Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Boundary Stabilization of Marine Structure

Tu Duc Nguyen *

* *Norwegian University of Science and Technology, Trondheim, Norway*
E-mail: Tu.Duc.Nguyen@itk.ntnu.no.

Abstract:
This note addresses the stabilization problem of a marine structure (i.e. cable/riser), connected to a surface vessel at one end and to a thruster unit at the other. Here, only lateral motion is considered. Based on boundary measurements, stabilizing control laws are designed. The controllers consist only on feedback from boundary measurements. The costs are thus minimized and the spillover instabilities are avoided. Simulation results are included.

1. INTRODUCTION

This note addresses the stabilization problem of a system consisting of a marine structure (i.e. cable/riser) connected to a surface vessel at the top end and to a thruster unit (e.g. robot system, ROV, mass modul, etc.) at the bottom end (Figure 1).The function of the thruster unit may be several, e.g. to perform maintenance and repair on underwater installations; while the marine structure is needed to provide power, control signals and other necessary signals for operating the thruster unit. Due to the motion of the surface vessel and fluid forces (i.e. wave and current forces), the marine structure undergoes deformations, which lead to reduced performance of the thruster unit. Thus, robust and high performance controllers for the thruster system are needed.

The dynamics of marine cable/riser have been studied by numerous authors, among others [2],[6],[7],[8],[9],[19],[20],[21],[22],[25],[26] and references therein. In [8],[20],[25],[26] modelling and analyzing of marine cable are studied. In [5],[11],[16] boundary control of elastic cable/beam are studied. Aamo and Fossen [1] considered modelling and control of mooring lines. Jensen et al. [13] study modelling and control of offshore marine pipeline, where the model of the system is based on the standard robot equation. In [17],[22],[27] modelling and control of towed marine cables are studied. As opposed to [17], the control design in [22],[27] are based on discretized models of the cables.

For discretized ordinary differential equation models of flexible system there exists many control design tools (see e.g. [3],[4],[14],[22],[27] and the references therein). A substantial difficulty in the design of these controllers is the choice of the discretization order. Reduction of the infinite dimensional continuum model to a finite dimensional (n^{th} order) discrete model means that certain motions ($\infty - n$) are neglected. Typically, modal analysis motivates the model reduction. With sufficient system damping, higher order modes can be neglected if the controller rolls off (i.e. the controller gain drops sharply) at high frequency. Choice of n too small results in spillover instability that occurs when the controller, designed for the finite dimensional model, senses and actuates higher order modes, driving them unstable (see e.g. [4]). Reduction of the control gain to eliminate spillover often results in poor performance. Choice of n too large results in a high order compensator that can be difficult and costly to implement. So to avoid the spillover instabilities and complexity associated with discretized and distributed controllers, the control design for flexible mechanical systems should be based on the distributed parameter models, which will be considered in this note. This note is an extension of [18], and is inspired by the work of Lindegaard et al. [15],[23], where *acceleration feedback* in *dynamic position* system (DP) was first introduced.

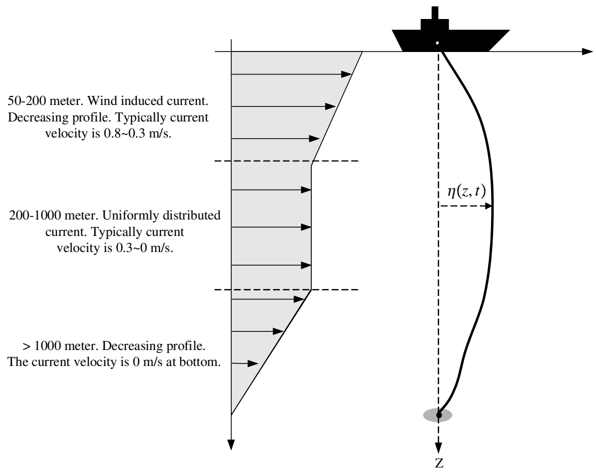

Fig. 1. Marine structure with marine vessels.

2. SYSTEM MODEL

Consider the system in Figure 1. The marine structure of length $L > 0$ is connected to a surface vessel of mass $M_{RB} > 0$ at one end and to a thruster unit of mass $m_{RB} > 0$ at the other. Here, only lateral motion is considered. The mathematical model of the system is adopted from [12],[24].

Let $\Omega =]0, L[$, $\overline{\Omega} = [0, L]$ and $\eta(z, t)$ denote the deflection of the marine structure at the point $z \in \overline{\Omega}$ and time $t \geq 0$. The equations of motion of the system are given as

$$\rho\ddot{\eta} = -\left(EI\eta_{zz}\right)_{zz} + \left(T\eta_z\right)_z - \mathtt{d}\dot{\eta} + \mathtt{c}_0\dot{\omega} + \mathtt{c}_2\left(\omega + U - \dot{\eta}\right)\left|\omega + U - \dot{\eta}\right|, \ z \in \Omega \tag{1}$$

$$M\ddot{\eta} = -\left(EI\eta_{zz}\right)_z + T\eta_z + \tau_0 + X_{wind} + X_{waves} + X_{disturb} - C_1\dot{\eta} + C_2\left(U - \dot{\eta}\right)\left|\dot{\eta} - U\right|, \ z = 0 \tag{2}$$

