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Approximations for state estimation in a plane using two two-axis accelerometers

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Abstract: This paper examines the estimation of the state of an object moving in the horizontal plane using two, two-axis accelerometers and no rate gyro. Second order approximations are developed for estimating the state noise covariance in the inertial reference frame from the known measurement covariance of the sensors. These approximations can be integrated with position and heading measurements into an extended Kalman filter..

1. INTRODUCTION

On the basis of cost or accuracy, there are applications where one might consider replacing and inertial platform with a three-axis rate gyro and a three-axis accelerometer with a set of three (or more) three-axis accelerometers. The three accelerometers are located in a plane at fixed distances apart and by resolving the position and orientation of plane of the accelerometers, the position of the object to which they are attached is known. As a partial solution to the problem, in this paper we will consider the problem sketched in Fig. 1: An object moves in the horizontal plane and has two, two-axis accelerometers mounted at different locations. For ease of notation in this paper, these points are selected at a distance R on each side of the centre of gravity. In addition, it is assumed that there is a lower frequency measurement of position and orientation (for example from a GPS and an electronic compass).

Clearly there is a non-linear relationship between the acceleration signals and the movement of the object. In addition, because of the physical constraint that the accelerometers remain at a fixed distance apart, there is redundancy in the measurements. The usual approach to state estimation is via an extended Kalman filter (see for example Brown & Hwang, 1992; Eitelberg, 1991), or an unscented Kalman filter (Julier & Uhlmann, 1997).

This paper will make use of second-order approximations for the mean and variance of functions of a random vector (Papoulis, 1965) to estimate the lateral and rotational accelerations in the plane. These signals can be integrated with position measurements in an extended Kalman filter. From the measurement redundancy of the accelerometers, the square of angular rate is estimated and this is used as an additional output equation in the Kalman Filter. In many application of the extended Kalman filter, only first order propagation of the noise signals is considered.

Ko and Bitmead (2007) have examined the issue of redundant measurements, imposed onto a problem by exact state constraints, in a linear problem setting. In this setting, one can leave the over-determined set of differential equations and apply the state constraint on the state noise covariance. In the present problem, it seems easier to resolve the measurements into a non-redundant set of equations.

The paper is set out as follows: Section 2.1 defines the problem to be solved. Section 2.2 provides approximations for the mean and variance of a function of random vector. Section 2.3 applies these results to the problem of approximating the lateral and rotational accelerations in the inertial frame from measurements of accelerations in the body frame. Section 2.4 shows how these results can be integrated into an extended Kalman filter. The accuracy of the rate estimate obtained from the configuration of accelerometers used in the paper is discussed in Section 3. Section 4 presents a numerical example to illustrate the results in the paper. Conclusions are drawn in Section 5.

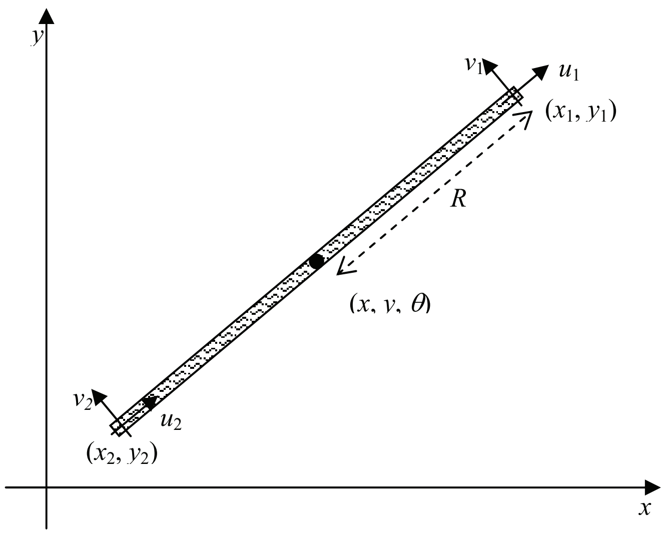

Fig. 1 – Object in a plane with two-axis accelerometers at each end

2. APPROXIMATE OPTIMAL ESTIMATION IN A PLANE

2.1 Problem statement

In Fig. 1, the linear position and orientation of the centre is given by (x, y, θ) with respect to the inertial frame, x-y. The position of the two sensor locations is given by

$$\begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x + R\cos\theta \\ y + R\sin\theta \\ x - R\cos\theta \\ y - R\sin\theta \end{pmatrix} \tag{1}$$

