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Integral Nested Sliding Mode Control for Robotic Manipulators

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Abstract: An Integral Nested Sliding Mode Control (INSMC) is proposed for n-link robotic manipulators tracking problem by employing Integral Sliding Mode (ISM) and Nested Sliding Mode (NSM) concepts. This controller has the robustness of NSM against matched and no matched perturbations, and the capability of ISM to reduce the sliding functions gains. Application to a two-link planar robot manipulator is presented as a simulation example.

1. INTRODUCTION

The robotic manipulator trajectory control problem has been studied extensively and an important number of methodologies have been used to solve it such as Computed Torque (see Khalil [1995]), Lyapunov Stability (see Canudas [1996]), Passivity (Canudas [1996]), Adaptive Control (see Craig [1988] and Slotine [1988]), Neural Control (see Ozaki [1992]), Fuzzy Control (see Choi [1997]) and Sliding Mode Control (SMC) (see Utkin [1999]). Among this methods SMC is one of the most effective approaches because its robustness to matched perturbations and low computational cost. However conventional SMC is not robust against no-matched perturbations.

In this work we design a controller on the basis of Nested Sliding Mode (NSM) (see Adhami [2005]) in combination with Integral Sliding Mode (ISM) in order to achieve robustness to matched, and no matched perturbations, and ensure output tracking. This Integral Nested Sliding Mode Control (INSMC) can guarantee the robustness of the system throughout the entire response starting from the initial time instance and reduce the sliding functions gains in comparison with NSM.

The rest of the work is organized as follows. The dynamics of an n-link robotic manipulator and its structural properties are formulated in Section 2. In Section 3 an INSMC for robotic manipulators is designed. The simulation results, based in a two-link planar robotic manipulator, are presented in Section 4 to verify the robustness and performance of the proposed control strategy. Finally, some conclusions are given in Section 5.

2. PROBLEM FORMULATION

Consider a non perturbed n-joint robotic manipulator system described by the following model:

$$M(q)\ddot{q} + C(q,q)\dot{q} + g(q) = \tau \qquad (1)$$

where $q(t)$ is an $n\times1$ vector of joint angular positions, τ is the $n\times1$ vector of applied joint torque, $M(q)$ is the $n\times n$ manipulator inertia matrix, $C(q,q)$ is the $n\times1$ vector of centripetal and coriolis torques and $g(q)$ is the $n\times1$ vector of gravitational torques. This model has the following important properties:

1) $M(q)$ is a symmetric positive definite matrix for all $q\in\Re^n$.

2) There exists a unique matrix $C(q,q)$ such that $M(q)-2C(q,\dot{q})$ is skew symmetric.

Defining $y_1 = q$, $y_2 = \dot{q}$ as the state variables and adding perturbations terms due to external disturbances, parameters variation and model uncertainties, we obtain the following state space representation:

$$\begin{aligned}\dot{y}_1 &= y_2 \\ \dot{y}_2 &= f_2(y_1,y_2) + b_2(y_1)u + \lambda(y_1,y_2,t)\end{aligned} \qquad (2)$$

with y_1 the output of the system and $u=\tau$ the vector of the torques applied to the joints of the robot, $f_2(y_1,y_2)$ and $b_2(y_1)$ are the continuous vector functions.

Throughout the development of the controller, we will use the following assumption:

A1) The unmatched $\lambda_u(y_1,t)$ and matched perturbation $\lambda_m(y_1,y_2,t)$ perturbation terms which will be defined later, are bounded by known positive scalar functions:

$$\begin{aligned}\|\lambda_u(y_1,t)\| &< \beta_1(y_1,t)\\ \|\lambda_m(y_1,y_2,t)\| &< \beta_2(y_1,y_2,t)\end{aligned} \tag{3}$$

.