$$m\ddot{\eta} = \left(EI\eta_{zz}\right)_z - T\eta_z + \tau_L + X_{\mathtt{waves}} + X_{\mathtt{disturb}} - \mathtt{C}_1\dot{\eta} + \mathtt{C}_2\left(U - \dot{\eta}\right)\left|U - \dot{\eta}\right|, \ z = L \tag{3}$$

for $t > 0$. Assume that the marine structure is connected to the vessels by means of ball-joints. This results in small angles of deflection and zero bending. Hence, the remaining boundary conditions of (1)-(3) are

$$EI\eta_{zz}|_0 = EI\eta_{zz}|_L = 0 \tag{4}$$

for $t \geq 0$. Here, $EI(z)$ and $T(z)$ denote the stiffness and tension of the structure at $z \in \overline{\Omega}$, respectively, $\mathtt{d}(z)$ represents the structural damping at $z \in \Omega$, $\omega(z,t)$ is the lateral wave velocity at $z \in \Omega$ and time $t \geq 0$, $U(z,t)$ denotes the lateral current velocity at $z \in \overline{\Omega}$ and time $t \geq 0$, respectively, $\mathtt{c}_0, \mathtt{c}_2, C_1, C_2, \mathtt{C}_1, \mathtt{C}_2 > 0$ are the hydrodynamic coefficients, X_{wind}, X_{waves} and $X_{\mathtt{waves}}$ represent the generalized forces acting on vessels due to the wind and waves, $X_{disturb}$ and $X_{\mathtt{disturb}}$ denote the generalized forces acting on the vessels due to unmodelled disturbances, $\tau_0, \tau_L : \mathbb{R}^+ \to \mathbb{R}$ denote the thruster forces generated by the surface vessel and the thruster unit, respectively, and

$$\rho = \rho_R + \rho_w\left[C_m(z) - 1\right]\frac{\pi D_0^2}{4}$$

$$M = M_{RB} + M_A\left(\omega|_0\right)$$

$$m = m_{RB} + m_A\left(\omega|_L\right)$$

$$\mathtt{c}_0(z) = \frac{\rho_w \pi D_0^2}{4}C_m(z) > 0, \quad z \in \Omega$$

$$\mathtt{c}_2(z) = \frac{\rho_w D_0}{2}C_d(z) > 0, \quad z \in \Omega$$

ρ_R is the mass per unit length of the structure, ρ_w is the mass density of the ambient water, C_m, C_d and D_0 denote the mass coefficient, drag coefficient and diameter of the structure, respectively, M_A denotes the added mass of the surface vessel, and m_A is the added mass of the thruster unit.

The generalized forces due to the wind and the waves, X_{wind} and X_{waves}, are modelled as [12],

$$X_{wind} = \frac{1}{2}C_X\rho_a A_T V_r^2 \tag{5}$$

$$X_{waves}(s) = \frac{K_e s}{s^2 + 2\zeta_e\omega_e s + \omega_e^2}w_1 + d_1 \tag{6}$$

where C_X is the empirical force coefficient, ρ_a is the mass density of the air, A_T is the transverse projected area of the surface vessel, V_r denotes the relative wind speed (i.e. $V_r = V_{wind} - \dot{\eta}|_0$, where V_{wind} is the speed of the wind), $K_e > 0$ is the wave constant, $\zeta_e > 0$ is the damping coefficient, ω_e is the encounter frequency, s denotes the Laplace variable, d_1 represents the wave drift force modelled as slowly-varying bias term

$$\dot{d}_1 = w_2, \quad t > 0 \tag{7}$$

and w_1, w_2 are Gaussian white noise processes. The encounter frequency ω_e is generally given as,

$$\omega_e\left(V_{vessel}, \omega_0, \beta\right) = \left|\omega_0 - \frac{\omega_0^2}{g}V_{vessel}\cos\beta\right| \tag{8}$$

where ω_0 is the dominating wave frequency, g is the acceleration of gravity, V_{vessel} is the total speed of the surface vessel, β is the angle between the heading and the direction of the wave. However, the wave frequency of a dynamically positioned vessel can be sufficiently described by $\omega_e = \omega_0$, since V_{vessel} is close to zero [12].

Similarly, $X_{\mathtt{waves}}$ is given as

$$X_{\mathtt{waves}}(s) = \frac{\mathtt{K}_L s}{s^2 + 2\zeta_L\omega_L s + \omega_L^2}w_3 \tag{9}$$

where $\mathtt{K}_L > 0$ is the wave constant, $\zeta_L > 0$ is the damping coefficient, ω_L is the encounter frequency, and w_3 is Gaussian white noise process.

The lateral wave velocity $\omega(z,t)$ below the water surface is given by [10],

$$\omega(z,t) = \sum_{i=0}^{\infty}\omega_i W_i e^{-\frac{2\pi}{\lambda_i}z}\sin(\omega_i t), \quad z \in \Omega, \ t \geq 0 \tag{10}$$

where W_i is the wave amplitude, ω_i is the wave frequency, and λ_i is the wave length. See [10],[12] for further discussion on the topics above.

Let the initial conditions be given as

$$\left[\eta(z,0), \dot{\eta}(z,0)\right]^\top = \left[W_0(z), V_0(z)\right]^\top, \quad z \in \overline{\Omega} \tag{11}$$

where W_0 and V_0 are the initial position and velocity functions of the structure, respectively. Throughout this note, the subscript $(\cdot)_z$ and *dot*, e.g. $\dot{\eta}$, denote the partial derivative with respect to z and t, respectively.

