Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Relay-Stabilization and Identification of Unstable Processes

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Abstract: A set of conditions are found for stabilization of unstable processes using relay feedback. The relay-stabilized process will exhibit limit cycles which can then be used to obtain parameters of the process. In this paper, we explore the solution of first- and second- order processes containing one unstable eigenvalue and time delay. The resulting necessary conditions found turn out to be very tight. Simulations are given to show the identification process as well as how the limit cycle conditions apply.

Keywords: Relay control; Unstable Processes; Fourier transforms; Feedback stabilization.

1. INTRODUCTION

For stable processes, knowledge of the critical values such as ultimate gain and the ultimate period have been used successfully to generate a set of tuning parameters for Proportional-Integral-Derivative (PID) controllers, such as those prescribed by Ziegler and Nichols [9] and those prescribed by Tyreus and Luyben [6]. The determination of both ultimate gain and period were made easier by a relay-feedback configuration now known as the auto-tune method as by Åström and Hägglund [1] as a quicker and safer alternative to the original approach proposed by Ziegler and Nichols.

However, for unstable processes, the procedure may not necessarily work. Luyben [2] and Tan et al. [5] have shown that the ultimate gain and the ultimate period for some unstable processes can still be identified using relay feedback. Although the Nyquist point at (-1,0) can yield the ultimate gain and period, unstable processes will need more Nyquist points to build a robust control system.

In this paper, we explore the necessary conditions for a limit cycle. We will first focus on how to identify the parameters of a first order process with delay, by building on the methods used by Tan, et al. [5]. We can also extend the results to a second order system which contains one unstable eigenvalue.

Not all systems are relay stabilizable. Parameter identification of unstable processes will generally consist of some form of trial-and-error. However, necessary conditions for limit cycle will aid in determining when relay-stabilization is at least possible. Once it is possible, we can determine how the process can be modified by lead-lag elements to help with the identification process.

After relay stabilization, one identification approach is the limit cycle information, with or without transient signals. By using Fourier analysis, different frequency response points can be identified [8]. In this paper, we will show that the model parameters can be initially obtained by measuring critical points from the limit cycle information.

2. RELAY FEEDBACK STABLIZATION OF FIRST ORDER WITH DELAY

Given an SISO unstable process whose process is given by

$$\tau \frac{dx}{dt} = x + u(t - \tau_d) \qquad x(0) = x_o \tag{1}$$

and relay control R described by

$$u = \begin{cases} u_m & \text{if } e > \epsilon \text{ and } u^{-\delta t} > 0 \\ -u_m & \text{if } e < -\epsilon \text{ and } u^{-\delta t} < 0 \\ u^{-\delta t} & \text{otherwise} \end{cases} \tag{2}$$

where u is the relay controller output, $u^{-\delta t}$ is the value of relay signal u before possible switching, $e = r - x$ is the error, ϵ is the level of hysteris, r is the reference signal and x is the process output. See Figure 1.

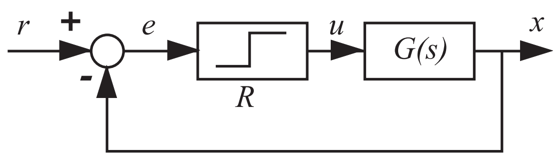

Fig. 1. Configuration for the relay feedback.

We now assume that the process has been stabilized by a relay and that a limit cycle has commenced. The analysis is then partitioned to several regions. A region will involve the time when the output x has experienced two intermediate switches. For the nth region, starting with $n = 0$, we will use $x^{*(n)}$ to denote the initial point and $x^{*(n+1)}$ to denote the end point. The corresponding nth switch time will be denoted by $t^{*(n)}$.

