Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Dual EKF State and Parameter Estimation in Multi-Class First-Order Traffic Flow Models ⋆

J.W.C. Van Lint, MSc, PhD *
Professor Serge P. Hoogendoorn, MSc, PhD **
A. Hegyi, MSc, PhD ***

* *Department of Transport & Planning, Faculty of Civil Engineering and Geosciences, Delft University of Technology (e-mail: j.w.c.vanlint@tudelft.nl).*
** *idem (e-mail: s.p.hoogendoorn@tudelft.nl).*
*** *idem (e-mail: a.hegyi@tudelft.nl).*

Abstract: In this paper we investigate a real-time traffic surveillance system based on a multi class first-order traffic flow model called FASTLANE. We demonstrate a dual extended Kalman Filtering approach in which the model state and parameters can be estimated simultaneously from real-time data. The innovation is that although FASTLANE maintains the dynamics of multiple vehicular classes (e.g. trucks, buses, cars), only the total mixed-class density is corrected by the filter, which is 'translated' into multi-class state corrections by means of state-dependent person car equivalents and class flow shares. Results on real data from a densely used freeway show that the DEKF procedure is able to reproduce accurate speeds and flows and physically plausible parameters.

1. INTRODUCTION

Modeling and predicting traffic conditions (speeds, travel times, flows, etc) in traffic networks is one of the main research areas in current traffic and transport science. Traffic prediction is also a crucial component in many real world applications within traffic and transport practice, such as travel information services, traffic management & decision support systems, freight and fleet management systems, etc. On a network scale, the most appropriate tools for predicting traffic conditions (and possibly optimizing traffic measures) are traffic simulation models (TSMs), which range from low-detail macroscopic (analytical) models to high detail microscopic simulation models. A good example of a real-time network surveillance and decision support system is the Renaissance system (Wang and Papageorgiou [2005]), which is centered on a so-called second order macroscopic analytical TSM.

For such a real-time traffic surveillance tool one requires - besides a network TSM - a data assimilation technique which connects the inputs (demand, turn fractions), state variables (densities, flows, speeds) and/or parameters used (e.g. capacities and critical densities) in the TSM to real time data from for example induction loops (Antoniou et al. [2005], Wang and Papageorgiou [2005], Wang et al. [2006]).

The differences between the most commonly used data assimilation techniques such as the EKF, UKF and PF (particle filter) lie in their assumptions. For example, EKF approaches assume that a TSM can be linearized around the prevailing state (e.g. cell densities), and that all model and measurement errors can be approximated by Gaussian white noise processes. These are clearly strong assumptions, which in many real traffic situations may not hold. UKF and (U)PF approaches relax these assumptions in various degrees, but are computationally much more demanding than the EKF, since they typically require Monte Carlo sampling procedures (or variations on these) to derive a reasonable approximation of the posterior (model and measurement) probability densities. As a result, the gain in estimation accuracy might pose constraints on the real-time applicability of such a TSM-UKF or PF traffic surveillance system. Munoz et al. [2006] provide other arguments which support the use of the EKF in real time traffic modeling over its more elaborate variants. In (Munoz et al. [2006]), it is shown that (local) linearization does not necessarily lead to an unrealistic (first order) traffic flow model. These authors describe and calibrate a linear version of the LWR model (Lighthill and Whitham [1955], Richards [1956]) and conclude that this linearized version in fact reproduces approximately the same (basic) phenomena as the original model.

In this paper we investigate a real-time traffic surveillance system, which uses a so-called *dual* extended Kalman filter (DEKF, see e.g. Haykin [2001]) in conjunction with a first-order multi-class traffic flow model FASTLANE. In a DEKF the network state (in terms of densities) is estimated, and subsequently some (or all) of the model parameters based on this corrected network state. This paper is organized as follows. The next section briefly introduces a new multi-class first order traffic flow model FASTLANE, to which this DEKF approach is applied. The section thereafter

⋆ This research is supported by the Dutch Technology Foundation STW, applied science division of NWO and the Technology Program of the Ministry of Economic Affairs under grant DCB 7814.

overviews the DEKF method, while in the remainder some results on real data are provided. The paper closes with a critical discussion, conclusions and recommendations for further research.

