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A performance comparison of an EKF and a High Gain observer for an electropneumatic positioning system: Simulation and practical results

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Abstract: The paper deals with an evaluation and comparison of the estimation performance of two non-linear observers in simulation as well as in a practical application. The objectives are to underline the different parameters synthesis of both observers, the choice of the numerical solver and the difficulty to obtain good performances with an electropneumatic system. A High Gain Observer and an Extended Kalman Filter (EKF) synthesis have been performed with Matlab, compared in the same Simulink model and finally implemented in the system.

NOMENCLATURE

y, v, a	position, velocity and acceleration of the actuator $[m][m/s][m/s^2]$
p_X	pressure in the chamber X $[Pa]$
p_E	exhaust pressure$[Pa]$
u_P, u_N	servodistributors voltages $[V]$
k	polytropic constant
F_{ext}	external force $[N]$
V_X	chamber X volume $[m^3]$
b	viscous friction coefficient $[N/m/s]$
M	total moving load mass $[Kg]$
T_X	chamber X temperature $[K]$
r	perfect gaz constant $[J/kg/K]$
S_X	piston X area $[m^2]$
q_m	mass flow rate provided from the servo- -distributor. $[kg/s]$
X	P or N

1. INTRODUCTION

The position or force control of electropneumatic actuators is well-known to be difficult. The main difficulties in modelling pneumatic actuators are their highly nonlinear behaviors Mc Cloy [1968], Shearer [1956], Blackburn et al. [1960]. These ones are associated with the nonlinear dynamic properties of pneumatic systems such as servodistributor flow characteristics, the thermodynamic properties of air compressed in a cylinder and the nonlinear friction between the contacting surfaces of the slider-piston system. In such a context, previous works Richard and Scavarda [1996] have shown the efficiency of nonlinear controls. Several nonlinear approaches have been investigated: input/output linearization Richard and Scavarda [1996], sliding mode Bouri and Thomasset [2001], higher order sliding mode Laghrouche et al. [2004], backstepping Smaoui et al. [2004]. These works have been developed for both Single Input-Single Output (SISO) system (only position or force is controlled) and Multi Input-Multi Output (MIMO) system (control of position and one pressure, for example). For the application of such control laws, all state variables need to be known. In order to minimize the number of sensors but also to attenuate the noise injected in the controller, there is a real interest to design observers Girin et al. [2006]. This paper presents a simulation and experimental comparison of an EKF and high gain observer on a pneumatic system. It is organized as follows: Section 2 deals with the modelling of the pneumatic servosystem. Section 3 gives a proof of the observability of the system when only the position and the pressure in a single chamber are measured. Sections 4 and 5 respectively describe the design of a high gain observer and an extended kalman filter. Section 6 exhibits the results. A simulation and experimental comparison with both algorithms is finally carried out.