2.1 Assumptions

The hydrodynamically added mass M_A for semi-submerged vessel depends in general on the frequency of motion due to the water surface effects. Here, since the surface vessel has low motion, the added mass $M_A > 0$ can be assumed to be constant [12]. Contrary, the hydrodynamically added mass for submerged vessels can generally be considered as constant [12]. Additionally, we assume that

A.1 the wave and current velocities are much larger then the velocity of the structure [6], i.e.

$$\omega + U + \dot{\eta} \approx \omega + U, \quad z \in \Omega, \ t \geq 0$$

A.2 the system parameters $\rho, \mathtt{d}, EI, T \in L_2(\Omega)$ are finite and strictly positive, i.e.

$$0 < \rho_{\min} \leq \rho(z) \leq \rho_{\max} < \infty, \quad z \in \Omega$$

$$0 < \mathtt{d}_{\min} \leq \mathtt{d}(z) \leq \mathtt{d}_{\max} < \infty, \quad z \in \Omega$$

$$0 < T_{\min} \leq T(z) \leq T_{\max} < \infty, \quad z \in \overline{\Omega}$$

for constants $\rho_{\min}, \rho_{\max}, \mathtt{d}_{\min}, \mathtt{d}_{\max}, T_{\min}, T_{\max} > 0$.

Application of A.1 to (1)-(3) yields

$$\rho\ddot{\eta} = -\left(EI\eta_{zz}\right)_{zz} + \left(T\eta_z\right)_z - \mathtt{d}\dot{\eta} + \mathtt{c}_0\dot{\omega} + \mathtt{c}_2\left(\omega + U\right)\left|\omega + U\right|, \ z \in \Omega \tag{12}$$

$$M\ddot{\eta} = -\left(EI\eta_{zz}\right)_z + T\eta_z + \tau_0 + X_{wind} + X_{waves} + X_{disturb} - C_1\dot{\eta} + C_2\left(U - \dot{\eta}\right)\left|U - \dot{\eta}\right|, \ z = 0 \tag{13}$$

$$m\ddot{\eta} = \left(EI\eta_{zz}\right)_z - T\eta_z + \tau_L + X_{\mathtt{waves}} + X_{\mathtt{disturb}} - \mathtt{C}_1\dot{\eta} + \mathtt{C}_2\left(U - \dot{\eta}\right)\left|U - \dot{\eta}\right|, \ z = L \tag{14}$$

for $t > 0$, with the boundary conditions (4) and initial conditions (11).

3. CONTROL FORMULATION

The objectives of the controllers are to control the position and velocity of the thruster unit and the surface vessel such that $\{\eta|_0, \dot{\eta}|_0, \eta|_L, \dot{\eta}|_L\} \to \{0,0,0,0\}$ as $t \to \infty$. Additionally, the designed controllers should also be able to attenuate the vibrations and oscillations in the system due to the sea loads, i.e. $\{|\eta(z,t)|, |\dot{\eta}(z,t)|\} < \infty$, $\forall z \in \overline{\Omega}$ and $t \geq 0$.

Inspired by the work of Lindegaard et al. [15],[23], where *acceleration feedback* in dynamic position system (DP) was first introduced, we propose the control laws

$$\tau_0(t) = -K_p\, \eta|_0 - K_d\, \dot{\eta}|_0 - K_m\, \ddot{\eta}|_0 - K_i \int_0^t \eta(0,\xi)\, d\xi \quad (15)$$

$$\tau_L(t) = -k_p\, \eta|_L - k_d\, \dot{\eta}|_L - k_m\, \ddot{\eta}|_L - k_i \int_0^t \eta(L,\xi)\, d\xi \quad (16)$$

for $t \geq 0$, where $k_p, k_d, k_m, k_i, K_p, K_d, K_m, K_i > 0$ are controller gains. The integral action is included to wield the influence of the sea current and wind, while the inertia term is to increase the robustness of the system against disturbances (see Remark 1).

Before one can proceed, it is necessary to assume that the system (12)-(14) with the boundary conditions (4), the initial conditions (11), and the control laws (15)-(16) is *well-posed*, i.e. the closed loop system has a unique solution and the solution is *sufficiently* smooth both in time and space.

Now, let the external disturbances be zero, i.e. $X_{wind}(t) = X_{waves}(t) = X_{disturb}(t) = X_{\mathtt{waves}}(t) = X_{\mathtt{disturb}}(t) = 0$, $U(z,t) = \omega(z,t) = 0$, $\forall z \in \Omega$, $t \geq 0$.

Define the state vector

$$\mathbf{q}(t) = \left[\int_0^t \eta|_0\, d\xi,\ \eta|_0,\ \dot{\eta}|_0,\ \eta, \dot{\eta},\ \int_0^t \eta|_L\, d\xi,\ \eta|_L,\ \dot{\eta}|_L\right]^\top$$

Consider now the storage functional

$$\begin{aligned}\mathcal{V}(\mathbf{q}) = &\frac{1}{2}\int_\Omega \left(T\eta_z^2 + EI\eta_{zz}^2 + \rho\dot{\eta}^2\right)\, dz + \gamma\int_\Omega \rho\dot{\eta}\eta\, dz \\ &+\frac{1}{2}(M+K_m)\,\dot{\eta}^2\big|_0 + \frac{1}{2}K_p\,\eta^2\big|_0 + \frac{\gamma}{2}K_i\left(\int_0^t \eta|_0\, d\xi\right)^2 \\ &+\gamma(M+K_m)\,\eta\dot{\eta}|_0 + K_i\,\eta|_0\int_0^t \eta|_0\, d\xi \\ &+\frac{1}{2}(m+k_m)\,\dot{\eta}^2\big|_L + \frac{1}{2}k_p\,\eta^2\big|_L + \frac{\gamma}{2}k_i\left(\int_0^t \eta|_L\, d\xi\right)^2 \\ &+\gamma(m+k_m)\,\eta\dot{\eta}|_L + k_i\,\eta|_L\int_0^t \eta|_L\, d\xi \qquad (17)\end{aligned}$$

where γ is the Lyapunov gain.