2. MULTI CLASS TRAFFIC FLOW MODELING: FASTLANE

In this paper we use an analytical macroscopic network traffic flow model FASTLANE (First-order fAST muLti-class mAcroscopic traffic flow model for simulation of NEtwork-wide traffic conditions - van Lint et al. [2007], van Lint et al. [2008]), which specifically addresses the heterogeneity of network traffic. Heterogeneity here depicts differences drive behavior and dynamics in (classes) of vehicles, e.g. person cars and trucks. The FASTLANE model differs from earlier multi-class models (see e.g. Logghe [2003] in that it calculates traffic dynamics (of the total heterogeneous traffic flow) in terms of *dynamic* person car equivalents (η's). These pce values are hence state-specific, that is, they are a function of the prevailing class-specific speeds.

2.1 Link dynamics

In this paper a traffic network is described by a directed graph $G = (N, A)$ of nodes $n \in N$ and links $a \in A$. Similar to earlier LWR based multi class traffic flow models (see e.g. Logghe [2003] for an overview), the core of the FASTLANE traffic flow model is the class-specific conservation of vehicle equation 1. In the ensuing, the subscript u will denote the user-class (e.g. person-cars, trucks). For each class u we have (for each link a) [1]:

$$\frac{\partial k_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0 \tag{1}$$

In (1), $k_u = k_u(t, x)$ denotes the class-specific density and $q_u = q_u(t, x) = k_u(t, x) \cdot v_u(t, x)$ denotes the class-specific flow at time instant t and location x. The boundary conditions (inflow at the entry of the link and the outflow at the link exit) are determined respectively by the upstream and the downstream links, expect for the case where a is an origin or a destination link. The class-specific speed is defined by $v_u(t, x) = V_u(K(t, x)), K(t, x) = \kappa(k_1(t, x), ..., k_U(t, x))$ where V_u denotes the class-specific *equilibrium* speed as a function of the *effective* density K. In turn, $K(t, x)$ is described by a function of the class-specific densities

$$K(t, x) = \sum_{u=1}^{U} \eta_u(t, x) k_u(t, x) \tag{2}$$

in which the person-car equivalents η_u, are state specific and specified by:

$$\eta_u(t, x) = \frac{s_u + T_u V_u(t, x)}{s_{u_0} + T_{u_0} V_{u_0}(t, x)} \tag{3}$$

In (3), s_u denotes the class-specific gross stopping distance (i.e. vehicle length + distance to predecessor at zero speed), and T_u denotes the class-specific minimum headway. The subscript u_0 depicts the reference class (person cars). The equilibrium speed V_u is chosen such that for effective densities K larger than some critical density K_c, the speeds of all classes u are equal to the critical speed v_c. In free-flow conditions, different classes move with different average speeds. More specifically, we have:

$$V_u(K) = \begin{cases} v_u^0 - K \dfrac{(v_u^0 - v_c)}{K_c} & , K < K_c \\ \dfrac{v_c K_c}{K} \left(1 - \dfrac{K - K_c}{K_{jam} - K_c}\right) & , K \geq K_c \end{cases} \tag{4}$$

in which the parameter K_{jam} denotes the effective jam density. Clearly, since

$$Capacity[pce/h] = max(q) = v_c K_c$$

the vehicle composition (e.g. percentage of trucks) and the prevailing speeds (governing the pce values) determine the capacity *in veh/h* of a link.

2.2 Numerical solution and node dynamics

To solve the FASTLANE equations a new numerical solution approach was developed based on the generalization of the well-known Godunov scheme Lebacque [1996]. Each link a is divided into a number of cells i (usually of 100 - 500 m). For each cell, the scheme determines a class-specific demand and a mixed-class supply. The mixed-class supply of downstream cell $i+1$ is then distributed over the classes according to the so-called shares in the demands of the upstream cell i.

$$q_u^{i \to i+1} = \frac{1}{\eta_u^i} \min\left(D_u^i, \lambda_u^i S^{i+1}\right) \tag{5}$$

In the network description, three node types are considered: link-to-link nodes, merges and diverges (Fig.1a, b and c). Below, a discussion of the traffic dynamics across these nodes will be given.