2. MODELLING

2.1 Dynamic model

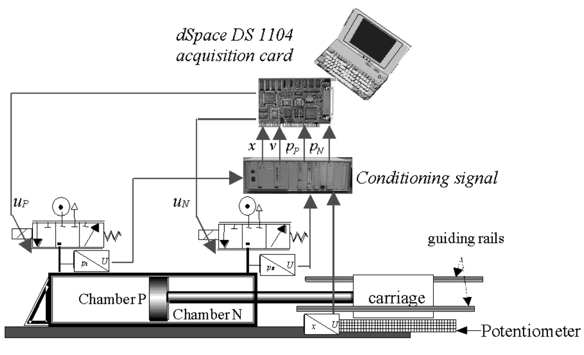

Fig. 1. The electropneumatic system

The electropneumatic system (figure 1) uses the following structure: two three-way proportional servodistributors/actuator/mass in translation. The actuator under consideration is an in-line electropneumatic cylinder using a simple rod. The electropneumatic system model can be obtained by using three physical laws: the mass flow rate through a restriction, the pressure behavior in a chamber with variable volume and the fundamental mechanical equations. The two servodistributors are supposed to be identical. This component is a pneumatic flow valve and consists of a matching spool-sleeve assembly and a proportional magnet directly controlling the movement of the spool against a spring. The spool is controlled in position by means of a position sensor. On the contrary of many other valve designs used in automotive or railway applications or in pneumatic circuits, the spool-sleeve technology has been preferred to the poppet technology. This means that pressure accuracy around zero opening has been set to the detriment of leakage. So this technology leads to characteristics without dead zone. In our case, the bandwidth of the Servotronic Joucomatic servodistributor and the actuator are respectively about 200 Hz and 1.5 Hz. Using the singular perturbation theory, the dynamics of the servodistributors are neglected and their models can be reduced to a static one described by two relationships $q_m(u_P, p_P)$ and $q_m(u_N, p_N)$ between the mass flow rate q_m , the input voltages u_P and u_N, and the output pressures p_P and p_N. The pressure and temperature evolution laws in a chamber with variable volume are obtained assuming the following assumptions Shearer [1956]: air is a perfect gas and its kinetic energy is negligible, the pressure and the temperature are homogeneous in each chamber. The temperature variations in each chamber can be neglected with respect to the supply temperature, *i.e.* $T_P = T_N = T$. Moreover the process is supposed to be polytropic and characterized by the coefficient k. The following electropneumatic system model is obtained:

$$\begin{cases} \dot{p}_P = \dfrac{krT}{V_P(y)}\left(q_m(u_P, p_P) - \dfrac{S_P}{rT}p_P v\right) \\ \dot{p}_N = \dfrac{krT}{V_N(y)}\left(q_m(u_N, p_N) + \dfrac{S_N}{rT}p_N v\right) \\ \dot{v} = \dfrac{1}{M}\left(S_P p_P - S_N p_N - F_{ext} - bv\right) \\ \dot{y} = v \end{cases} \tag{1}$$

Where:

$$F_{ext} = (S_P - S_N)\, p_{ext} \tag{2}$$

And:

$$\begin{cases} V_P(y) = V_P(0) + S_P y \\ V_N(y) = V_N(0) - S_N y \end{cases} \text{With} : \begin{cases} V_P(0) = V_{DP} + S_P \dfrac{l}{2} \\ V_N(0) = V_{DN} + S_N \dfrac{l}{2} \end{cases} \tag{3}$$

The main difficulty for model (1) is to know the mass flow rate $q_m(u_X, p_X)$. In order to establish a mathematical model of the power modulator flow stage, many works present approximations based on physical laws Araki [1981], Mo [1989] by the modelling of the geometrical variations of the restriction areas of the servodistributor as well as by the experimental local characterization Richard and Scavarda [1996]. These methods are based on approximations of fluid flow through a convergent nozzle in turbulent regime, corrected by a coefficient C_q Mc Cloy and Martin [1980] or on the norm ISO 6358. The authors in Belgharbi et al. [1999] have developed analytical models for both simulation and control purposes. In this paper we will used one of proposed models where the mass flow rate q_m is considered as an algebraic function:

$$q_m(u_X, p_X) = \varphi(p_X) + \psi(p_X, \text{sign}(u_X))\, u_X \tag{4}$$

$\varphi(p_X)$ in (4) is a 5$^{\text{th}}$ polynomial function whose evolution corresponds to the mass flow rate leakage, it is identical for all input control value u. $\psi(p_X, sign(u_X))$ is a 5$^{\text{th}}$ polynomial function whose evolution is similar to the one described by the methods based on approximations of mass flow rate through a convergent nozzle in turbulent regime Mc Cloy and Martin [1980]. It is a function of the input control *sign* because the behaviour of the mass flow rate characteristics is clearly different for the inlet ($u_X > 0$) and the exhaust ($u_X < 0$). For a discussion and more details on the choice of functions and their degrees please refer to Belgharbi et al. [1999]. For the sake of clarity, let k_i $(1 \le i \le 10)$ define as : $k_1 = krT\varphi(p_P)$; $k_2 = krT\psi(p_P, \text{sign}(u_P))$; $k_3 = -kS_P$; $k_4 = krT\varphi(p_N)$; $k_5 = krT\psi(p_N, \text{sgn}(u_N))$; $k_6 = kS_N$; $k_7 = \dfrac{S_P}{M}$; $k_8 = -\dfrac{S_N}{M}$; $k_9 = \dfrac{-b}{M}$; $k_{10} = \dfrac{-F_{ext}}{M}$

