i)}(x, u, \dot{u}, \cdots, u^{(i-1)})$ is defined as

$$f^{(1)}(x, u) = f(x, u)$$

$$\begin{aligned} f^{(i)}(x, u, .., u^{(i-1)}) &= \frac{\partial f^{(i-1)}(x, u, \dot{u}, \cdots, u^{(i-2)})}{\partial x} f(x, u) \\ &+ \frac{\partial f^{(i-1)}(x, u, \dot{u}, \cdots, u^{(i-1)})}{\partial u}\dot{u} + \cdots \\ &+ \frac{\partial f^{(i-1)}(x, u, \dot{u}, \cdots, u^{(i-1)})}{\partial u^{(i-2)}} u^{(i-1)}. \end{aligned}$$

Taking the solution (3) around $t = k\delta$ and considering a piecewise constant input u_k for $k\delta \le t < (k+1)\delta$, we can write the discrete solution as

$$x_{k+1} = \sum_{i=0}^{\infty} \frac{\delta^i}{i!}\left[L^i_{f(x,u)}(x)\right]_{\substack{x=x_k \\ u=u_k}} = e^{\delta L^i_{f(x,u)}}(x)\Big|_{\substack{x=x_k \\ u=u_k}} \quad (4)$$

where $L^i_{f(x,u)}(\cdot)$ is defined as

$$L^i_{f(x,u)}(x) = \frac{\partial L^{i-1}_{f(x,u)}}{\partial x} f(x, u), \quad L^0_{f(x,u)}(x) = x.$$

From the previous expression, if for a finite i the term $L^i_{f(x,u)}$ is zeroed, namely the nilpotency condition is fulfilled, and the discretization becomes exact. Otherwise, only an approximation up to a certain degree can be obtained. The condition of nilpotency is a sufficient condition for exact discretization.

3. THE TAKAGI–SUGENO FUZZY MODEL

Let us consider a continuous time nonlinear system described by

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x) \end{aligned} \quad (5)$$

where $x \in \mathbb{R}^n$, $u, y \in \mathbb{R}^m$, and f and h are analytic functions of their arguments. It is well known that it is possible to describe, at least in a certain region of interest, the behavior of the nonlinear system (5) by a suitable aggregation of local linear subsystems. One of these approaches is the Takagi–Sugeno modeling. In particular, the subsystems are defined as follows

Plant rule i: IF z_j is F_{ji}, $j = 1, \cdots, p$

$$\text{THEN } \Sigma\text{: } \begin{cases} \dot{x} = A_i x + B_i u \\ y = C_i x, \qquad i = 1, \cdots, r \end{cases}$$

where $z_1, \cdots, z_p$ are measurable premise variables, which may coincide with the state vector or with a partial set of this vector through the output signals y_i. Moreover, F_{1i} are the corresponding fuzzy sets. Usually, these linear subsystems are obtained from some knowledge of the process dynamics or by their linearization about some point of interest.

For a given pair $(x(\cdot), u(\cdot))$, the aggregate fuzzy model is obtained by using a singleton fuzzifier, a product inference and a center of gravity defuzzifier, giving a Continuous Fuzzy Model (*CFM*) described by

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} \mu_i(z) A_i x + \sum_{i=1}^{r} \mu_i(z) B_i u \\ y &= \sum_{i=1}^{r} \mu_i(z) C_i x. \end{aligned} \quad (6)$$

with $z = \begin{pmatrix} z_1 & \cdots & z_p \end{pmatrix}^T$. Here $\mu_i(z)$ is the normalized weight for each rule calculated from the membership functions for z_j in F_{ji}, and such that $\mu_i(z) \geq 0$, and

$$\sum_{i=1}^{r} \mu_i(z) = 1.$$

This modeling procedure allows studying nonlinear systems by introducing tools valid in the linear setting. In particular, several results are known for stabilization. One of these provides sufficient conditions for the asymptotic stability of the equilibrium of the aggregate fuzzy model (Tanaka and Wang [2001]).