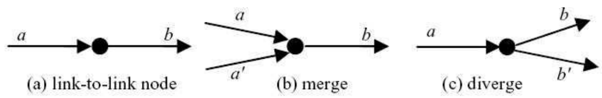

Fig. 1. Normal nodes, bifurcations and diverges

A link-to-link node (Fig.1a) depicts a simple interface between two network links a and b, describing a discontinuity, such as a lane drop, or a change in speed limits. To determine the resulting traffic dynamics, the class-specific demands D_u^a of an incoming link a are compared to the mixed-class supply S^b of the outgoing link b. Put simply, traffic demand is transferred from link a to link b proportional to the traffic composition on link a, and constraint by the maximum possible (total) flow which can enter in link b. This is modeled by means of class-specific shares (proportions) λ_u, which are calculated as follows

$$\lambda_u(t, x) = \frac{\eta_u(t, x) \cdot q_u(t, x)}{\sum_u \eta_u(t, x) \cdot q_u(t, x)} \tag{6}$$

At the origins, these shares are equal to the traffic composition (e.g. truck percentage) set by the user. As a result, we get the following expression for the class-specific flow between link a and b:

$$q_u^{a \to b} = \frac{1}{\eta_u^a} \min\left(D_u^a, \lambda_u^a S^b\right) \tag{7}$$

[1] We omit the link index a to keep notation simple

At bifurcation nodes (Fig.1b) with two incoming links a and a' and one outgoing link b, we need to determine how the supply (the amount of traffic able to enter outgoing link b) is distributed not only across the classes u through shares λ_u^a, but also across the incoming links a and a' through proportions κ_u^a. In the current version of the model, we make the (rough) assumption that the available supply is distributed according to the effective capacity of the incoming links, that is

$$q_u^{a'\to b} = \frac{1}{\eta_u^{a'}} \min\left(D_u^{a'}, \kappa^{a'} \lambda_u^{a'} S^b\right) \tag{8}$$

with

$$\kappa^{a'} = \frac{C^{a'}}{C^a + C^{a'}}, \kappa^a = 1 - \kappa^{a'}$$

Furthermore, if the demand of one of the incoming links is less than the assigned supply, the remaining supply will be assigned to the other link. Finally, the traffic dynamics at a diverge or bifurcation node (Fig.1c) are described by the turn fractions γ_n^b, which depict the distribution of the total flow over the outgoing links. As a result, for merges we have

$$q_u^{a\to b} = \frac{1}{\eta_u^a} \min\left(\gamma_n^b D_u^a, \lambda_u^a S^b\right) \tag{9}$$

3. A DUAL EXTENDED KALMAN FILTER FOR STATE AND PARAMETER ESTIMATION

3.1 General structure of the Dual EKF

Since Fastlane - like most analytical traffic flow models - can be cast into a discrete state-space form, it can be connected to real-time data with an (D)EKF in a straightforward manner.

Consider the following discretized non-linear state space system (note that from here on t depicts discrete time instants)

$$\mathbf{x}_t = \mathbf{f}\left(\mathbf{x}_{t-1}, \mathbf{w}_{t-1}, \mathbf{u}_{t-1}\right) + \mathbf{r}_{x,t-1} \tag{10}$$

$$\mathbf{d}_t = \mathbf{h}\left(\mathbf{x}_t, \mathbf{w}_t\right) + \mathbf{r}_{d,t} \tag{11}$$

in which (10) depicts the process equation which describes the dynamics of state $\mathbf{x}_{t+1}$ as a function of $\mathbf{x}_t$ and external disturbances $\mathbf{u}_t$ plus a zero mean Gaussian error term $\mathbf{r}_t$. The function f contains (possibly time-varying parameters) $\mathbf{w}_t$. Equation (11) depicts the observation equation h which relates the system state to (observable) outputs $\mathbf{d}_t$. Details on the EKF algorithm can be found in many textbooks (e.g. Simon [2006]), here we briefly highlight the main issues.