First, using the inequalities

$$|\eta(z,t)| \leq |\eta(0,t)| + \left[L\int_\Omega |\eta_z|^2\, dz\right]^{\frac{1}{2}}, \quad z \in \overline{\Omega}$$

$$(a+b) \leq 2a^2 + 2b^2, \quad \forall a, b \in \mathbb{R}$$

yields

$$|\eta(z,t)|^2 \leq 2|\eta(0,t)|^2 + 2L\int_\Omega |\eta_z|^2\, dz, \quad z \in \overline{\Omega}$$

Thus,

$$\int_\Omega |\eta_z|^2\, dz \geq \frac{1}{2L^2}\int_\Omega |\eta|^2\, dz - \frac{1}{L}|\eta(0,t)|^2 \quad (18)$$

Application of (18) to (17) gives

$$\begin{aligned}\mathcal{V} \geq &\frac{1}{2}\int_\Omega \left(EI\eta_{zz}^2 + \rho\dot{\eta}^2\right)\, dz + \gamma\int_\Omega \rho\dot{\eta}\eta\, dz \\ &+\frac{T_{\min}}{4L^2}\int_\Omega \eta^2\, dz - \frac{T_{\min}}{2L}\,\eta^2\big|_0 \\ &+\frac{1}{2}(M+K_m)\,\dot{\eta}^2\big|_0 + \frac{1}{2}K_p\,\eta^2\big|_0 + \frac{\gamma}{2}K_i\left(\int_0^t \eta|_0\, d\xi\right)^2 \\ &+\gamma(M+K_m)\,\eta\dot{\eta}|_0 + K_i\,\eta|_0\int_0^t \eta|_0\, d\xi \\ &+\frac{1}{2}(m+k_m)\,\dot{\eta}^2\big|_L + \frac{1}{2}k_p\,\eta^2\big|_L + \frac{\gamma}{2}k_i\left(\int_0^t \eta|_L\, d\xi\right)^2 \\ &+\gamma(m+k_m)\,\eta\dot{\eta}|_L + k_i\,\eta|_L\int_0^t \eta|_L\, d\xi \qquad (19)\end{aligned}$$

Given $k_i, k_m, K_i, K_m > 0$. Choose the remaining gains as

$$0 < \gamma < \min\left\{\frac{\mathtt{d}_{\min}}{\rho_{\max}}, \frac{T}{L^2\mathtt{d}_{\max}}, \sqrt{\frac{T_{\min}}{2L^2\rho_{\max}}}\right\} \quad (20)$$

$$k_d > 2\gamma(m+k_m) \quad (21)$$

$$K_d > 2\gamma(M+K_m) \quad (22)$$

$$k_p > \max\left\{\gamma^2(m+k_m) + \frac{k_i}{\gamma}\ ,\ 2\gamma(k_d + \mathtt{C}_1)\right\} \quad (23)$$

$$K_p > \max\left\{\gamma^2(M+K_m) + \frac{K_i}{\gamma} + \frac{T_{\min}}{L}\ ,\ 2\gamma(K_d + C_1)\right\} \quad (24)$$

It follows thus

$$\mathcal{V}(\mathbf{q}) > 0, \quad \forall \mathbf{q} \neq 0$$

Note that there are several ways to select the Lyapunov gain and controller gains. The selection (20)-(24) is just one of the possibilities, and is based on the analysis below and the way the expression of $\mathcal{V}$ and the time derivative of $\mathcal{V}$ are written on (cf. eq. (19) and (27)).

Next, taking the time derivative of (17) along solution trajectories of (12)-(14) gives

$$\begin{aligned}\dot{\mathcal{V}} = &-\int_\Omega [\mathtt{d} - \gamma\rho]\,\dot{\eta}^2\, dz - \gamma\int_\Omega \mathtt{d}\dot{\eta}\eta\, dz \\ &-\gamma\int_\Omega EI\eta_{zz}^2\, dz - \gamma\int_\Omega T\eta_z^2\, dz \\ &-[K_d + C_1 - \gamma(M+K_m)]\,\dot{\eta}^2\big|_0 \\ &-\gamma[K_d + C_1]\,\eta\dot{\eta}|_0 - [\gamma K_p - K_i]\,\eta^2\big|_0 \\ &-[k_d + \mathtt{C}_1 - \gamma(m+k_m)]\,\dot{\eta}^2\big|_L \\ &-\gamma[k_d + \mathtt{C}_1]\,\eta\dot{\eta}|_L - [\gamma k_p - k_i]\,\eta^2\big|_L \\ &-C_2\,\dot{\eta}|_0^2\,\left|\dot{\eta}|_0\right| - \gamma C_2\,\eta|_0\,\dot{\eta}|_0\,\left|\dot{\eta}|_0\right| \\ &-\mathtt{C}_2\,\dot{\eta}|_L^2\,\left|\dot{\eta}|_L\right| - \gamma\mathtt{C}_2\,\eta|_L\,\dot{\eta}|_L\,\left|\dot{\eta}|_L\right| \qquad (25)\end{aligned}$$