Theorem 1. *Let us assume that the pairs A_i, B_i of (11) are stabilizable, $i = 1, \cdots, r$, namely there exist matrices K_i such that $A_i + B_i K_i$ are Hurwitz. Then, the equilibrium of the continuous fuzzy control system (6) is globally asymptotically stable if there exist a common positive definite matrix P such that*

$$P(A_i + B_i K_i) + (A_i + B_i K_i)^T P < 0 \tag{7}$$

for $i = 1, 2, \cdots, r$, and

$$G_{ij}^T P + P G_{ij} < 0 \tag{8}$$

for $i < j$, and $G_{ij} = \dfrac{(A_i + B_i K_j) + (A_j + B_j K_i)}{2}$. ◁

Theorem 1 expresses the fact that if each linear subsystem can be stabilized, and if there exists a matrix P satisfying the matrix Lyapunov equations (7), (8), then the following continuous fuzzy controller

$$u = \left(\sum_{i=1}^{r} \mu_j(x) K_j \right) x \tag{9}$$

stabilizes the fuzzy system (6).

The sampled version of system (5), using zero order holders, is given by

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k &= h(x_k). \end{aligned} \tag{10}$$

Also in the discrete time context one can consider a description of the sampled system (10) by means of an aggregation of linear subsystems which, in the Takagi–Sugeno modelization, are defined as

Plant rule i: IF $z_{j,k}$ is F_{ji}, $j = 1, \cdots, p$

$$\text{THEN } \Sigma\text{:} \begin{cases} x_{k+1} = A_i^d x_k + B_i^d u_k \\ y_k = C_i^d x_k, \qquad i = 1, \cdots, r. \end{cases}$$

With a procedure analogous to that for continuous systems, one can get a Discrete Fuzzy Model (*DFM*), described by

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} \mu_i(z_k) A_i^d x_k + \sum_{i=1}^{r} \mu_i(z_k) B_i^d u_k \\ y_k &= \sum_{i=1}^{r} \mu_i(z_k) C_i^d x_k. \end{aligned} \tag{11}$$

Sufficient conditions for the asymptotic stability of the equilibrium of the *DFM* are given by the following (Tanaka and Wang [2001]).

Theorem 2. *Let us assume that the pairs A_i^d, B_i^d of (11), are stabilizable, $i = 1, \cdots, r$, namely there exist matrices K_i^d such that $A_i^d + B_i^d K_i^d$ are Schur. Then, the equilibrium of the discrete fuzzy control system (11) is globally asymptotically stable if there exist a common positive definite matrix P such that*

$$(A_i^d + B_i^d K_i^d)^T P (A_i^d + B_i^d K_i^d) - P < 0 \tag{12}$$

for $i = 1, 2, \cdots, r$, and

$$(G_{ij}^d)^T P G_{ij}^d - P < 0 \tag{13}$$

for $i < j$ and $G_{ij}^d = \dfrac{(A_i^d + B_i^d K_j^d) + (A_j^d + B_j^d K_i^d)}{2}$. ◁

Hence, if each linear subsystem can be stabilized, and if there exists a matrix P satisfying the matrix Lyapunov equations (12), (13), then the discrete fuzzy controller

$$u_k = \left(\sum_{j=1}^{r} \mu_j(x_k) K_j^d \right) x_k \tag{14}$$

stabilizes the fuzzy system (11). An interesting question arises at this point: if the discrete system (10) is the exact discretization of the corresponding continuous system (5), and the fuzzy system (11) is the exact description of system (5), will the controller (14) stabilize the continuous system (6) as well? This question will be studied in the following section.

4. THE FUZZY DISCRETE STABILIZATION PROBLEM

Let us consider the nonlinear system (5), defined globally in a region D, and let us suppose to determine its exact discretization

$$x_{k+1} = A_d x_k + B_d u_k + f_{2d}(\delta, x_k, u_k). \tag{15}$$

We also assume that it its possible to obtain an exact TS fuzzy model (11) of (15), defined in the same region D. The problem of global discretized stabilization can be formulated as follows.

Global Discretized Stabilization Problem (GDSP). Given a nonlinear system (5), the GDSP consist of finding a discrete system (11) and a piece–wise constant controller (14) such that, for any initial condition $x_0 = x(0) \in D$, the solution of the close–loop system

$$\dot{x} = f(x, \sum_{j=1}^{r} \mu_j(x_k) K_j^d x_k)$$

$$x_{k+1} = \sum_{i=1}^{r} \mu_i(x_k) A_i^d x_k + \sum_{i=1}^{r} \mu_i(x_k) B_i^d \left(\sum_{j=1}^{r} \mu_j(x_k) K_j^d \right) x_k$$

satisfies that

$$\lim_{t\to\infty} x(t) = 0. \qquad \diamond$$

The following result expresses a condition for the existence of a solution to the GDSP.