Then, the electropneumatic actuator is modelized through the following nonlinear system:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{V_P(x_4)}\left[k_1(x_1) + k_2(x_1, u_P)u_P + k_3 x_1 x_3\right] \\ \dot{x}_2 = \dfrac{1}{V_N(x_4)}\left[k_4(x_2) + k_5(x_2, u_N)u_N + k_6 x_2 x_3\right] \\ \dot{x}_3 = k_7 x_1 + k_8 x_2 + k_9 x_3 + k_{10} \\ \dot{x}_4 = x_3 \end{cases} \tag{5}$$

with $x = [x_1\ x_2\ x_3\ x_4]^T = [p_P\ p_N\ v\ y]^T \in \mathcal{X}$, and $u = [u_P\ u_N]^T \in \mathcal{U}$, $\mathcal{X} = \{x \in I\!R^4 \mid x_{min} \le x_i \le x_{MAX},\ 1 \le i \le 2,\ \ x_{imin} \le |x_i| \le x_{iMAX},\ 3 \le i \le 4\}$ and $\mathcal{U} = \{u \in I\!R^2 \mid |u_P| \le u_{MAX}, |u_N| \le u_{MAX}\}$. $x_{1,2min}$ and $x_{1,2MAX}$ are the minimum/maximum values of pressure in each chamber, x_{3min} and x_{3MAX} (resp. x_{4min} and x_{4MAX}) the minimum/maximum values of velocity (and position) of the rod actuator. u_{MAX} is the maximum value of servodistributors voltage.

3. OBSERVABILITY ANALYSIS

3.1 Preliminaries

Consider the nonlinear system:

$$\begin{array}{l} \dot{x} = f(x) + g(x)u \\ \tilde{y} = h(x) \end{array} \tag{6}$$

with $x \in \mathcal{X} \subset I\!R^n$ the state vector, $u \in \mathcal{U} \subset I\!R^m$ the input vector and $\tilde{y} \in I\!R^p$ the measurement vector, such that: $\tilde{y} = h(x) = [\tilde{y_1} \cdots \tilde{y_p}]^T = [h_1(x) \cdots h_p(x)]^T$.

Definition 1. System (6) is observable for $x \in \mathcal{X}$ and $u \in \mathcal{U}$ if there exists p integers l_i $(1 \le i \le p)$ such that

(1) $l_1 \ge l_2 \ge \cdots \ge l_p$,

(2) $\sum_{i=1}^{p} l_i = n$,

(3) The transformation

$$\Phi = \left[\tilde{y}_1 \; \dot{\tilde{y}}_1 \; \cdots \; \tilde{y}_1^{(l_1-1)} \; \cdots \; \tilde{y}_p \; \cdots \; \tilde{y}_p^{(l_p-1)} \right]^T \quad (7)$$

is a state transformation for $x \in \mathcal{X}$ and $u \in \mathcal{U}$. The integers $\{l_1 \dots l_p\}$ are called *observability indices.*

3.2 Application to the electropneumatic system

Consider the nonlinear system (5). Only the position of the actuator and the pressure in P chamber are measured, *i.e.* $\tilde{y} = [\tilde{y}_1 \;\; \tilde{y}_2]^T := [y \; p_P]^T = [x_4 \; x_1]^T$. State $l_1 = 3$ and $l_2 = 1$: items 1 and 2 of Definition 1 are fulfilled. It yields

$$\Phi = \begin{bmatrix} \tilde{y}_1 \\ \dot{\tilde{y}}_1 \\ \ddot{\tilde{y}}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_3 \\ k_7 x_1 + k_8 x_2 + k_9 x_3 + k_{10} \\ x_1 \end{bmatrix} \quad (8)$$

Consider its Jacobian matrix

$$\frac{\partial \Phi}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ k_7 & k_8 & k_9 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

This matrix is a full rank one for all $x \in \mathcal{X}$ and $u \in \mathcal{U}$, which implies that item 3 of Definition 1 is fulfilled. Then, system (5) is observable from the knowledge of only carriage position and pressure p_P.

Remark 2. With the previously considered output vector, system (5) is *observable for every input* $u(t)$, viewed that jacobian matrix (9) does not depend on $u(t)$.

Remark 3. The choice of output vector is not unique: for example, system (5) is also observable with measure of p_N instead of p_P.