3.2 Step 1: state estimation

In the *prediction step* a prior state estimate $\mathbf{x}_t^-$ is calculated with (10). A prior estimate for the state error covariance $\mathbf{P}$ is calculated with

$$\mathbf{P}_t^- = \mathbf{F}\mathbf{P}_{t-1}\mathbf{F}^T + \mathbf{Q}_t \tag{12}$$

in which $\mathbf{F} = \frac{df}{d\mathbf{x}}|_{\mathbf{x}=\mathbf{x}_t^-}$ depicts the derivative of the process equation (10) to the state.

In the *correction step* a posterior estimate of $\mathbf{x}$ is calculated through

$$\hat{\mathbf{x}}_t = \mathbf{x}_t^- + \mathbf{G}_t\left(\mathbf{d}_t - \mathbf{h}\left(\mathbf{x}_t^-\right)\right) \tag{13}$$

$$\mathbf{P}_t = \left(\mathbf{I} - \mathbf{G}_t\right)\mathbf{P}_t^- \tag{14}$$

in which $\mathbf{G}$ depicts the so-called Kalman Gain matrix calculated by

$$\mathbf{G}_t = \mathbf{P}_t^-\mathbf{H}^T\left(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T - \mathbf{R}\right)^{-1} \tag{15}$$

where $\mathbf{H} = \frac{dh}{d\mathbf{x}}|_{\mathbf{x}=\mathbf{x}_t^-}$ depicts the derivative of the observation equation (11) to the state

3.3 Step 2: parameter estimation

Let us assume the parameters $\mathbf{w}_t$ in equations (10) and (11) are also time-varying, according to a known Gaussian noise process, in the simplest case a random walk

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \mathbf{r}_{w,t} \tag{16}$$

Equations (16) and (11) again constitute a discrete state-space system which can be solved in the same manner as above.

There are roughly two approaches to combine online state and parameter estimation. In the first approach (*joint* estimation) the two systems are solved in *parallel*. The state vector in that case is augmented as in $[\mathbf{x}_t^T, \mathbf{w}_t^T]^T$. The second approach updates the state and parameter vectors *sequentially*. This sequential procedure is referred to as *dual* estimation (Haykin [2001]). Fig.2 schematically illustrates this procedure.

The main benefit of the dual EKF (DEKF) procedure is that the parameters are now estimated on the basis of a corrected (filtered) state estimate, which as argued in (Haykin [2001]) leads to smoother and more stable parameter estimates. Implementation-wise there are also benefits. Separating state and parameter estimation allows the modeler to use coarser time and space discretisation for the parameters. This makes sense physically, since these parameters reflect average drive behavior which fluctuates on a much coarser (space-time) scale than the state variables (densities). This is also reflected by the fact that the process noise for the parameters (16) is typically an order of magnitude smaller than that for the state (10). On the downside, the DEKF procedure ignores the (possibly strong) cross-correlation between state-variables and parameters, which *are* captured in the joint estimation procedure.

Implementation of the DEKF procedure is straightforward. In the parameter correction step two modifications are required. First, equation (11) is replaced by

$$\mathbf{d}_t = \mathbf{h}\left(\hat{\mathbf{x}}_t, \mathbf{w}_t\right) + \mathbf{r}_{d,t} \tag{17}$$

Secondly, the matrices $\mathbf{F}$ and $\mathbf{H}$ in equations (12) and (15) now depict the derivatives with respect to the parameters, that is, $\frac{df}{d\mathbf{w}}$ and $\frac{dh}{d\mathbf{w}}$ respectively.

3.4 Application of the DEKF to Fastlane

Recall from (1) that Fastlane maintains user class specific densities and flows $k_{u,t}$ and $q_{u,t}$ (t now depicts discrete