Theorem 3. *Assume the following assumptions hold*

(H_0) *The nonlinear system (5) is exactly discretizable.*

(H_1) *The function f satisfies the Lipschitz condition*

$$\|f(t, x) - f(t, y)\| \le L\|x - y\|$$

in a region D, and the solution $x(t)$ of the continuous system (5) exists globally on D.

(H_2) *There exist matrices $P > 0$, and K_i^d such that the matrix inequalities*

$$(A_i^d + B_i^d K_i^d)^T P (A_i^d + B_i^d K_i^d) - P < 0 \qquad (16)$$

for $i = 1, 2, \cdots, r$, and

$$\left(G_{ij}^d\right)^T P G_{ij}^d - P < 0 \qquad (17)$$

hold.

Under (H_0), (H_1), (H_2), the controller (14) solves the GDSP. ◁

Proof. From the previous discussion, since by (H_2) the controller (14) stabilizes the system (11), then it stabilizes also the exact discretized system (15). Hence, we have that

$$\lim_{k\to\infty} x(k\delta) = \lim_{k\to\infty} x_k = 0.$$

Now, by assumption (H_1), the solution $x(t)$ of the continuous system (5) can be expressed as

$$x(k\delta + \theta) = F(x_k, \theta)$$

with

$$F(0, \theta) = 0$$

where $t = k\delta + \theta$ and $\theta \in [0, \delta)$ a linear function. In fact, this solution can be written as

$$x(t) = x(t_0) + \int_{t_0}^{t} f(\tau, x(\tau)\, d\tau.$$

Setting $t_0 = k\delta$, $t = k\delta + \theta$, $\theta \in [0, \delta)$, $x_k = x(k\delta)$

$$x(k\delta + \theta) = x_k + \int_{k\delta}^{k\delta+\theta} f(\tau, x(\tau)\, d\tau = F(x_k, \theta)$$

and

$$\begin{aligned}\|x(k\delta + \theta)\| = \|F(x_k, \theta)\| &\le \|x_k\| + \int_{k\delta}^{k\delta+\theta} \|f(\tau, x(\tau)\|\, d\tau \\ &\le \|x_k\|_+ L \int_{k\delta}^{k\delta+\theta} \|x(\tau)\, d\tau \\ &\le (\alpha + L\theta)\|x_k\|_c\end{aligned}$$

where

$$\|x_k\|_c = \max_{\tau\in[k\delta, k\delta+\theta]} \|x(\tau)\|$$

and $\|x_k\| \le \alpha\|x_k\|_c$ for an appropriate constant α. Hence, if x_k goes to zero, then $F(x_k, \theta)$ goes to zero as well. Therefore, one can conclude that

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} x(k\delta + \theta) = 0. \qquad \triangleleft$$

Remark 1. *Note that assumption (H_1) is the extension to the nonlinear setting of the corresponding result for linear systems. In fact, recalling that the solution of a linear system can be written as*

$$x(k\delta + \theta) = e^{A\theta} x_k + \int_{k\delta}^{k\delta+\theta} e^{A(k\delta+\theta-s)} B u(s)\, ds$$

if a piece–wise constant controller $u(t) = K x_k$ is used for $t \in [k\delta, (k + 1)\delta)$, then the solution takes the form

$$\begin{aligned}x(k\delta + \theta) &= \left(e^{A\theta} + \int_{k\delta}^{k\delta+\theta} e^{A(k\delta+\theta-s)}\, ds\, BK \right) x_k \\ &= \left(e^{A\theta} + \int_{0}^{\theta} e^{A\tau}\, d\tau\, BK \right) x_k\end{aligned}$$

and from this it follows that if the linear discrete system is stabilized, then the linear continuous system is stabilized as well. It is worth noting that when taking an approximate solution, this property is no longer guaranteed, as pointed out before. ◁

4.1 The Particular Case of a Class of Feedback Discretizable Nonlinear Systems

In general there is not a special form for nonlinear system to guarantee that there exists an exact discretization. For particular cases, however, it is possible to show that there exists an exact discretization. For example, the class of systems described by

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_2, x_3, \cdots, x_n, u)\\
\dot{x}_2 &= f_2(x_3, x_4, \cdots, x_n, u)\\
&\vdots\\
\dot{x}_{n-1} &= f_{n-1}(x_n, u)\\
\dot{x}_n &= f_n(u)
\end{aligned} \tag{18}
$$

where $f_i(\cdot)$ are polynomials, can be discretized exactly.