The relay itself will have switched before $t^{*(n+1)}$. we denote this point as $t^{(n+1)}$. The difference is the time delay, i.e.

$$\tau_d = t^{*(n+1)} - t^{(n+1)} \tag{3}$$

Using the initial conditions that $x(0) = x_o$ and $u_m = -\text{sign}(x_o)$, we can solve the relay process and arrive at the following results at the nth region: $t^{*n} \leq t < t^{*(n+1)}$,

$$\begin{aligned} x(t) &= (-1)^{n+1} u_m + \left(x^{*(n)} + (-1)^n u_m\right) \exp\left(\frac{t - t^{*(n)}}{\tau}\right) \\ t^{*(n+1)} &= t^{*(n)} + \tau_d + \tau \ln\left(\frac{u_m + \epsilon}{u_m + (-1)^n x^{*(n)}}\right) \\ x^{*(n+1)} &= (-1)^{n+1} u_m + \left(x^{*(n)} + (-1)^n u_m\right) \exp\left(\frac{\Delta t^{*(n)}}{\tau}\right) \end{aligned} \tag{4}$$

where

$$\Delta t^{*(n)} = t^{*(n+1)} - t^{*(n)} \tag{5}$$

is the time width of the nth region. As $t \to \infty$, this value will settle to equal half the period of the limit cycle. For the first region, we use $x^{*0} = x_o$ and $t^{*0} = 0$.

We could reduce equation (4) to the limiting values. Let x^* be the amplitude of x at the limit cycle, and Δt^* be half the period of the limit cycle. Then equation (4) becomes

$$\Delta t^* = \tau_d + \tau \ln\left(\frac{u_m + \epsilon}{u_m - x^*}\right) \tag{6}$$

$$\frac{u_m + x^*}{u_m - x^*} = \exp\left(\frac{\Delta t^*}{\tau}\right) \tag{7}$$

From the process data, we could measure the period, P, and amplitude x^*. These, plus the knowledge of relay values u_m and ϵ, we arrive the parameter identification equations:

$$\begin{aligned} \Delta t^* &= \frac{P}{2} \\ \tau &= \Delta t^* \left[\ln\left(\frac{u_m + x^*}{u_m - x^*}\right)\right]^{-1} \\ \tau_d &= \Delta t^* \ln\left(\frac{u_m + x^*}{u_m + \epsilon}\right) \left[\ln\left(\frac{u_m + x^*}{u_m - x^*}\right)\right]^{-1} \end{aligned} \tag{8}$$

Note that another simpler method to determine delay τ_d is to measure the time at which the relay has switched, to the switch point of x. Nonetheless, we will use equation (8) to obtain our necessary conditions.

By observing the equation for τ, the denominator limits u_m to be

$$u_m > x^* \tag{9}$$

Combining equation for τ and τ_d, we get

$$\frac{\tau_d}{\tau} = \ln\left(\frac{u_m + x^*}{u_m + \epsilon}\right) \tag{10}$$

Thus, with (9),

$$\frac{\tau_d}{\tau} < \ln\left(\frac{2u_m}{u_m + \epsilon}\right) \tag{11}$$

The condition given in (11) is a much tighter condition than the one given in [5], where they specified $\tau_d/\tau < \ln(2)$.

Example 1:

Simulating the system given in (1) and (2)with $u_m = 1$, $x_o = -0.9$, $\epsilon = 0.2$, $\tau = 0.25$ and $\tau_d = 0.1$, we have the result given in Figure 2.

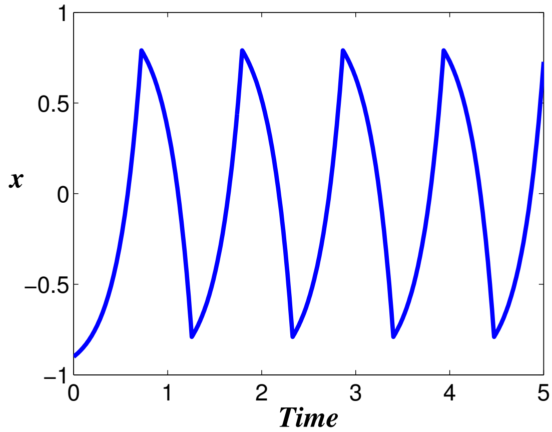

Fig. 2. Simulation results of Example 1.