where integration by parts has been successively applied. Application of (18) and the assumption A.2 to (25) yields

$$
\begin{aligned}
\dot{\mathcal{V}} \leq & -\int_{\Omega} [\mathtt{d} - \gamma\rho]\, \dot{\eta}^2 \, dz - \gamma \int_{\Omega} \mathtt{d}\dot{\eta}\eta \, dz \\
& -\gamma \int_{\Omega} EI\eta_{zz}^2 \, dz - \frac{\gamma T_{\min}}{2L^2} \int \eta^2 \, dz \\
& - \left[\gamma K_p - K_i - \frac{\gamma T_{\min}}{L}\right] \eta^2\Big|_0 \\
& -\gamma \left[K_d + C_1\right] \eta\dot{\eta}|_0 - \left[K_d + C_1 - \gamma (M + K_m)\right] \dot{\eta}^2\Big|_0 \\
& - \left[k_d + \mathtt{C}_1 - \gamma (m + k_m)\right] \dot{\eta}^2\Big|_L \\
& -\gamma \left[k_d + \mathtt{C}_1\right] \eta\dot{\eta}|_L - \left[\gamma k_p - k_i\right] \eta^2\Big|_L \\
& -C_2 \, \dot{\eta}|_0^2 \left|\dot{\eta}|_0\right| - \gamma C_2 \, \eta|_0 \, \dot{\eta}|_0 \left|\dot{\eta}|_0\right| \\
& -\mathtt{C}_2 \, \dot{\eta}|_L^2 \left|\dot{\eta}|_L\right| - \gamma \mathtt{C}_2 \, \eta|_L \, \dot{\eta}|_L \left|\dot{\eta}|_L\right|
\end{aligned} \tag{26}
$$

Let

$$\mathbf{p}(z,t) = \left[\eta(z,t), \dot{\eta}(z,t)\right]^\top, \quad z \in \overline{\Omega}, \ t \geq 0$$

The right-hand side of (26) can be rewritten as

$$
\begin{aligned}
\dot{\mathcal{V}} \leq & -\gamma \int_{\Omega} EI\eta_{zz}^2 \, dz - \frac{C_1}{2} \, \dot{\eta}^2\Big|_0 - \frac{\mathtt{C}_1}{2} \, \dot{\eta}^2\Big|_L \\
& - \int_{\Omega} \mathbf{p}^\top \mathbf{P} \mathbf{p} \, dz - \mathbf{p}^\top\Big|_0 \mathbf{P}_0 \, \mathbf{p}|_0 - \mathbf{p}^\top\Big|_L \mathbf{P}_L \, \mathbf{p}|_L \\
& - \left[\frac{K_d}{2}\dot{\eta}^2 + C_2\dot{\eta}^2 |\dot{\eta}| + \gamma \frac{K_p}{2}\eta^2 + \gamma C_2 \eta\dot{\eta} |\dot{\eta}|\right]_{z=0} \\
& - \left[\frac{k_d}{2}\dot{\eta}^2 + \mathtt{C}_2\dot{\eta}^2 |\dot{\eta}| + \gamma \frac{k_p}{2}\eta^2 + \gamma \mathtt{C}_2 \eta\dot{\eta} |\dot{\eta}|\right]_{z=L}
\end{aligned} \tag{27}
$$

where

$$\mathbf{P} = \begin{bmatrix} \frac{\gamma}{2}\frac{T_{\min}}{L^2} & \frac{\gamma}{2}\mathtt{d} \\ \frac{\gamma}{2}\mathtt{d} & \mathtt{d} - \gamma\rho \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{P}_0 = & \begin{bmatrix} \frac{\gamma}{4}K_p & \frac{\gamma (K_d + C_1)}{2} \\ \frac{\gamma (K_d + C_1)}{2} & \frac{K_d + C_1}{2} \end{bmatrix} \\
& + \begin{bmatrix} \frac{\gamma}{4}K_p - K_i - \frac{\gamma T_{\min}}{L} & 0 \\ 0 & \frac{K_d + C_1}{2} - \gamma (M + K_m) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P}_L = & \begin{bmatrix} \frac{\gamma}{4}k_p & \frac{\gamma (k_d + \mathtt{C}_1)}{2} \\ \frac{\gamma (k_d + \mathtt{C}_1)}{2} & \frac{k_d + \mathtt{C}_1}{2} \end{bmatrix} \\
& + \begin{bmatrix} \frac{\gamma}{4}k_p - k_i & 0 \\ 0 & \frac{k_d + \mathtt{C}_1}{2} - \gamma (m + k_m) \end{bmatrix}
\end{aligned}
$$