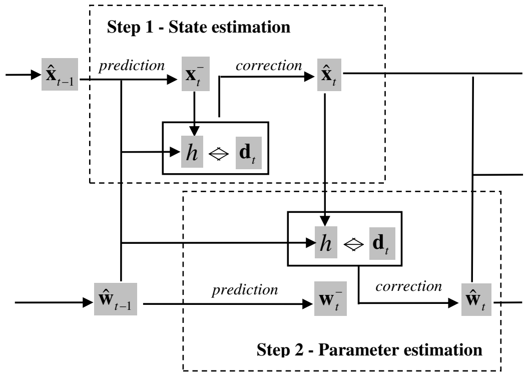

Fig. 2. Schematic representation of dual EKF procedure

time). We define the effective density K_t^i (2) as the system state on each cell i and $[v_c^{A'}, K_c^{A'}, K_{jam}^{A'}]$ as the parameters to be estimated for each set of links $A' \subset A$. These subsets A' are chosen on the basis of common characteristics (e.g. number of lanes, slope, etc). Assuming a traffic network is discretized into N cells i, and M subsets A' this implies a $N \times 1$ state vector and a $3M \times 1$ parameter vector.

The process equation in the state estimation step of the DEKF-FASTLANE procedure involves applying the class-specific conservation laws for each cell i, which in discretized form read

$$f : k_{u,t+1}^i = k_{u,t}^i + \frac{\Delta t}{\Delta x^i}\left(q_{u,t}^{i-1\to i} - q_{u,t}^{i\to i+1}\right) \qquad (18)$$

In (18) the fluxes $q_{u,t}^{i-1\to i}$ and $q_{u,t}^{i\to i+1}$ are calculated by (7)-(9). The effective density K_{t+1} and class-specific speeds and flows are then recomputed with (2)-(4). Calculating the error covariance $\mathbf{P}_t^-$ requires the calculation of matrix $\mathbf{F}$. Due to (2)-(4) this would involve applying the chain rule

$$\frac{df}{dK} = \frac{df}{dk_u}\frac{dk_u}{d\eta_u}\frac{d\eta_u}{dV_u}\frac{dV_u}{dK}$$

which is a $[N \times NN_u] \cdot [NN_u \times NN_u] \cdot [NN_u \times NN_u] \cdot [NN_u \times N]$ matrix operation (N_u denotes the number of user classes). A cheap and reasonable approximation is to assume that on those cells where observations are available the multi-class dynamics can be described in terms of a single (average) class, so that

$$\frac{df}{dK} \approx \frac{df}{dk}\frac{dk}{dK} = C\frac{df}{dk}$$

in which $C < 1$ is a constant equal to $1/\bar{\eta}_t$ where

$$\bar{\eta}_t^i = \frac{\sum_u k_{u,t}^i \eta_{u,t}^i}{\sum_u \eta_{u,t}^i}$$

denotes the average pce value η of each vehicle in the flow. After each update, the average corrected mixed-class state is distributed over the class-specific states by using the prior pce values and class flow shares. This approximation is particularly usefull in cases where only mixed-class average speeds and (total) aggregated flows are measured. This is for example the case in the Dutch dual loop motorway monitoring system. The mixed-class average speed and total flow are related to the class-specific speeds and flows as follows

$$v_t^i = \frac{\sum_u k_{u,t}^i v_{u,t}^i}{\sum_u k_{u,t}^i}, \; q_t^i = \sum_u q_{u,t}^i \qquad (19)$$

On the basis of these quantities the observation function h for speeds (the speed density relation of equation (4)) now becomes a single (average) class function which reads

$$V(K) = \begin{cases} v^0 - K\dfrac{(v^0 - v_c)}{K_c} & , K < K_c \\ \dfrac{v_c K_c}{K}\left(1 - \dfrac{K - K_c}{K_{jam} - K_c}\right) & , K \geq K_c \end{cases} \qquad (20)$$

with an average weighted free-speed

$$v^0 = \frac{\sum_u k_{u,t}^i v_u^0}{\sum_u k_{u,t}^i} \qquad (21)$$

The observation equation for flows follows directly from (20) and reads

$$q_t = k_t V(K_t)$$

4. EXPERIMENTAL SETUP

4.1 Data and network

We will test the DEKF-FASTLANE model on a 3.7 km 3-lane stretch of the southbound A13 freeway between Delft-Zuid and Rotterdam Airport in the Netherlands. This freeway stretch is equipped with dual carriage way inductive loops which measure total flows and average speeds in one minute intervals. For this example a typical congested afternoon peak was selected (March 12, 2006) Fig.3 schematically outlines this freeway stretch and depicts the detector locations with small white numbered blocks. In the next section the DEKF-FASTLANE estimator is ran with data from detectors 2, 5 and 6. Loop detector 1 is used to derive the upstream traffic demand. In this example we have roughly estimated the demand at onramp Rotterdam Airport at on average 5% of the main carriage way flow, whereas the outflow to Rotterdam Airport is also estimated at on average 5% of the main carriage way flow.