The corresponding accelerations are

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} \ddot{x} - R\sin\theta\,\ddot{\theta} - R\cos\theta\,\dot{\theta}^2 \\ \ddot{y} + R\cos\theta\,\ddot{\theta} - R\sin\theta\,\dot{\theta}^2 \\ \ddot{x} + R\sin\theta\,\ddot{\theta} + R\cos\theta\,\dot{\theta}^2 \\ \ddot{y} - R\cos\theta\,\ddot{\theta} + R\sin\theta\,\dot{\theta}^2 \end{pmatrix} \tag{2}$$

The sensors are located in the body frame (u-v), so the sensor (acceleration) signals, (a_{ui}, a_{vi}), $i = 1{,}2$ are related to accelerations via the rotation,

$$\begin{pmatrix} a_{ui} \\ a_{vi} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \ddot{x}_i \\ \ddot{y}_i \end{pmatrix} \tag{3}$$

Therefore,

$$\begin{pmatrix} a_{u1} \\ a_{v1} \\ a_{u2} \\ a_{v2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & -R \\ -\sin\theta & \cos\theta & R & 0 \\ \cos\theta & \sin\theta & 0 & R \\ -\sin\theta & \cos\theta & -R & 0 \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \dot{\theta}^2 \end{pmatrix} \tag{4}$$

The accelerations in the inertial frame can be solved explicitly from the accelerometer signals as,

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \dot{\theta}^2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cos\theta & -\sin\theta & \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta & \sin\theta & \cos\theta \\ 0 & 1/R & 0 & -1/R \\ -1/R & 0 & 1/R & 0 \end{pmatrix} \begin{pmatrix} a_{u1} \\ a_{v1} \\ a_{u2} \\ a_{v2} \end{pmatrix} \tag{5}$$

This paper will consider the application of the transformation in eq(5) to state estimation in the inertial frame. To do this, the mean and covariance of the signals in the inertial frame from the (assumed known) covariance of the sensor signals is required.

2.2 Results for mean and variance of functions of a random variable

Scalar approximations given by Papoulis (1965, Section 5.4), can be extended to approximate the mean and variance of $y = h(\boldsymbol{x})$, a function of a random vector, $\boldsymbol{x} \in \Re^{n\times1}$, with density, $f(\boldsymbol{x})$. The great benefit of the approximations is that they are independent of the density.

Mean

Approximate y using the first three terms of the Taylor series expansion around $\bar{\boldsymbol{x}}$, the mean of $\boldsymbol{x}$. This is reasonable if $h(\boldsymbol{x})$ is smooth as the density $f(\boldsymbol{x})$ takes significant values near the mean.

$$y \approx h(\bar{\boldsymbol{x}}) + \left.\frac{\partial h}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\bar{\boldsymbol{x}}} (\boldsymbol{x} - \bar{\boldsymbol{x}}) + \frac{1}{2}(\boldsymbol{x} - \bar{\boldsymbol{x}})^{\mathrm{T}} \left.\frac{\partial^2 h}{\partial \boldsymbol{x}^2}\right|_{\boldsymbol{x}=\bar{\boldsymbol{x}}} (\boldsymbol{x} - \bar{\boldsymbol{x}}) \tag{6}$$

$\frac{\partial h}{\partial \boldsymbol{x}}$ is the Jacobian and $\frac{\partial^2 h}{\partial \boldsymbol{x}^2}$ is the Hessian of h with respect to $\boldsymbol{x}$.

Using eq(6), the mean of y can be approximated by,

$$\begin{aligned} \mathcal{E}\{y\} &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h(\boldsymbol{x}) f(\boldsymbol{x}) dx_1 \cdots dx_n \\ &\approx h(\bar{\boldsymbol{x}}) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\left.\frac{\partial^2 h}{\partial \boldsymbol{x}^2}\right|_{\bar{\boldsymbol{x}}} \circ \boldsymbol{C}_{\boldsymbol{x}} \right]_{ij} \end{aligned} \tag{7}$$

where $^{\mathrm{o}}$ denotes the point-wise (Schur or Hadamard) product of the matrices, $[\bullet]_{ij}$ refers to the individual matrix elements, and $\boldsymbol{C}_{\boldsymbol{x}} = \mathrm{cov}(\boldsymbol{x})$ is the covariance of $\boldsymbol{x}$. Notice that when calculating the expectation from eq(6) to eq(7), the linear term with $(\boldsymbol{x} - \bar{\boldsymbol{x}})$ vanishes and the quadratic term is calculated by observing that the result is a scalar, allowing the order of calculations to be swapped to isolate terms which give cov($\boldsymbol{x}$).