A2) The signum function can be approximated by the sigmoid function as showed by the following limit:

$$\lim_{\varepsilon\to\infty} sigm(\varepsilon S) = sign(S) \tag{4}$$

The following figure shows the approximation for various values of the sigmoid function slope

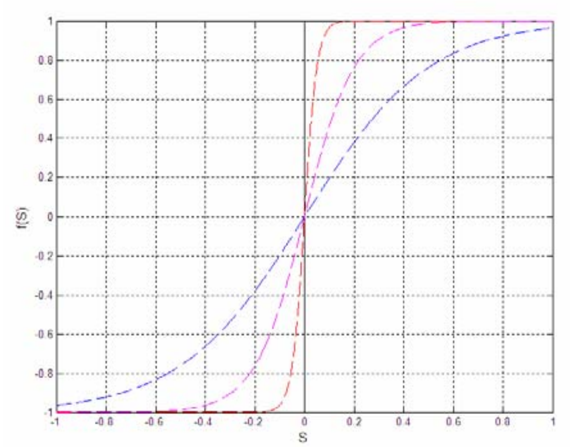


Fig. 1. Sigmoid function for various values of the slope ε.

A3) $rank\left[b_2(y_1)\right] = n$,

where n denotes the degrees of freedom of the manipulator.

Let $y_{1ref}(t)$ presents the desired trajectory of the joint positions vector. The considered problem is to design an Integral Nested Sliding Mode controller that obtains output trajectory tracking in despite of the perturbations of the system.

3. INSMC FOR ROBOTIC MANIPULATORS

Let $y_{1ref}(t)$ be a twice differentiable function, but with unknown derivatives, then define the output tracking error as $e_1 = y_1 - y_{1ref}(t)$ and its derivative as

$$\dot{e}_1 = y_2 + \lambda_u(y_1,t) \tag{5}$$

where $\lambda_u(y_1,t)$ is the unmatched term defined by the following equality

$$\lambda_u(y_1,t) = -\dot{y}_{1ref}\,. \tag{6}$$

Then define the pseudo-sliding function s_1 for the first block (5) as

$$s_1 = e_1 + z_1\,,\; z_1(0) = -e_1(0) \tag{7}$$

where z_1 is the integral variable that will be defined later. The dynamics of s_1 can be obtained of the form

$$\dot{s}_1 = y_2 + \dot{z}_1 + \lambda_u(y_1,t) \tag{8}$$

Considering y_2 as virtual control in (5), we propose

$$y_{2ref} = y_{2,0ref} + y_{2,1ref} \tag{9}$$

where $y_{2,0ref}$ is the nominal part of the control and $y_{2,1ref}$ is the control which will be designed to reject the perturbation in (5) (see Utkin [1999]). To obtain y_{2ref} and replace it in (6) we must define the sliding function for the second block as

$$s_2 = e_2 + z_2, \quad e_2 = y_2 - y_{2ref}\,,\; z_2(0) = -e_2(0) \tag{10}$$

with z_2 the integral variable. From the equation (10) we obtain

$$y_2 = s_2 + y_{2ref} - z_2 \tag{11}$$

Then using (9) and (11) the first transformed block (8) becomes as

$$\dot{s}_1 = s_2 - z_2 + y_{2,0ref} + y_{2,1ref} + \dot{z}_1 + \lambda_u \tag{12}$$

Now $\dot{z}_1$ of the form

$$\dot{z}_1 = -(s_2 - z_2 + y_{2,0ref}) \tag{13}$$

with the initial condition $z_1(0) = -e_1(0)$, and defining $y_{2,0ref}$ as follows

$$y_{20ref} = -c_1 e_1 \tag{14}$$

where $c_1 > 0$, the dynamics for z_1 and s_1 are represented as

$$\begin{aligned}\dot{z}_1 &= -e_2 - y_{2,0ref}\\ \dot{s}_1 &= y_{2,1ref} + \lambda_u.\end{aligned} \tag{15}$$