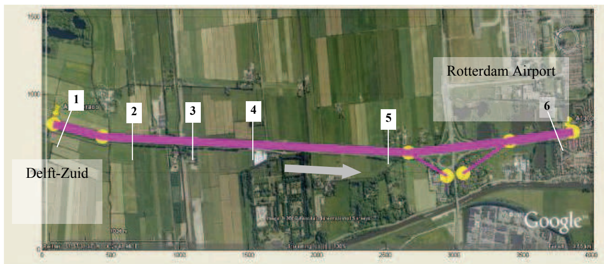

Fig. 3. Freeway stretch coded in FASTLANE: A13 from Delft-Zuid to Rotterdam Airport. The white blocks indicate the locations of dual loop detectors.

To asses the quality of the state estimate, the results of the DEKF-FASTLANE estimator at the locations of detectors

3 and 4 are compared to the actual detector data on the basis of the root mean of squared error (RMSE)

$$RMSE = \sqrt{\frac{1}{N}\sum_{N}(d_i - y_i)^2}$$

where d_i and y_i depict the measured and predicted speed or flow respectively. Also a more qualitative comparison is made on the basis of graphs depicting empirical and estimated speed-flow relationships on these detectors.

4.2 Model parameter settings

In this example, three classes of vehicles, trucks, vans and person cars (the reference class) are defined according to the parameters listed in Table 1. The initial values for the dynamic mixed-class parameters were chosen as follows: $v_c = 85$ km/h, $K_c = 25$ pce/km, and $K_{jam} = 1/s_{u_0} \approx 133$ pce/km. As a result, the a priori capacity of a single lane equals 2125 (pce/h).

Table 1. Class-specific parameters *Fastlane*

	person cars	vans	trucks
percentage at origins (%)	75	15	10
s_u (gross distance gap in m)	7.5	10	18
T_u (time headway in s)	1.2	1.4	1.8
v_u (free speed in km/h)	120	100	85

4.3 Filter scenarios

The Kalman Gain G_t in (16) governs the magnitude with which both parameters and state variables are adjusted by balancing the degree of uncertainty in the measurements (r_d) and the in the parameters and state variables (r_x). This implies that the ratio between these noise factors β ($\propto r_d/r_x$) influences the responsiveness of the filter to new measurements. A very small β will keep the model "tight" to the data, but might lead to an overly responsive and (in terms of parameters) overfitted model. Vice versa, a large β will lead to a less responsive and smoother filter, albeit that the state and parameter may become biased.

In this example we assess the DEKF-*Fastlane* state / parameter estimator for three values of β (1, 10^{-2} and 10^{-4}), where we fix the process noise and scale the measurement noise accordingly. In the example, we used $r_x = 1$ [veh/km] for the state process noise. The parameter process noise is chosen two orders of magnitude smaller (10^{-2}). In total we assess 3 (number of noise (β) settings) × 3 (measurement data used: speeds, flows or both) × 3 (state estimator setup: only state, state and K_c, and state and K_c, v_c, K_{jam}) = 27 scenarios.

5. RESULTS

Fig.4 summarizes the RMSE results of the DEKF-FASTLANE estimator with respect to speed measured at detectors 3 and 4. There are a number of interesting observations to be made. First of all using just the speed data (from detectors 1,2,5) yields by far the best results. Moreover, when using just speed data it appears that neither filter sensitivity (different β settings) nor dually estimating one or three parameters have much effect on the state estimation results. When also the flow data is used,