As an example of eq(7), if the mean value of θ is $\bar{\theta}$, and the covariance is σ_θ^2, from eq(7),

$$\begin{aligned} \mathcal{E}\{\cos\theta\} &\approx \cos\bar{\theta}\left(1 - \tfrac{1}{2}\sigma_\theta^2\right) \\ \mathcal{E}\{\sin\theta\} &\approx \sin\bar{\theta}\left(1 - \tfrac{1}{2}\sigma_\theta^2\right) \end{aligned} \tag{8}$$

(For two r.v.'s x_1 and x_2, $\mathcal{E}\{x_1 x_2\} \approx \bar{x}_1 \bar{x}_2 + \mathrm{cov}(x_1 x_2)$.)

Variance and co-variance

Given $y_i = h_i(\boldsymbol{x})$ and $y_j = h_j(\boldsymbol{x})$ and $y = h(\boldsymbol{x})$, the co-variance $\sigma_{y_i y_i}^2$ is approximated using a similar approach,

$$\begin{aligned} \sigma_{y_i y_i}^2 &= \mathcal{E}\{(y_i - \bar{y}_i)(y_j - \bar{y}_j)\} \\ &\approx \left.\frac{\partial h_i}{\partial \boldsymbol{x}}\right|_{\bar{\boldsymbol{x}}} \boldsymbol{C}_{\boldsymbol{x}} \left(\left.\frac{\partial h_j}{\partial \boldsymbol{x}}\right|_{\bar{\boldsymbol{x}}} \right)^{\mathrm{T}} \end{aligned} \tag{9}$$

Taking the example in eq(8),

$$\begin{aligned} \mathrm{cov}(\cos\theta) &\approx \sigma_\theta^2 \sin^2\bar{\theta} \\ \mathrm{cov}(\sin\theta) &\approx \sigma_\theta^2 \cos^2\bar{\theta} \end{aligned} \tag{10}$$

If random variables x and θ are not correlated,

$$\begin{aligned}\operatorname{cov}(x\cos\theta) &\approx \sigma_\theta^2\,\bar{x}^2\sin^2\bar{\theta} + \sigma_x^2\cos^2\bar{\theta}\\ \operatorname{cov}(x\sin\theta) &\approx \sigma_\theta^2\,\bar{x}^2\cos^2\bar{\theta} + \sigma_x^2\sin^2\bar{\theta}\end{aligned} \tag{11}$$

2.3 Mean and covariance for the signals in the inertial frame

To keep the exposition simple, take the simplest case that the accelerometer signals are uncorrelated with each other, have white additive noise and covariance, σ_a^2. The white noise sensor signal can be assumed uncorrelated with the (low pass filtered) angle estimates.

$$\mathcal{E}\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{\theta}\\ \dot{\theta}^2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\mathcal{E}\{\cos\theta\}(a_{u1}+a_{u2}) - \mathcal{E}\{\sin\theta\}(a_{v1}+a_{v2})\\ \mathcal{E}\{\sin\theta\}(a_{u1}+a_{u2}) + \mathcal{E}\{\cos\theta\}(a_{v1}+a_{v2})\\ (a_{v1}-a_{v2})/R\\ (-a_{u1}+a_{u2})/R\end{pmatrix} \tag{12}$$

$$\operatorname{cov}\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{\theta}\\ \dot{\theta}^2\end{pmatrix} = \begin{pmatrix}\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14}\\ \lambda_{12} & \lambda_{22} & \lambda_{23} & \lambda_{24}\\ \lambda_{13} & \lambda_{23} & \lambda_{33} & \lambda_{34}\\ \lambda_{14} & \lambda_{24} & \lambda_{34} & \lambda_{44}\end{pmatrix} \tag{13}$$