Since the Lyapunov gain and the controller gains are chosen according to (20)-(24), it follows that $\mathbf{P}, \mathbf{P}_0, \mathbf{P}_L > 0$. Furthermore, it is straightforward to verify that the last two terms in (27) can be made negative semi-definite for sufficiently large design parameters $k_p, k_d, K_p, K_d > 0$ and sufficiently small $\gamma > 0$. Hence,

$$\dot{\mathcal{V}} \leq 0 \tag{28}$$

It follows thus from *Lyapunov*'s stability theorem that the equilibrium point

$$
\begin{aligned}
\mathbf{q}^* = \Bigg[& \frac{1}{K_i} \Big(C_2 U_\infty |U_\infty| + \frac{1}{2} C_X \rho_a A_T V_{wind,\infty}^2 \\
& - (EI\eta_{zz}^*)_z + T\eta_z^* \Big)\Big|_{z=0}, 0, 0, \eta^*(z), 0, \\
& \frac{1}{k_i} \left(\mathtt{C}_2 U_\infty |U_\infty| + (EI\eta_{zz}^*)_z - T\eta_z^* \right)\Big|_{z=L}, 0, 0 \Bigg]^\top
\end{aligned}
$$

is stable and the solution $\mathbf{q}(t)$ is bounded for $t \geq 0$, where

$$
\begin{aligned}
U_\infty(z) &= \lim_{t\to\infty} U(z,t) < \infty, \quad z \in \overline{\Omega} \\
V_{wind,\infty} &= \lim_{t\to\infty} V_{wind}(t) < \infty
\end{aligned}
$$

and η^* can be obtained by solving the equation

$$(EI\eta_{zz}^*)_{zz} + (T\eta_z^*)_z + \mathtt{c}_2 U_\infty |U_\infty| = 0, \quad z \in \Omega$$

with the boundary conditions

$$\eta^*|_0 = \eta^*|_L = EI\eta_{zz}^*|_0 = EI\eta_{zz}^*|_L = 0$$

Moreover, from the *LaSalles*'s theorem it follows that the equilibrium point $\mathbf{q}^*$ is globally uniformly asymptotically stable.

Remark 1. It should be noticed that beside increasing the masses from M and m to $M + K_m$ and $m + k_m$, respectively, the *acceleration feedback* also reduces the gain in front of the disturbances $X_{wind} + X_{waves} + X_{disturb}$ and $X_{\mathtt{waves}} + X_{\mathtt{disturb}}$ from $1/M$ and $1/m$ to $1/(M + K_m)$ and $1/(m + k_m)$, respectively. The system is thus less sensitive to external disturbances. The design can be further improved by introducing a frequency dependent *virtual mass* (see [12],[15],[23]), i.e. replacing K_m and k_m with transferfunctions $H_m(s)$ and $h_m(s)$ in (15)-(16), where s denotes the Laplace-variable. If $H_m(s)$ and $h_m(s)$ are chosen as low-pass filters,

$$
\begin{aligned}
H_m(s) &= \frac{K_m}{1 + T_m s} \\
h_m(s) &= \frac{k_m}{1 + t_m s}
\end{aligned}
$$

with the gains $K_m, k_m > 0$ and time constants $T_m, t_m > 0$, then the total masses are $M + K_m$ and $m + k_m$ at low frequencies ($s \to 0$), respectively, while at high frequencies ($s \to \infty$) the total masses $M + K_m$ and $m + k_m$ reduce to M and m, respectively.

4. SIMULATION

To simulate the system (12)-(14) with the feedback control laws (15)-(16), the *finite-element method* with *Hermitian* basis functions has been applied. The marine structure was divided into 20 elements. For simplicity, the system parameters are set to be constant. The state variables were initially set to zero. The system parameters are summarized below. We let the unmodelled disturbances be zero, i.e. $X_{disturb}(t) = X_{\mathtt{disturb}}(t) = 0, \ \forall t \geq 0$. Simulation results are shown in Figure 2 - 5. The position of the vessels is shown in Figure 3, and the 2-norm of the state vector $\left[\eta|_0, \dot{\eta}|_0, \eta|_L, \dot{\eta}|_L\right]^\top$ is shown in Figure 5. Obviously, $\{|\eta(z,t)|, |\dot{\eta}(z,t)|\} < \infty, \ \forall z \in \overline{\Omega}$ and $t \geq 0$, and $\{\eta|_0, \dot{\eta}|_0, \eta|_L, \dot{\eta}|_L\} \to 0$ as $t \to \infty$.

<u>Marine structure:</u>

$$
\begin{aligned}
[\rho, L]^\top &= [1 \text{ kg/m}, \ 600 \text{ m}]^\top \\
[\mathtt{c}_0, \mathtt{c}_2]^\top &= [80, \ 82]^\top \\
[\mathtt{d}, EI, T]^\top &= [0.1 \text{ Ns/m}^2, 4.2 \cdot 10^8 \text{ Nm}^2, 1.1 \cdot 10^6 \text{ N}]^\top
\end{aligned}
$$

Wind, wave and current:

$$
\begin{array}{ll}
[i, \lambda_i, W_i]^\top & = [1,\ 10\ \mathrm{m},\ 0.1\ \mathrm{m}]^\top \\
[K_e, \zeta_e, \omega_e]^\top & = [0.07,\ 0.11,\ 0.64]^\top \\
[\mathtt{K}_L, \zeta_L, \omega_L]^\top & = [0.035,\ 0.11,\ 0.64]^\top \\
[C_X, A_T]^\top & = \left[2.15,\ 10^3\mathrm{m}^2\right]^\top \\
[\rho_a, V_{wind}]^\top & = \left[1.25\mathrm{kg/m}^3,\ 20\ \mathrm{m/s}\right]^\top \\
U(z) & = \frac{U|_0 - U|_L}{L} z + U|_0
\end{array}
$$

$$\text{where } [U|_0 \,, U|_L]^\top = [1, 0.1]^\top \ \mathrm{m/s}$$

Surface vessel:

$$
\begin{array}{ll}
M & = 9.6 \cdot 10^7 \ \mathrm{kg} \\
[C_0, C_1, C_2]^\top & = [4.8,\ 0.9,\ 1]^\top \cdot 10^6
\end{array}
$$