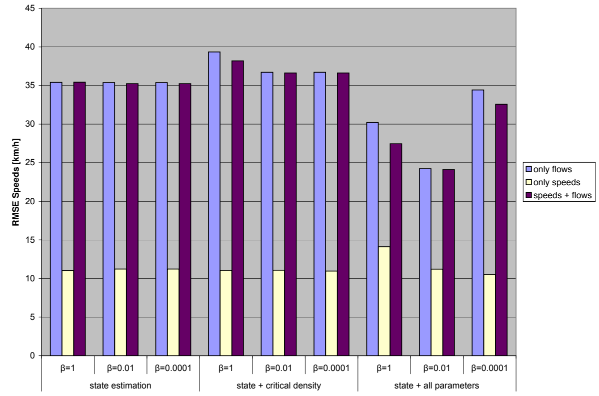

Fig. 4. RMSE speeds (on detectors 3 & 4) in different filter and measurement scenario's

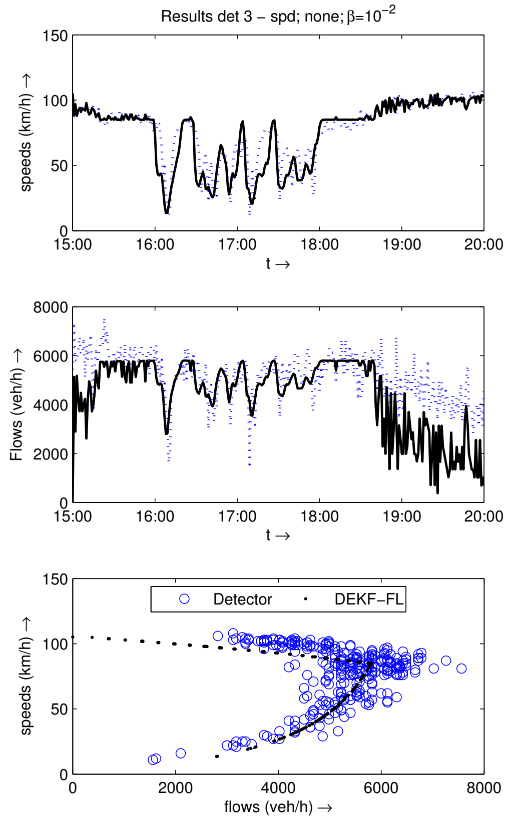

Fig. 5. Speed estimation results at detector 3 in case only state variables are estimated.

dual estimation *is* beneficial, but only in case all mixed-class parameters (K_c, v_c, K_{jam}) are estimated. Overall, on the basis of RMSE on speeds, a state estimator with fixed parameters outperforms all other (dual estimation) configurations by far. In a real-time control setting where an accurate mean state estimate is relevant, this clearly is the preferable filter scenario.

Fig.5 and 6 show the estimated speeds (top), flows (middle) and speed-flow relationship (bottom) at detector 3,

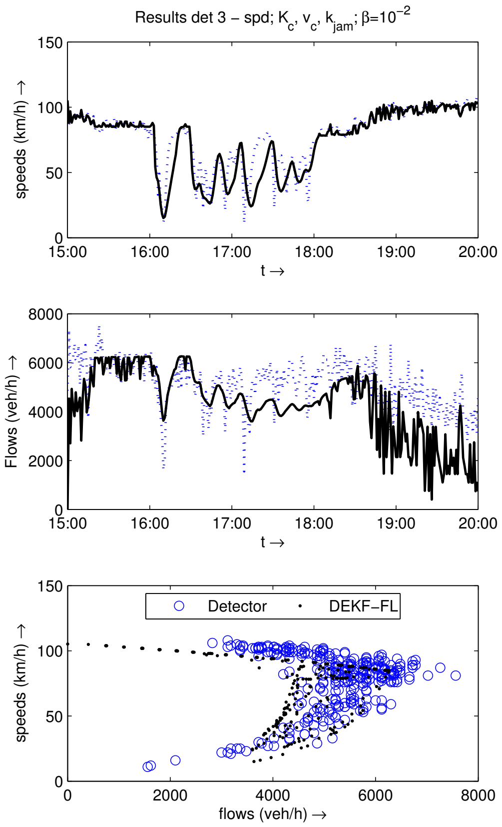

Fig. 6. Speed estimation results at detector 3 in the dual estimation case (state and parameters).