More in general, a special class of systems for which exact discretization can be obtained is that of the nonlinear system (Monaco et al. [1996])

$$\dot{x} = f(x) + g(x)u_k, \quad g(x) = \begin{pmatrix} g_1(x) & \cdots & g_m(x) \end{pmatrix}^T \tag{19}$$

where $x \in \mathbb{R}^n$, $f(x)$, $g_1(x), \cdots, g_m(x)$ are real analytic vector fields on $\mathbb{R}^n$, and the control $u_k \in \mathbb{R}^m$ is piece–wise constant over the sampling period δ. The sampled version of (19) is described by (Monaco and Normand-Cyrot [1985], Monaco et al. [1996])

$$x_{k+1} = e^{f(x)+g(x)u_k}(I_d)\Big|_{x_k} = \sum_{k=0}^{\infty} L^k_{f(x)+g(x)u_k}(I_d)\Big|_{x_k}. \tag{20}$$

This expression is the formal exponential function, calculated as an infinite series in x_k. Thus, the problem is to determine the function $F(\delta, x_k, u_k)$: $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, expressing the sampled closed form and sum of the infinite exponential series. If the sum of (20) is finite, i.e. if there exists a $\bar{k}$ such that for any $(x, u) \in \mathbb{R}^n \times \mathbb{R}^m$

$$x_{k+1} = \sum_{k=0}^{\bar{k}} L^k_{f(x)+g(x)u_k}(I_d)\Big|_{x_k} \tag{21}$$

then $F(\delta, x_k, u_k)$ is the finite discretization of the continuous system (19). This finite discretization is not *coordinate free*, but in closed form. These concepts are related to the nilpotency of the Lie algebra associated to the continuous system. In fact, if this algebra is nilpotent and of dimension n at some point, then there exists locally a sampled closed form and, in *suitable coordinates*, the system is finite discretizable. Hence one tries to induce the finite discretizability property by applying first a continuous feedback. For, following Monaco et al. [1996], we will consider the following assumption.

(H_3) There exist a feedback

$$u = \alpha(x) + \beta(x)v$$

and a transformation

$$z = \Phi(x)$$

such that the system (5) is transformed into a nonlinear system

$$
\begin{aligned}
\dot{z} &= \tilde{f}(z, v)\\
y &= h(\Phi^{-1}(z))
\end{aligned} \tag{22}
$$

which is exactly discretizable. ◇

This condition relaxes assumption (H_0) in the sense that it covers a broader set of nonlinear systems. The following corollary can be thus derived.

Corollary 1. *Assume conditions (H_1), (H_2) and (H_3) hold. Then the GDEP is solved.* ◁

In this case, the controller to be implemented is an hybrid controller, given by

$$u(t) = \alpha(x) + \beta(x)\left(\sum_{j=1}^{r} \mu_j(\Phi^{-1}(z_k))\tilde{K}^d_j\right)\Phi^{-1}(z_k)$$

where the membership functions $\mu_j(\Phi^{-1}(z_k)$ and the gains $\tilde{K}^d_j$ are calculated for the transformed system (22).

5. AN ILLUSTRATIVE EXAMPLE

Let us consider the system given by

$$
\begin{aligned}
\dot{x}_1 &= x_2 + x_3^2\\
\dot{x}_2 &= x_3\\
\dot{x} &= u
\end{aligned}
$$

which is in the special form (18). Its discretization is

$$x_{k+1} = \begin{pmatrix} 1 & \delta & \delta x_3 + \delta^2 \frac{(1+2u_k)}{2}\\ 0 & 1 & \delta\\ 0 & 0 & 1 \end{pmatrix} x_k + \begin{pmatrix} \delta^3 \frac{(1+2u_k)}{6}\\ \frac{\delta^2}{2}\\ \delta \end{pmatrix} u_k.$$

To obtain the TS discrete fuzzy model, we rewrite this system as

$$x_{k+1} = \begin{pmatrix} 1 & \delta & z_1\\ 0 & 1 & \delta\\ 0 & 0 & 1 \end{pmatrix} x_k + \begin{pmatrix} z_2\\ \frac{\delta^2}{2}\\ \delta \end{pmatrix} u_k$$

where the minimum and maximum values for z_1 and z_2 are chosen as

$$
\begin{aligned}
z_{1,\min} &= -0.28125, & z_{1,\max} &= 0.34375\\
z_{2,\min} &= -2.6042 \times 10^{-3}, & z_{2,\max} &= 7.8125 \times 10^{-3}
\end{aligned}
$$

and $u \in [-1, 1]$, $x_3 \in [-1, 1]$, $\delta = 0.25$. The membership functions are obtained as in Tanaka and Wang [2001], namely

$$
\begin{aligned}
M_1(z_1) &= \frac{z_1 + 2.21875}{4.5}, & M_2(z_1) &= \frac{-z_1 + 2.21875}{4.5}\\
N_1(z_2) &= \frac{z_2}{0.0052}, & N_1(z_2) &= \frac{-z_2 + .0052}{0.0052}\\
\mu_1(x_k) &= M_1 N_1, & \mu_2(x_k) &= M_2 N_1\\
\mu_1(x_k) &= M_1 N_2, & \mu_1(x_k) &= M_2 N_2.
\end{aligned}
$$