Then the second part $y_{2,1ref}$ $y_{2,1ref}$ of (9) is selected of the form

$$y_{2,1ref} = -k_1 sigm(\varepsilon_1 s_1) \tag{16}$$

where $k_1 > 0$. Proceeding with the second block, its dynamics can be obtained by differentiating (10) along the trajectories of the system (2):

$$\dot{s}_2 = f_2(y_1, y_2) + b_2(y_1)u - \dot{y}_{2ref} + \dot{z}_2 + \lambda_m \quad (17)$$

where $\dot{y}_{2ref}$ is defined as

$$\dot{y}_{2ref} = -k_1\varepsilon_1 P y_{2,1ref} - c_1 y_2$$
$$P = \begin{pmatrix} 1-\tanh^2(\varepsilon_1 s_1(1)) & 0 \\ 0 & 1-\tanh^2(\varepsilon_1 s_1(2)) \end{pmatrix} \quad (18)$$

and the matched perturbation term λ_m is given by

$$\lambda_m = -\left(c_1 + k_1\varepsilon P\right)\lambda_u + \lambda(y_1, y_2, t) \quad (19)$$

Designing $u = u_0 + u_1$ we obtain

$$\dot{s}_2 = f_2(y_1, y_2) + b_2(y_1)u_0 + b_2(y_1)u_1 - \dot{y}_{2ref} + \dot{z}_2 + \lambda_m \quad (20)$$

we choose $\dot{z}_2$ as follows

$$\dot{z}_2 = -f_2(y_1, y_2) - b_2(y_1)u_0 + \dot{y}_{2ref} \quad (21)$$

with

$$z_2(0) = -e_2(0) \quad (22)$$

to ensure sliding mode occurrence from initial instance: Then choosing

$$u_0 = b_2(y_1)^{-1}(-f_2(y_1, y_2) + \dot{y}_{2ref} - c_2 e_2) \quad (23)$$

where $c_2 > 0$, and using (21) and (23) the equation (20) is reduced to

$$\dot{s}_2 = b_2(y_1)u_1 + \lambda_m. \quad (24)$$

To induce sliding mode in (24) we choose

$$u_1 = -k_2 b_2(y_1)^{-1} sign(s_2) \quad (25)$$

with $k_2 > 0$. Using (15), (16), (24 and (25), the dynamics of the variables s_1 and s_2 are derived as follows

$$\begin{aligned} \dot{s}_1 &= -k_1 sigm(\varepsilon_1 s_1) + \lambda_u \\ \dot{s}_2 &= -k_2 sign(s_2) + \lambda_m \end{aligned} \quad (26)$$

while the tracking errors e_1 and e_2 dynamics are obtained from (5), (9)-(11), (14) and (10), (21)-(24), respectively, of the form λ_m

$$\begin{aligned} \dot{e}_1 &= -c_1 e_1 + e_2 + y_{2,1ref} + \lambda_u \\ \dot{e}_2 &= -c_2 e_2 + b_2(y_1)u_1 + \lambda_m. \end{aligned} \quad (27)$$

Establishing the following set of conditions:

$$k_1 > \frac{\beta_1}{1-\delta},\ 1 > \delta > 0,\ \ k_2 > \beta_2 \quad (28)$$

$$\left\|\dot{\lambda}_u\right\| \le \delta_u \left\|\dot{s}_1\right\|,\ \ \delta_u > 0, \quad (29)$$

we can enunciate a theorem as follows

Theorem 1. *If the assumptions A1), A2) and A3) hold, the conditions (28) are satisfied and the control law*

$$u = b_2(y_1)^{-1}(-f_2(y_1, y_2) + \dot{y}_{2ref} - c_2 e_2) - k_2 b_2(y_1)^{-1} sign(s_2)$$

is constructed; then a solution of the error dynamics (27) is asymptotically stable.

The proof of the Theorem 1 is described in **Appendix A**.

Therefore the control objectives are fulfilled, and the desired performance of the robotic manipulator is obtained.