Where,

$$\begin{aligned}
\lambda_{11} &\approx \Big(\sigma_\theta^2\left((a_{u1}+a_{u2})\sin\bar{\theta} + (a_{v1}+a_{v2})\cos\bar{\theta}\right)^2\\
&\quad + \left(\sigma_{u1}^2+\sigma_{u2}^2\right)\cos^2\bar{\theta} + \left(\sigma_{v1}^2+\sigma_{v2}^2\right)\sin^2\bar{\theta}\Big)/4\\
\lambda_{12} &\approx \Big(\sigma_\theta^2\left(-(a_{u1}+a_{u2})\sin\bar{\theta} - (a_{v1}+a_{v2})\cos\bar{\theta}\right)\\
&\qquad \times\left((a_{u1}+a_{u2})\cos\bar{\theta} - (a_{v1}+a_{v2})\sin\bar{\theta}\right)\\
&\quad + \left(\sigma_{u1}^2+\sigma_{u2}^2-\sigma_{v1}^2-\sigma_{v2}^2\right)\sin\bar{\theta}\cos\bar{\theta}\Big)/4\\
\lambda_{13} &\approx \left(-\sigma_{v1}^2+\sigma_{v2}^2\right)\sin\bar{\theta}/(4R)\\
\lambda_{14} &\approx \left(-\sigma_{u1}^2+\sigma_{u2}^2\right)\cos\bar{\theta}/(4R)\\
\lambda_{22} &\approx \Big(\sigma_\theta^2\left((a_{u1}+a_{u2})\cos\bar{\theta} - (a_{v1}+a_{v2})\sin\bar{\theta}\right)^2\\
&\quad + \left(\sigma_{u1}^2+\sigma_{u2}^2\right)\sin^2\bar{\theta} + \left(\sigma_{v1}^2+\sigma_{v2}^2\right)\cos^2\bar{\theta}\Big)/4\\
\lambda_{23} &\approx \left(\sigma_{v1}^2-\sigma_{v2}^2\right)\cos\bar{\theta}/(4R)\\
\lambda_{24} &\approx \left(-\sigma_{u1}^2+\sigma_{u2}^2\right)\sin\bar{\theta}/(4R)\\
\lambda_{33} &\approx \left(\sigma_{u1}^2+\sigma_{u2}^2\right)/\left(4R^2\right)\\
\lambda_{34} &\approx 0\\
\lambda_{44} &\approx \left(\sigma_{v1}^2+\sigma_{v2}^2\right)/\left(4R^2\right)
\end{aligned}$$

Note that if the sensor covariance is the same in all channels, $\lambda_{13} = \lambda_{14} = \lambda_{23} = \lambda_{24} = 0$.

2.4 Building an (approximate) optimal state estimator

The above estimates of the linear and rotational accelerations in the inertial frame and of the angular rate squared can be used to build a Kalman filter. Supposing that there are measurements of the position and yaw angle, $m = (x, y, \theta)^T$.

The additional estimate of $\dot{\theta}^2$ from the accelerometer measurements will be treated as an additional (non-linear) measurement. The model is given with the following notation (and some ambiguity with respect to the position, x and state vector, $\boldsymbol{x}$). Notice that because the accelerometer signals are used in both the state and output equations, the state noise and measurement noise are correlated. Also note that the state differential equation is linear in the co-ordinate system used while the output equation is non-linear because of the inclusion of the angular rate squared term.

$$\begin{aligned}\boldsymbol{x}_{i+1} &= \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{B}(\boldsymbol{u}_i + \boldsymbol{v}_i)\\ \boldsymbol{y}_i &= \begin{pmatrix}x & y & \theta & \dot{\theta}^2\end{pmatrix}^{\mathrm{T}} + \boldsymbol{w}_i\end{aligned} \tag{14}$$

Where,

$\boldsymbol{x} = \begin{pmatrix}\dot{x} & \dot{y} & \dot{\theta} & x & y & \theta\end{pmatrix}^{\mathrm{T}}$ - the state vector

$\boldsymbol{u} = \begin{pmatrix}a_x & a_y & a_\theta\end{pmatrix}^{\mathrm{T}}$ - the input vector