Thruster unit:

$$
\begin{array}{ll}
m & = 30 \ \mathrm{kg} \\
[\mathtt{C}_0, \mathtt{C}_1, \mathtt{C}_2]^\top & = [1.5,\ 100,\ 820]^\top
\end{array}
$$

Controller gains:

$$
\begin{array}{ll}
[K_p, K_d, K_i, K_m]^\top & = [1,\ 3.75,\ 0.085,\ 0.4]^\top \cdot 10^7 \\
[k_p, k_d, k_i, k_m]^\top & = [1,\ 3.75,\ 0.085,\ 0.4]^\top \cdot 10^5
\end{array}
$$

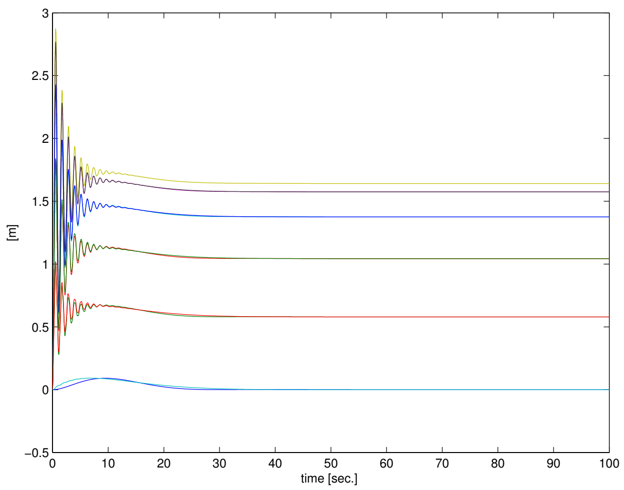

Fig. 2. Deflection of the marine structur $\eta(z, t)$ at selected nodes.

5. CONCLUSIONS

The stabilization problem of a marine structure connected to a surface vessel at one end and a thruster unit at the other end is considered. The dynamics of the marine structure and the vessels are described by a partial differential equation and ordinary differential equations, respectively. The control laws consist only of feedback from boundary measurements. The measurement- and implementation-cost are thus minimized and spillover instabilities are avoided. The theoretical results are verified by simulation results, and they are in agreement.

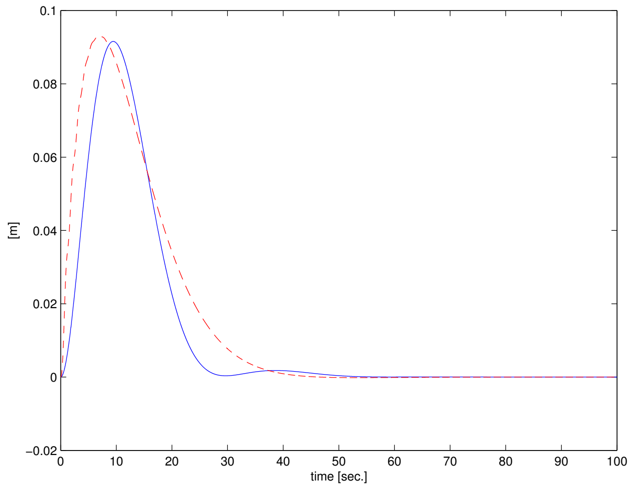

Fig. 3. Position of the surface vessel [solid line] and the thruster unit [dashed line].

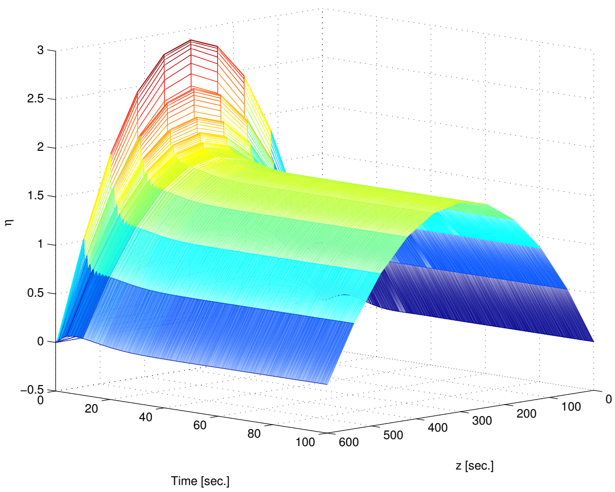

Fig. 4. Evolution of the state $\eta(z, t)$.

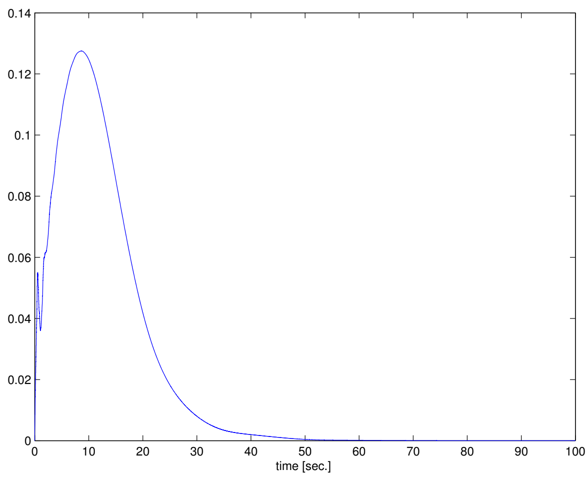

Fig. 5. 2-norm of $[\eta(0, t), \dot{\eta}(0, t), \eta(L, t), \dot{\eta}(L, t)]^\top$.