3. SIMULATIONS

As a testing illustration of the designed algorithm, it will be applied to a two-link planar robot manipulator with perturbations due to external disturbances, model uncertainties, parameters variation and the load that the robot manipulate. To completely define the state-space representation in (2), we will define the following terms as

$$\begin{aligned} f_2(y_1, y_2) &= -M\left(y_1\right)^{-1} N\left(y_1, y_2\right) \\ b_2(y_1) &= M\left(y_1\right)^{-1} \end{aligned},$$

$$M\left(y_1\right) = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},$$

$$N\left(y_1, y_2\right) = \begin{pmatrix} -L_1 L_2 M_2\left(2y_2(1)y_2(2) - y_2(2)^2\right) \\ L_1 L_2 M_2 y_2(1)^2 \sin\left(y_1(2)\right) \end{pmatrix},$$

$$\begin{aligned} m_{22} &= L_2^2 M_2, \\ m_{12} &= m_{21} = m_{22} + L_1 L_2 M_2 \cos\left(y_1(2)\right), \\ m_{11} &= L_1^2\left(M_1 + M_2\right) + 2m_{12} - m_{22}, \end{aligned}$$

where L_1, L_2, M_1 and M_2 are the lengths and masses of the the first and second links, respectively. The values of these manipulator parameters used in the simulations were

$$M_1 = 10\ kg,\ \ M_2 = 1\ kg,\ \ L_1 = 1\ m,\ \ L_2 = 1\ m;$$

To fulfil all the design conditions, the control parameters were adjusted to

$$K_1 = 7,\ \ K_2 = 35,\ \ c_1 = 4,\ \ c_2 = 4,\ \ \varepsilon = 20.$$

The perturbations terms used in simulation are

$$\lambda_u = \begin{bmatrix} 4 \\ 2\sin(.5t) \end{bmatrix} \quad \lambda_m = \begin{bmatrix} 5\cos(2t) \\ 5+3\sin(t) \end{bmatrix},$$

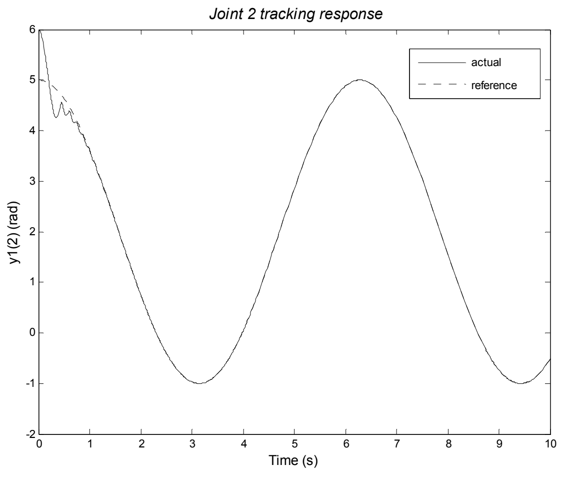
and the references for the angular joint positions are

$$y_{1ref} = \begin{bmatrix} 2\sin(2t) \\ 2+3\cos(t) \end{bmatrix}.$$

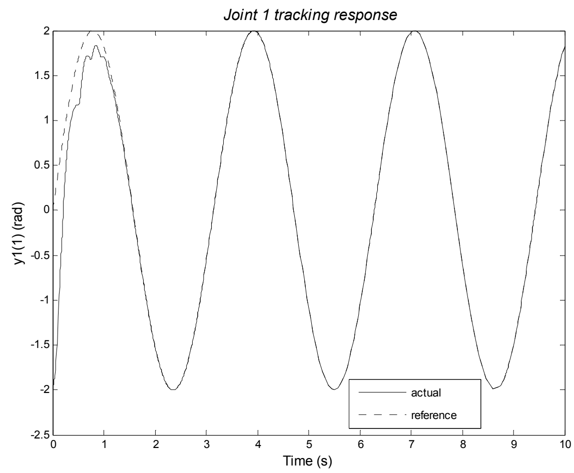
Figure 2 shows the results for the tracking responses, in Figure 2a and 2b are shown the responses for joint 1 and joint 2 respectively. It can be appreciated that the performance of the control defined in (29) is satisfactory, since the objectives are accomplished, rejecting the external disturbances, model uncertainties and parameters variation as well. The tracking errors converge to a neighbourhood of zero, it can be observed in Figure 3. This also can be observed in Figure 4 where the phase portrait of the tracking errors is shown. In Figure 5 the input controls for joint 1 and joint 2, 5a and 5b respectively, can be observed.