$\boldsymbol{A}_c = \begin{pmatrix}\boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times3}\end{pmatrix}$ - continuous state transition matrix

$\boldsymbol{A} = \exp\{\boldsymbol{A}_c\Delta t\}$ - discrete state trans. matrix

$\boldsymbol{B} = \int_0^{\Delta t}\exp\{\boldsymbol{A}_c t\}dt\begin{pmatrix}\boldsymbol{I}_{3\times3}\\ \boldsymbol{0}_{3\times3}\end{pmatrix}$ - discrete input matrix

the state noise covariance:

$$\boldsymbol{Q}_i = \mathcal{E}\left\{\boldsymbol{v}_i\,\boldsymbol{v}_i^{\mathrm{T}}\right\} = \begin{pmatrix}\lambda_{11} & \lambda_{12} & \lambda_{13} & \\ \lambda_{12} & \lambda_{22} & \lambda_{23} & \boldsymbol{0}_{3\times3}\\ \lambda_{13} & \lambda_{23} & \lambda_{33} & \\ & \boldsymbol{0}_{3\times3} & & \boldsymbol{0}_{3\times3}\end{pmatrix}$$

the state - output noise cross covariance:

$$\boldsymbol{S}_i = \mathcal{E}\left\{\boldsymbol{v}_i\,\boldsymbol{w}_i^{\mathrm{T}}\right\} = \begin{pmatrix} & \lambda_{14}\\ \boldsymbol{0}_{3\times3} & \lambda_{24}\\ & \lambda_{34}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times1}\end{pmatrix}$$

the output noise covariance:

$$\boldsymbol{R}_i = \mathcal{E}\left\{\boldsymbol{w}_i\,\boldsymbol{w}_i^{\mathrm{T}}\right\} = \begin{pmatrix}\operatorname{cov}\left(\begin{pmatrix}n_{ax} & n_{ay} & n_{a\theta}\end{pmatrix}^{\mathrm{T}}\right) & \boldsymbol{0}_{3\times1}\\ \boldsymbol{0}_{1\times3} & \lambda_{44}\end{pmatrix}$$

When output measurements are not available, the state vector and state covariance are updated via,

$$\begin{aligned}\hat{\boldsymbol{x}}_{i+1} &= \boldsymbol{A}\hat{\boldsymbol{x}}_i + \boldsymbol{B}\boldsymbol{u}_i\\ \boldsymbol{P}_{i+1} &= \boldsymbol{A}_i\boldsymbol{P}_i\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{Q}_i\end{aligned} \tag{15}$$

With measurements, the state vector and covariance are updated via the (extended) Kalman filter,

$$\begin{aligned}\boldsymbol{K}_i &= \left(\boldsymbol{A}_i\boldsymbol{P}_i\boldsymbol{C}_i^{\mathrm{T}} + \boldsymbol{S}_i\right)\left(\boldsymbol{R}_i + \boldsymbol{C}_i\boldsymbol{P}_i\boldsymbol{C}_i^{\mathrm{T}}\right)^{-1}\\ \hat{\boldsymbol{x}}_{i+1} &= \boldsymbol{A}\hat{\boldsymbol{x}}_i + \boldsymbol{B}\boldsymbol{u}_i + \boldsymbol{K}(\boldsymbol{y} - \hat{\boldsymbol{y}})\\ \boldsymbol{P}_{i+1} &= \boldsymbol{A}_i\boldsymbol{P}_i\boldsymbol{A}_i^{\mathrm{T}} - \boldsymbol{K}_i\left(\boldsymbol{C}_i\boldsymbol{P}_i\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{S}_i^{\mathrm{T}}\right) + \boldsymbol{Q}_i\end{aligned} \tag{16}$$

3. ACCURACY OF ANGULAR RATE ESTIMATE

This section will consider the quality of the rate estimate that can be derived from the two, two-axis accelerometer arrangement discussed in the paper.

For this, consider the sub-model for estimating the angular rate, $\omega_{i+1} = \omega_i + \Delta t \ddot{\theta}$ with output, $y = \dot{\theta}^2$, and the sensor derived signals' $\left(\ddot{\theta}, \dot{\theta}^2\right)$ expected value and variance estimated via eq(12) and eq(13) above.