on the basis of single (only state) and dual (state + parameters) estimation respectively. The differences in the estimated speeds and flow time series appear small (compare the top and middle graphs in Fig.5 and 6). However, the dual estimation procedure does result in a very different speed-flow relationship (fundamental diagram). In case the parameters are estimated sequentially, the estimated speed-flow relationship is - like the one based on detector data - widely scattered (Fig.6), which is a result of the time varying fundamental diagram poarameters (K_c, v_c, K_{jam}) . Fig.7 shows the resulting effective capacity $(C_{eff} = v_c K_c)$ at detector 3. Clearly, during the congested period (16:00-18:00) a significant drop in capacity is estimated, which makes sense from a physical point of view.

6. CONCLUSION

This paper showed that a first-order multi-class traffic flow model (FASTLANE) can be used in a traffic surveillance system by means of a dual extended Kalman filter (DEKF). Only the total (effective) density is corrected by the DEKF, which is "translated" into multi-class state corrections by means of state-dependent person car equivalents and class flow shares. Results on real data from a densely used freeway show that the DEKF procedure is able to reproduce speeds and flows in between detectors accurately. Estimating the model parameters sequentially does not improve the state estimation (between measurements), but does lead to more plausible physical results, such as a drop in capacity during congestion, and a widely scattered fundamental diagram.

ACKNOWLEDGEMENTS

The authors thank the anonymous reviewers, whos suggestions were of great help to improve this paper.

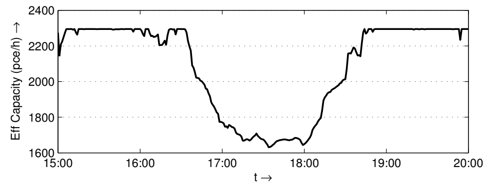

Fig. 7. Effective capacity (in pce/h per lane) at detector 3 in the dual estimation case (state and parameters).

REFERENCES

Constantinos Antoniou, Moshe Ben-Akiva, and Haris N. Koutsopoulos. On-line calibration of traffic prediction models. In *Transportation Research Board Annual Meeting, CD-Rom*, Washington, D.C., USA, 2005. National Academies Press.

S. Haykin. *Kalman Filtering and Neural Networks*. John Wiley & Sons, New York, 2001.

J. P. Lebacque. The gudunov scheme and what it means for first order traffic flow models. In J. B. Lesort, editor, *Proceedings of the 13th International Symposium on Transportation and Traffic Theory*, pages 647–677, Lyon, France, 1996.

M. Lighthill and G. Whitham. On kinematic waves ii: A theory of traffic flow on long crowded roads. *Proc. R. Soc*, A 229(1178):317–345, 1955.

Steven Logghe. *Dynamic Modelling of Heterogeneous Vehicular Traffic*. Faculty of Applied Science, Katholieke Universiteit Leuven, Leuven, 2003.

Laura Munoz, Xiaotian Sun, Roberto Horowitz, and Luis Alvarez. A piecewise-linearized cell transmission model and parameter calibration methodology. In *85th Transportation Research Board Annual Meeting*, Washington, D.C., 2006. The National Academies.

P.J. Richards. Shock waves on the highway. *Operations Research*, 4:4251, 1956.

Dan Simon. *Optimal State Estimation: Kalman, H-infinity, and Nonlinear Approaches*. John Wiley & Sons, New York, 2006.

J.W.C. van Lint, S. P. Hoogendoorn, and M. Schreuder. Multi-class first order traffic flow modeling. In *Traffic and Granular Flow '07*, Paris-Sud University, Orsay, 2007. Springer.

J.W.C. van Lint, S. P. Hoogendoorn, and M. Schreuder. Fastlane - a new multi-class first order traffic flow model. In *Transportation Research Board Annual Meeting, CD-Rom*, Washington D.C., 2008. National Academies.

Y. Wang, M. Papageorgiou, and A. Messmer. A real time freeway network traffic surveillance tool. *IEEE Transactions on control systems technology*, 14(1), 2006.

Yibing Wang and Markos Papageorgiou. Real-time freeway traffic state estimation based on extended kalman filter: a general approach. *Transportation Research Part B*, 39:141167, 2005.