Using the equations in (8) yielded values that are less than 0.1% relative error.

To explore the necessary condition, we can adjust the value of τ_d by increasing it until x^* approaches u_m. From (11), we have

$$\tau_d < \tau \ln\left(\frac{2u_m}{u_m + \epsilon}\right) = 0.1277$$

Figure 3 and 4 shows that with $\tau_d = 0.1276$ the relay feedback system is stable while using $\tau_d = 0.1278$ made it unstable.

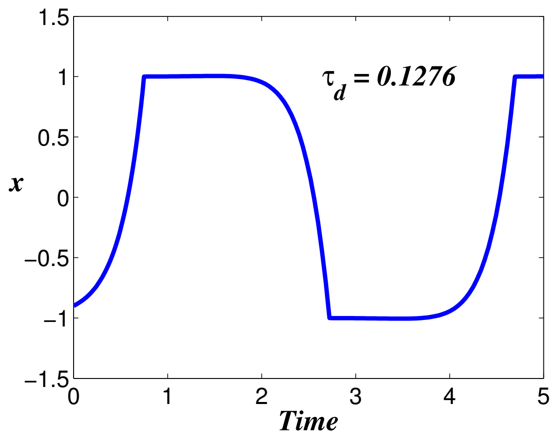

Fig. 3. Simulation results of Example 1 using $\tau_d = 0.1276$.

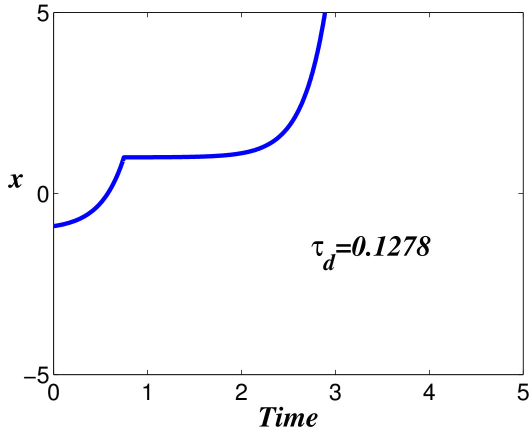

Fig. 4. Simulation results of Example 1 using $\tau_d = 0.1278$.

3. SECOND ORDER PROCESS WITH DELAY

We will limit our analysis to the following systems in series,

$$\begin{aligned} \tau_1 \frac{dx}{dt} &= x + u(t - \tau_d) \qquad & x(0) = x_o \\ \tau_2 \frac{dy}{dt} &= x - y \qquad & y(0) = y_o \end{aligned} \tag{12}$$

subject to the same relay controller given in (2) where the error is based on y, i.e. $e = r - y$. We will assume that we can measure both x and y directly. One situation where this is applicable is when y is a first-order filtering of x to be fed back to the relay controller - thus τ_2 can be thought of as a robustness filter parameter.

Following the procedures and notations as before, we obtain the following results:

$$\begin{aligned} x(t) &= (-1)^{n+1} u_m + (x^{*n} + (-1)^n u_m) \exp\left(\frac{t - t^{*n}}{\tau_1}\right) \\ y(t) &= c^{*n} \exp\left(\frac{-(t - t^{*n})}{\tau_2}\right) + (-1)^{n+1} u_m \\ &\quad + a\left(x + (-1)^n u_m\right) \\ \Delta t^{*(n)} &= \tau_d + \bar{t}^{(n)} \end{aligned} \tag{13}$$

where

$$\begin{aligned} a &= \frac{\tau_1}{\tau_1 + \tau_2} \\ c^{*n} &= (y^{*n} + (-1)^n u_m) - a\left(x^{*n} + (-1)^n u_m\right) \\ \bar{t}^{(n)} &= \tau_1 \ln\left(\frac{-\epsilon + (-1)^n u_m - c^{*n} e^{-\bar{t}^{(n)}/\tau_2}}{a\left[x^{*n} + (-1)^n u_m\right]}\right) \end{aligned} \tag{14}$$

Note that $\bar{t}^{(n)}$ appear on both sides equation (14). This requires a nonlinear solver, or for the form given in (14), the method of successive substitution is sufficient.