REFERENCES

[1] Aamo, O. M. and Fossen, T. I., *Controlling Line Tension in Thruster Assisted Mooring Systems*, Proc. IEEE Int. Conf. on Control Applications, Hawaii, USA, 1999.

[2] Ablow C., and Schechter S., *Numerical simulation of undersea cable dynamics*, Ocean Engineering, Vol. 10, No. 3, pp. 443-457, 1983.

[3] Atluri, S. M. and Amos A. K. (Eds.), *Large Space Structures: Dynamics and Control*, Springer-Verlag, 1988.

[4] Balas, M., *Active Control of Flexible Systems*, Journal of Optimalization and Applications, Vol. 25, No. 3, pp. 415-436, 1978.

[5] Baicu, C., Rahn, C. and Nibali, B., *Active Boundary Control of Elastic Cables: Theory and Experiment*, Journal of Sound and Vibration, Vol. 198, No. 1, pp. 17-26, 1996.

[6] Blevins, R. D., *Flow-induced Vibration*, Van Nostrand Reinhold, 1990.

[7] Chakrabarti, D. W. and Frampton, R. E., *Review of Riser Analysis Techniques*, Applied Ocean Research, Vol. 4, No.2, pp. 73-90, 1982.

[8] Dowling, A. P., *The dynamics of towed flexible cylinders Part 1. Neutrally buoyant elements*, Journal of Fluid Mechanics, Vol. 187, pp. 507-532, 1988.

[9] Ersdal, S., *An Experimental Study of Hydrodynamic Forces on Cylinders and Cables in Near Axial Flow*, Ph.D. thesis, Department of Marine Technology, Norwegian University of Science and Technology, 2004.

[10] Faltinsen, O. M., *Sea Loads on Ships and Offshore Structures*, Cambridge University Press, New York, 1990.

[11] Fard, M. P. and Sagatun, S. I., *Exponential Stabilization of a Transversely Vibrating Beam via Boundary Control*, Journal of Sound and Vibration, Vol. 240, No. 4, pp. 613-622, 2001.

[12] Fossen, T. I., *Marine Control Systems: Guidance, Navigation, and Control of Ships, Rigs and Underwater Vehicles*, Marine Cybernetics, Trondheim, Norway, 2002.

[13] Jensen, G. A., Transeth, A. A., and Nguyen, T. D., *Modelling and Control of Offshore Marine Pipeline during Pipelay*, Proc. IFAC World Congress, Seoul, Korea, 2008.

[14] Joshi, S. M., *Control of Large Flexible Space Structures*, Springer-Verlag, 1989.

[15] Lindegaard, K. P., *Acceleration Feedback in Dynamic Positioning*, Ph.D. thesis, Department of Eng. Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway, 2003.

[16] Morgul, O., Rao, B. P. and Conrad, F., *On Stabilization of a Cable with a Tip Mass*, IEEE Transactions on Automatic Control, Vol. 39, No. 10, 1994.

[17] Nguyen, T. D. and Egeland, O., *Stabilization of Towed Marine Seismic Streamer Cables*, Proc. IEEE Int. Conf. on Decision and Control, Bahamas, 2004.

[18] Nguyen, D. T., *Stabilization of Marine Structure*, Proc. Int. Conf. on Robotics, Vision, Information, and Signal Processing, Penang, Malaysia, 2007.

[19] Ortloff, C. R. and Ives, J., *On the dynamic motion of a thin flexible cylinder in a viscous stream*, Journal of Fluid Mechanics, Vol. 38, pp. 713-736, 1969.

[20] Paidoussis, M. P., *Dynamics of cylindrical structures subjected to axial flow*, Journal of Sound and Vibration, Vol. 29, No. 3, pp. 365-385, 1973.

[21] Paidoussis, M. P., *Fluid-structure interactions : slender structures and axial flow*, Elsevier Academic Press, 2004.

[22] Pedersen, E. and Sørensen, A., *Modelling and Control of Towed Marine Seismic Streamer Cables*, Proc. IFAC Conf. on Control Applications in Marine Systems, Glasgow, Scotland, 2001.

[23] Sagatun, S. I., Fossen, T. I., and Lindegaard, K. P., *Inertance Control of Underwater Installations*, Proc. IFAC Conf. on Control Applications in Marine Systems, Glasgow, Scotland, 2001.

[24] Schjølberg, I. and Egeland, O., *Control of an Underwater Robot System Connected to a Ship by a Slender Marine Structure*, Proc. IEEE Int. Conf. on Control Applications, Dearbon, MI, US, pp. 43-48, 1996.

[25] Triantafyllou, G. and Chryssostomidis, C., *The dynamics of towed arrays*, Proc. Int. Offshore Mechanics and Arctic Engineering Symp. 7th, Houston, Texas, US, 1988.

[26] Triantafyllou, M. S., *Cable Mechanics with Marine Applications*, Technical report, MIT, Cambridge, US, 1990.

[27] Turkyilmaz, Y. and Egeland, O., *Boundary Control Design for Towed Cables via Backstepping*, Proc. 7th European Control Conf., Cambridge, UK, 2003.

[28] Yttervik, R., *Ocean current variability in relation to offshore engineering*, Ph.D. thesis, Departement of Marine Technology, Norwegian University of Science and Technology, 2005.