If an extended Kalman filter is developed to estimate the rate (i.e. the signal that would be available from a rate gyro), from eq(16) with $\boldsymbol{A}$=1, $\boldsymbol{C} = \left.\frac{\partial y}{\partial \dot{\theta}}\right|_{\dot{\theta}=\omega_0} = 2\omega_0$, (with ω_0 the current estimated rate) the following calculations (scalar with lower case symbols) apply to the state error covariance:

$$p_{i+1} = p_i - \frac{\left(p_i c_i + s_i\right)^2}{\left(r_i + p_i c_i^2\right)} + q_i \tag{17}$$

$q_i = \lambda_{33}(\Delta t)^2$, $s_i = \lambda_{34} \approx 0$ and $r_i = \lambda_{44}$. At constant, non-zero rate, the steady state solution of the Riccati equation is

$$p = q/2 + \sqrt{q^2/4 + qr/c^2} \tag{18}$$

Furthermore, if the noise covariance in each channel of the sensor is the same,

$$q \approx (\Delta t)^2 \sigma^2 / \left(2R^2\right), r \approx \sigma^2 / \left(2R^2\right), \text{ and}$$

$$p \approx (\Delta t)^2 \sigma^2 / \left(4R^2\right)\left(1 + \sqrt{1 + 1/(c\Delta t)^2}\right) \tag{19}$$

The Δt^2 in eq(19) cancels the typical dependence of the sensor noise. Notice that without rotation, the rate estimate error covariance will increase without bound.

To put some numbers to the above, consider the Analog Devices ADIS16080 80 °/sec rate gyroscope which has a datasheet noise figure of 0.05 °/sec/√Hz and the ADIS16003 1.7g dual-axis accelerometer with a datasheet noise figure of 110 μg/√Hz. (Analog Devices, 2008). Suppose R = 0.1 m (giving a full scale rate of 740 °/sec), and Δt = 0.01 s (which with appropriate low-passing filtering of the sensor signal gives an accelerometer noise standard deviation of 1.4 mg). At 80 °/sec rate, the rate standard deviation from eq(19) is 0.24 °/sec, comparable to the rate noise at the same sampling rate (and filtering assumption) of 0.63 °/sec from the ADIS16080 rate sensor. The noise figure to full scale ratio of the accelerometer is about 10 times smaller than that of the rate gyro.

4. EXAMPLE

The approach is illustrated below a numerical example. The centre of gravity of an object with R=0.5 m moves in a circle of radius 10 m while rotating at $\text{sgn}(t - t_{\text{end}}/2) \times 0.1 \times t$ rad/s. The simulation parameters shown in Table 1 are assumed. Figure 2 illustrates the effect of wrong initial conditions and Figure 3 illustrates the behaviour with correct initial conditions after the filter covariance has settled.

Table 1. Simulation Parameters

Compass Standard deviation	$\sigma_{y\theta} = 0.1$	[rad] (~5°)
Sampling rate	1.0	[s]
Accelerometers Standard deviation	$\sigma_a = 9.8 \times 10^{-3}$	[m/s^2] (~1 mG)
Sampling rate	h=0.01	[s]
Position Standard deviation	$\sigma_{xy} = 1.0$	[m]
Sampling rate	1.0	[s]

Observations

As the simulation shows, the filter is able to extrapolate position measurements (for example, 1 s updates from a GPS system) over a number of accelerometer. In the numerical investigation, the square root of the diagonal elements of the state covariance matrix, $\sqrt{\boldsymbol{P(i,i)}}$ (the standard deviation), was examined to get some idea of the accuracy. The position is estimated with a standard deviation of around 0.5 m (Fig. 3c) and this agrees with the measurement accuracies used. On the other hand, the angle is estimated with a standard deviation of only 0.01 rad. As this is far better than the assumed heading measurement of 0.1 rad, it must be because of the assumed high accuracy of the acceleration measurements.

Simulation is "doomed to succeed" but the simulation illustrates the principle presented in the paper. Any practical application would have to deal with accelerometer offset (bias) and drift, and if in a gravitational field, the effect of gravity on any sensor misalignment. The high accuracy of the angle estimation is suspicious and further study is required to identify the effect of accelerometer bias on the angle estimate and its accuracy.