For the limiting equations for this case, we have with x^* as the amplitude of x at the limit cycle and $P = 2\Delta t^*$ as the period of oscillation. The parameter τ_1 gives the same equation as before,

$$\tau_1 = \Delta t^* \left[\ln\left(\frac{u_m + x^*}{u_m - x^*}\right)\right]^{-1} \tag{15}$$

which means one of the necessary condition for the case of the first order case reappear, i.e.

$$u_m > x^* \tag{16}$$

Let y^* be the value of y when $x = x^*$. This value can be measured from the responses. Note that this is not the amplitude of y. (In fact, it is often less than the amplitude of y at the limit cycle). The value of y^* satisfies

$$\frac{y^*(1+q) + u_m(1-q)}{x^*(1+q) + u_m(1-q)} = a = \frac{\tau_1}{\tau_1 + \tau_2} \tag{17}$$

where

$$q = e^{-\Delta t^*/\tau_2}$$

The parameter τ_2 can now be found from (17) by using a nonlinear solver such as successive substitution.

The other limiting values are:

$$c^* = a\left(u_m - x^*\right) - \left(u_m - y^*\right) \tag{18}$$

$$\bar{t} = \tau_1 \ln\left(\frac{u_m + c^* e^{-\bar{t}/\tau_2} + \epsilon}{a\left[u_m - x^*\right]}\right) \tag{19}$$

From (19) for $\bar{t}$, the argument inside the logarithm function has to be positive. This fact, combined with the formula for c^*, yields the condition

$$u_m > \frac{e^{-\bar{t}/\tau_2}\left(ax^* - y^*\right) - \epsilon}{1 - (1-a)\, e^{-\bar{t}/\tau_2}} \tag{20}$$

where the denominator is always positive since ($0 < a < 1$). Since this constraint includes the case where τ_2 is very small, it is not a tight condition.

Following the development before and letting x^* approach u_m in the limit, we obtain the following condition:

$$\frac{\tau_d}{\tau_1} < \ln\left(a \frac{2u_m}{u_m + \epsilon}\right) \tag{21}$$

Example 2:

Simulating the system given in (12)with $u_m = 2$, $x_o = -0.9$, $y_o = -0.1$, $\tau_1 = 1$, $\epsilon = 0$, $\tau_2 = 0.5$ and $\tau_d = 0.1$, we have the result given in Figure 5 and 6.

Using (15), we can find $\tau_1 = 1.001$. Reading the value from the response, we have $y^* = 0.1801$ and we can solve numerically for τ_2 and obtain 0.4982, which is close to actual value of $\tau_2 = 0.5$.

To explore the necessary condition, we can again adjust the value of τ_d by increasing it until x^* approaches u_m. From (21), we have

$$\tau_d < \tau \ln\left(a \frac{2u_m}{u_m + \epsilon}\right) = 0.2877$$

Figure 7 and 8 shows that with $\tau_d = 0.2872$, the relay feedback has stabilized the process, while for $\tau_d = 0.2878$, the relay feedback had not stabilized the process.

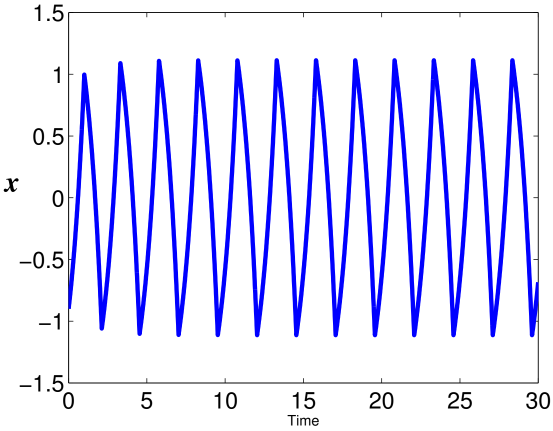

Fig. 5. Simulation results for x of Example 2.