The aggregate model is thus written as

$$x_{k+1} = \sum_{i=1}^{4} \mu_i(x_k) A_i^d x_k + \sum_{i=1}^{4} \mu_i(x_k) B_i^d u_k$$

$$y_k = \sum_{i=1}^{4} \mu_i(x_k) C_k x_k$$

where

$$A_1 = A_3 = \begin{pmatrix} 1 & \delta & 2.28125 \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = A_4 = \begin{pmatrix} 1 & \delta & -2.28125 \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = B_2 = \begin{pmatrix} 0.0052 \\ \frac{\delta^2}{2} \\ \delta \end{pmatrix}$$

$$B_2 = B_4 = \begin{pmatrix} 0 \\ \frac{\delta^2}{2} \\ \delta \end{pmatrix}.$$

The results of the application of the discrete stabilizer (14) are shown in Figures 1 and 2. Figure 1 shows the response of the discretized system with the discrete controller, while Figure 2 shows the response of the continuous time nonlinear system when driven by the digital controller. As it can be observed, the continuous systems is stabilized by the proposed digital controller.

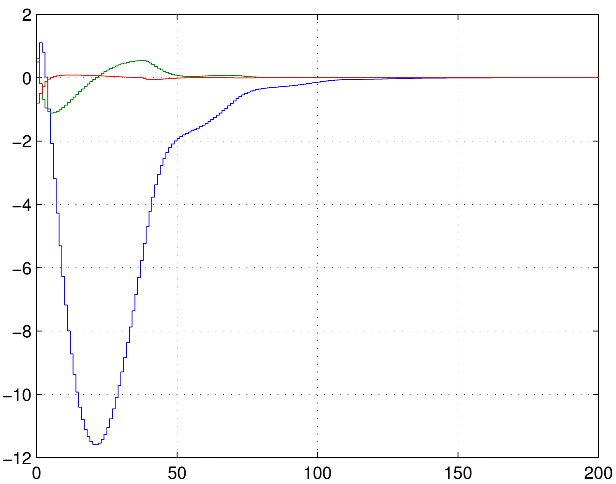

Fig. 1. Response of the discretized system

6. CONCLUSIONS

In this paper a scheme that guarantees the global stabilization of a continuous time nonlinear system by means of a digital controller has been proposed. This scheme is based on the existence of an exact discretization and an exact Discrete Takagi–Sugeno Fuzzy Model for which a discrete stabilizer is calculated. This result can be extended to the class of nonlinear systems exactly discretizable via a continuous feedback. The proposed scheme can be seen as an extension of the well known result for linear time invariant systems, and can be seen as a possible way of stabilizing nonlinear systems by a digital controller calculated on the basis of a discrete TS fuzzy model. An illustrative example suggests the validity of the result.

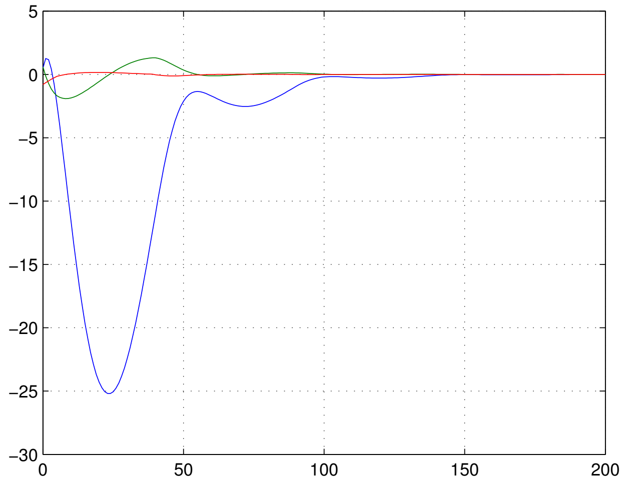

Fig. 2. Response of the continuous nonlinear system

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