5. CONCLUSIONS

This paper has presented approximations to translate body frame acceleration measurements and corresponding covariance into the values and covariance in the inertial frame. The approximations for mean and variance of functions of random vectors may be developed to suit many similar engineering problems where often the density of the underlying distributions are unknown and a simple tool is required to make sensible progress on a particular problem.

The method has been illustrated via a numerical problem.

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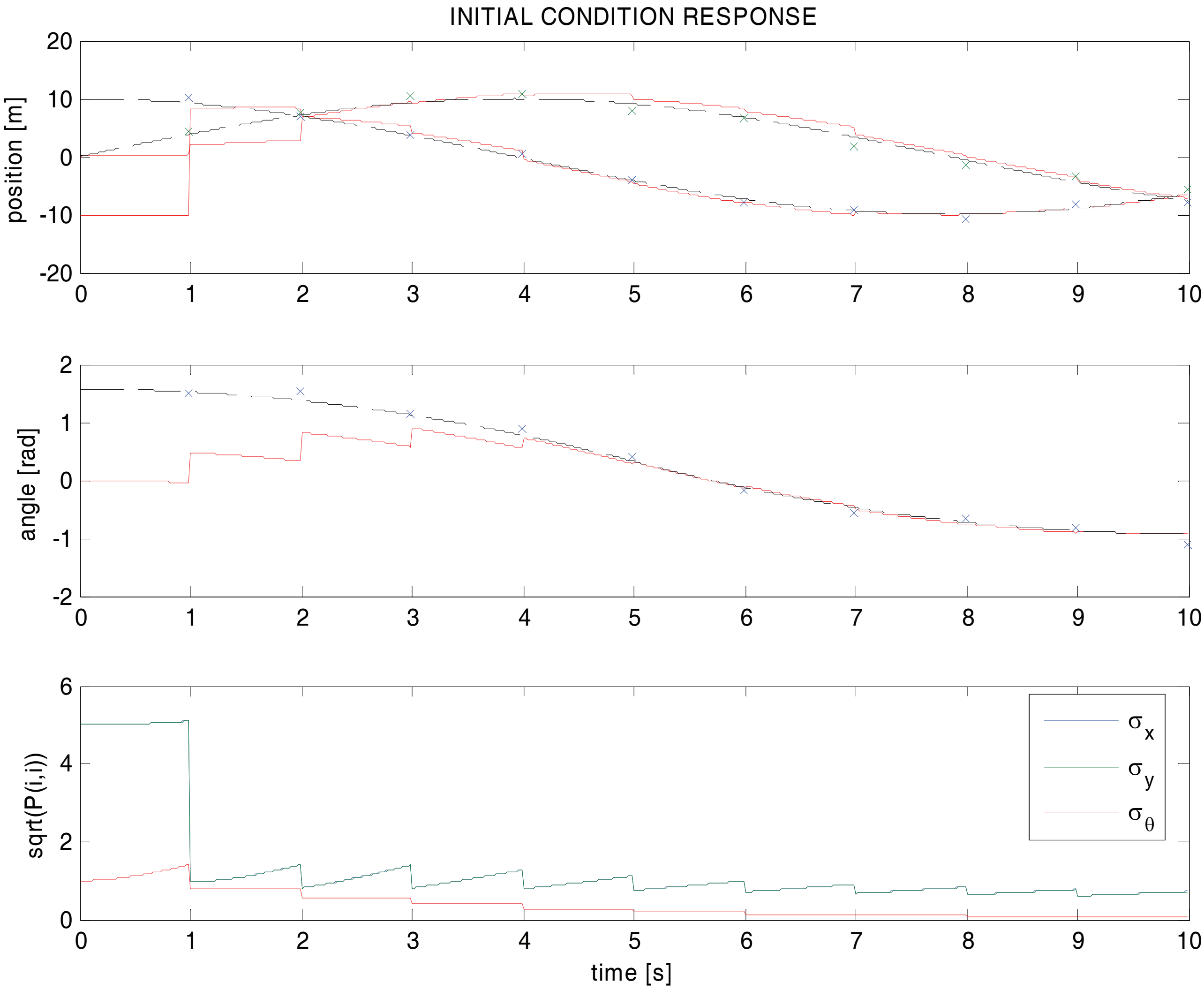

Fig. 2 – Transient response of filter. Solid lines: . Dashed lines: x. Position and angle measurements: 'x'