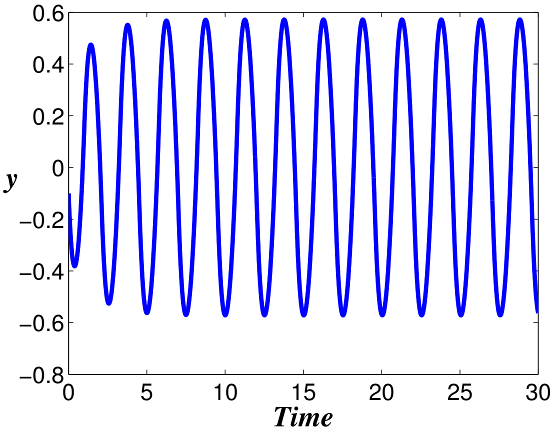

Fig. 6. Simulation results for x of Example 2.

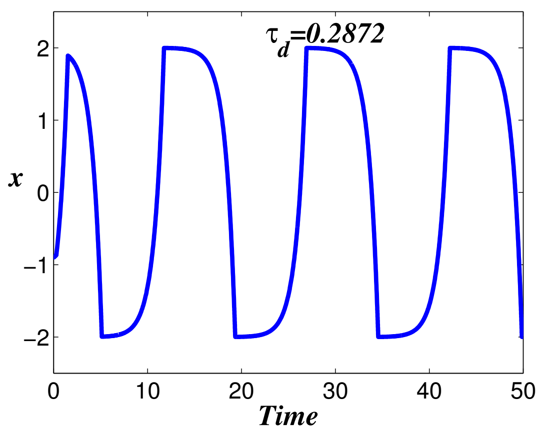

Fig. 7. Simulation results of Example 2 using $\tau_d = 0.2872$.

Actually, the simulation had become unstable at $\tau_d = 0.2873$, which could be due to round-off errors. Nonetheless, the estimate upper bound for τ_d remained close to the simulation result.

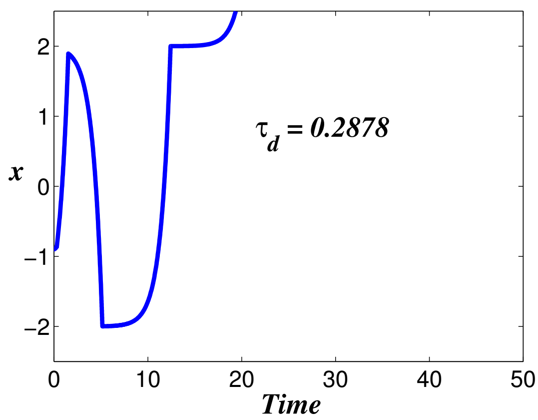

Fig. 8. Simulation results of Example 1 using $\tau_d = 0.1278$.

4. CONCLUSION

Stability analysis of relay feedback systems was obtained for two simple unstable systems: a first order process with delay and a second order with delay. In both cases, we were able to determine the solution inside regions sandwiched by switching points. From the analytical results, we were able to find some tight necessary conditions for limit cycles.

Several issues such as noise need to be addressed. Nonetheless, the conditions found can be used to prune out systems that can not be stabilized using relay. If the process turn out to be relay-stabilizable, other methods can hopefully be used to investigate the parameters and behavior of the process. Some of the results given in the paper show a set of equations which can be used to estimate the model parameters. These values can then be used as initial estimates. Due to the presence of noise, a method such as the Fourier analysis should provide more accurate values of points in the Nyquist plot.

Another point is that, in this paper, we adjusted the values of τ_d for the purpose of checking the tightness of the limit-cycle conditions. The results show that the maximum ratio of τ_d to τ depends on the ratio of u_m to ϵ.

Note that if the process contained process gains $K_p \neq 1$, we simply need to replace u_m by $K_p u_m$ in the conditions. For identification of K_p, we can apply the method in [1] once the system has been stabilized to complement the methods provided in this paper.

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