4. CONCLUSIONS

An INSMC is designed for rigid robotic manipulators by the combination of Nested and Integral SMC concepts. The proposed algorithm is robust against matched and no matched perturbations due to external disturbances, model uncertainties and parameters variations. The INSMC demonstrates a satisfactory performance in output tracking problem of robotic manipulators, moreover it obtains a reduced steady tracking error in comparison with standard SMC.



(a)



(b)
Fig. 2. Tracking responses

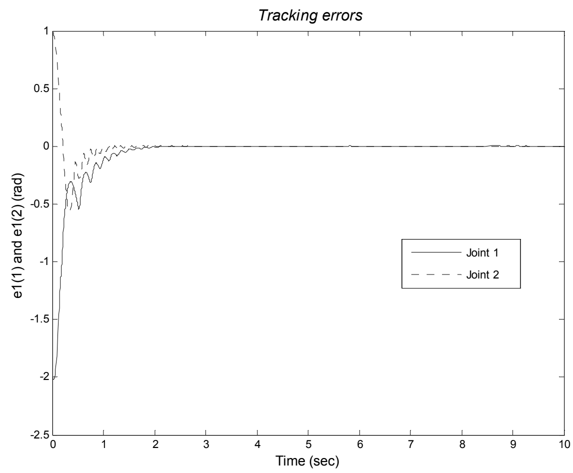


Fig. 3. Tracking errors.

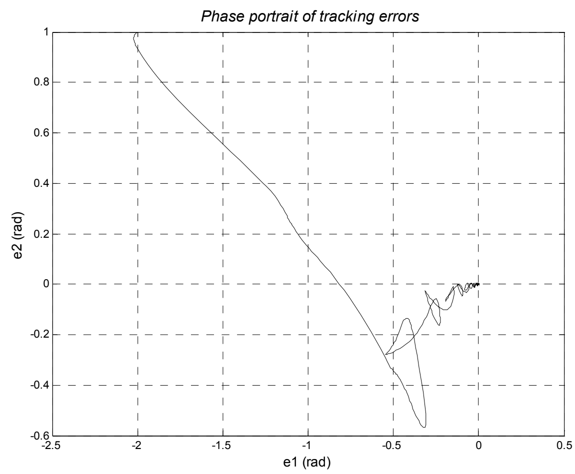
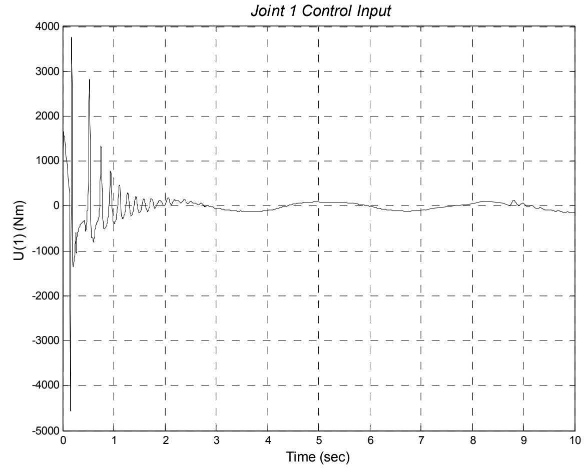
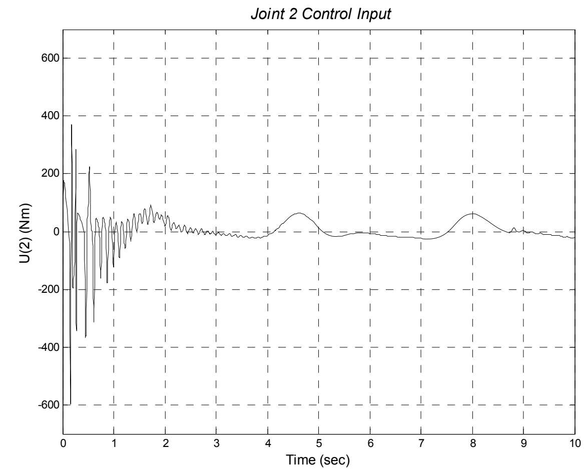


Fig. 4. Phase portrait of the tracking errors.



a)



b)

Fig. 5 Input controls for: a) joint 1, b) joint 2.