Initial ***P*** = diag {12, 12, 12, 52, 52, 12}

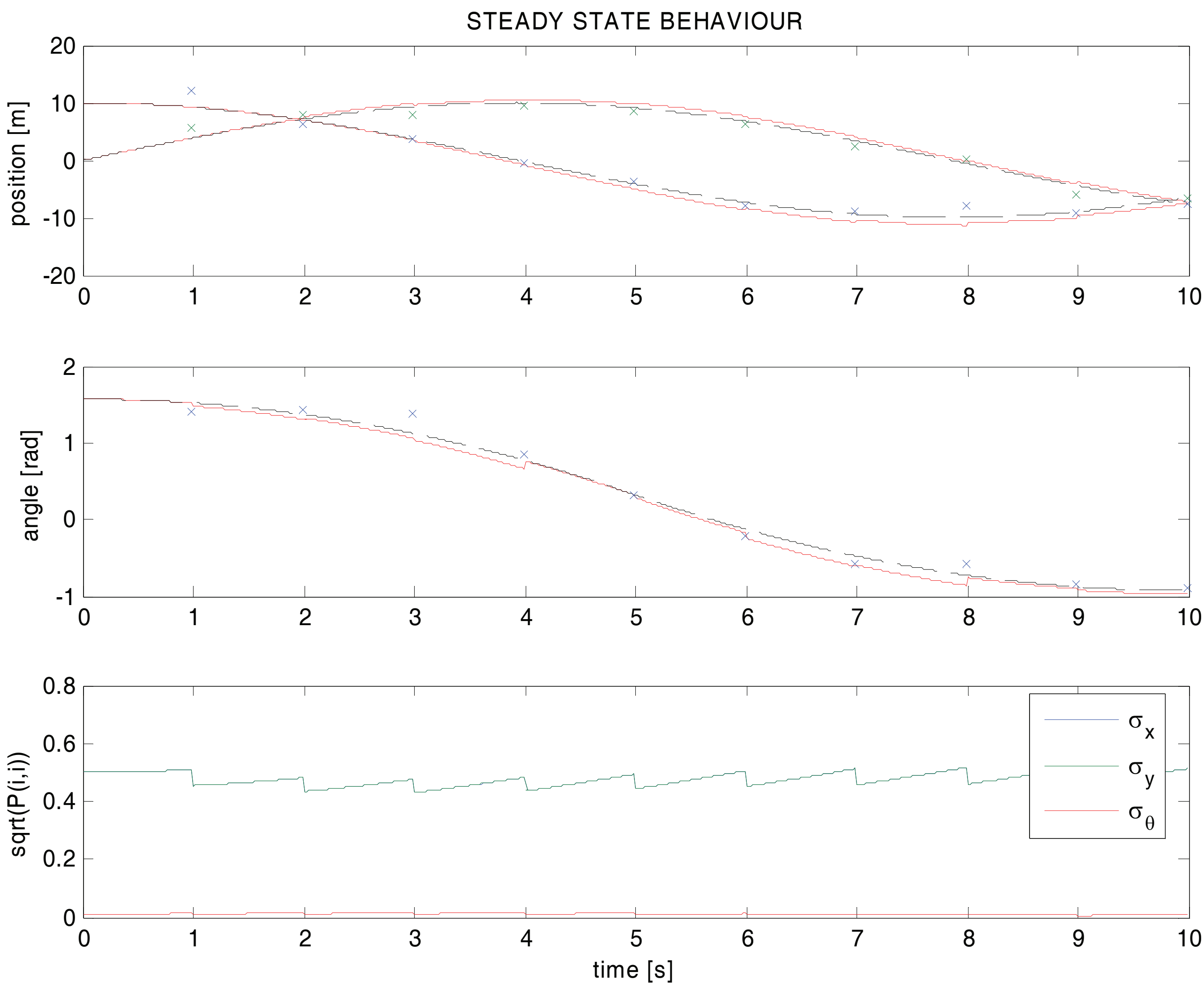

Fig. 3 – Steady response of filter. Solid lines: . Dashed lines: x. Position and angle measurements: 'x'

Initial ***P*** = diag {0.12, 0.12, 0.012, 0.52, 0.52, 0.012}