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Appendix A. PROOF OF THEROEM 1

In order to prove Theorem 1, it must be demonstrated first stability of the system (26). We choose the following Lyapunov function:

$$V_2 = \tfrac{1}{2} s_2{}^T s_2 \qquad \text{(a1)}$$

Differentiating (A1) along the trajectories of the system (26) and using (3) yields

$$\begin{aligned}\dot{V}_2 &= s_2{}^T \left(-k_2 sign(s_2) + \lambda_m \right) \\ &\le \|s_2\| \left(-k_2 + \beta_2(y_1, y_2, t) \right)\end{aligned}.$$

Under the conditions (28) we obtain $\dot{V}_2 < 0$, therefore, due to the condition (10) sliding mode occurs on $s_2 = 0$ from initial time. If we do not know the initial condition we assure at least finite time convergence of s_2 to zero.

Proceeding in similar way for the first block, we define

$$V_1 = \tfrac{1}{2} s_1{}^T s_1 ,$$

Thus

$$\dot{V}_1 = s_1{}^T \left(-k_1 sigm(\varepsilon_1 s_1) + \lambda_u \right). \qquad \text{(a2)}$$

Establishing the following equality

$$sigm(\varepsilon_1 s_1) = sign\left(s_1\right) - \Delta\left(\varepsilon_1, s_1\right), \qquad \text{(a3)}$$

where $\Delta\left(\varepsilon_1, s_1\right)$ is the difference between the signum and sigmoid functions, then using (a3) the derivative (a2) becomes as

$$\dot{V}_1 < \|s_1\| \left(-k_1 \left(1 - \left\|\Delta\left(\varepsilon_1, s_1\right)\right\| \right) + \|\lambda_u\| \right).$$

It is evidently that $\Delta\left(\varepsilon_1, s_1\right)$ is bounded that is for a given ε_1, there is, a positive constant $1 > \delta > 0$ such that

$$\left\|\Delta\left(\varepsilon_1, s_1\right)\right\| = \delta$$

Therefore under the condition (28) we have $\dot{V}_1 < 0$, and hence s_1 converges to a region $\|s_1\| \le \Omega$ given by

$$\Omega = \frac{\ln\left(\frac{2-\delta}{\delta}\right)}{2\varepsilon_1}.$$

Now, defining $\varphi = \dot{s}_1$ and $V_\varphi = \frac{1}{2}\varphi^T\varphi$ and then using (15) and (18) the straightforward calculations gives

$$\begin{aligned}\dot{V}_\varphi &= \varphi^T\dot{\varphi}\\ &= \varphi^T\left(-k_1\varepsilon P\varphi + \dot{\lambda}_u\right)\end{aligned}$$

Under the condition (29) or

$$\|\dot{\lambda}_u\| \le d_u \|\varphi\|$$

$\varphi(t)$ converges asymptotically to zero. Therefore, from (15) and (24) we obtain

$$\begin{aligned}y_{2,1ref} &= -\lambda_u\\ b(y_1)u_1 &= -\lambda_m\end{aligned}$$

and hence the system (27) can be reformulated as

$$\begin{aligned}\dot{e}_1 &= -c_1e_1 + e_2\\ \dot{e}_2 &= -c_2e_2\end{aligned} \qquad \text{(a2)}$$

If $c_1 > 0$ and $c_2 > 0$ then a solution of (a2) asymptotically tends to zero provided then

$$\lim_{t\to\infty} e_1(t) = 0 .$$

and Theorem 1 is proved.