4. HIGH GAIN OBSERVER

4.1 Preliminaries

The synthesis of this observer is based on uniform observability property of (6) Bornard and Hammouri [1991]. This class of observers is interesting due to its applicability to a large class of nonlinear systems, and its easy design. Suppose that system (6) is uniformly observable: then, there exists a state coordinate transformation (7) $\zeta = \Phi(x)$ such that system (6) is locally equivalent to:

$$\begin{aligned} \dot{\zeta} &= A\zeta + \Theta(\zeta, u) \\ \tilde{y} &= C\zeta \end{aligned} \quad (10)$$

with $A = \mathrm{diag}\,[A_1 \; \cdots \; A_p]_{n \times n}$, A_i $(1 \le i \le p)$ defined as:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{l_i \times l_i}, \quad (11)$$

$C = \mathrm{diag}\,[C_1 \; \cdots \; C_p]_{p \times n}$, $C_i = I_{1 \times l_i}$, $\Theta(\zeta, u) = [\Theta_1^T \; \Theta_2^T \; \cdots \; \Theta_p^T]^T$, and $\Theta_i = [0 \; \cdots \; 0 \; h_i^{(l_i-1)}]^T$. Furthermore, suppose that:

H1. The function Θ is globally Lipschitz with respect to ζ, uniformly with respect to u.

Let K denote a gain matrix such that $(A - KC)$ is Hurwitz, and $\Lambda(T) = \mathrm{diag}[\Lambda_1 \; \Lambda_2 \; \cdots \; \Lambda_p]^T$ with $\Lambda_i = \mathrm{diag}[T_i \; T_i^2 \; \cdots \; T_i^{k_i}]$, with $T_i > 0$. Then, system:

$$\dot{\hat{\zeta}} = A\hat{\zeta} + \Theta(\hat{\zeta}, u) + \Lambda^{-1} K (\tilde{y} - C\hat{\zeta}) \quad (12)$$

with $\hat{\zeta} \in I\!R^n$, is an asymptotic observer for (10). Furthermore, dynamics of this observer can be made arbitrarily fast through K and T_i. From (8)-(12), an observer for: (6) reads as

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + \left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}} \right]^{-1} \Lambda^{-1} K (\tilde{y} - h(\hat{x})) \quad (13)$$

4.2 Application to the electropneumatic system

Consider nonlinear system (5). Only the position of the actuator and the pressure in chamber are measured, *i.e.* $\tilde{y} = [\tilde{y}_1 \;\; \tilde{y}_2]^T := [y \; p_P]^T = [x_4 \; x_1]^T$. State $l_1 = 3$ and $l_2 = 1$ $(l_1 + l_2 = 4)$. Through state coordinates transformation (8), system (5) is equivalent to (with A_1 and A_2 defined by (11)):

$$\dot{\zeta} = \begin{bmatrix} A_1 & 0_{3\times1} \\ 0_{1\times3} & A_2 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ 0 \\ \varphi_1(\zeta, u) \\ \varphi_2(\zeta, u) \end{bmatrix} \; \tilde{y} = \begin{bmatrix} C_1 & 0_{1\times1} \\ 0_{1\times3} & C_2 \end{bmatrix} \zeta \quad (14)$$

with $\varphi_1 = k_7\beta_1 + k_8\beta_2 + k_9\zeta_2$, $\varphi_2 = \beta_1$,

$$\begin{aligned} \beta_1 &= \frac{1}{V_P(\zeta_1)} \left[k_1(\zeta_4) + k_2(\zeta_4, u_P)u_P + k_3\zeta_4\zeta_2 \right], \\ \beta_2 &= \frac{1}{V_N(\zeta_1)} \left[k_4(\zeta_3) + k_5(\zeta_3, u_N)u_N + \frac{k_6}{k_8}\zeta_2\zeta_3 \right. \\ & \left. \quad -\frac{k_6 k_7}{k_8}\zeta_2\zeta_4 - \frac{k_6 k_9}{k_8}\zeta_2^2 - \frac{k_6}{k_8}k_{10}\zeta_2 \right]. \end{aligned} \quad (15)$$

System (5) is locally uniformly observable. For $x \in \mathcal{X}$ and $u \in \mathcal{U}$, hypothesis H1 is fulfilled. Thus, from (12), an observer for (14) reads as:

$$\begin{aligned} \dot{\hat{\zeta}} &= \begin{bmatrix} A_1 & 0_{3\times1} \\ 0_{1\times3} & A_2 \end{bmatrix} \hat{\zeta} + \begin{bmatrix} 0 \\ 0 \\ \varphi_1(\hat{\zeta}, u) \\ \varphi_2(\hat{\zeta}, u) \end{bmatrix} \\ & + \Lambda^{-1} \begin{bmatrix} K_{11} & 0 \\ K_{12} & 0 \\ K_{13} & 0 \\ 0 & K_2 \end{bmatrix} \left(\tilde{y} - \begin{bmatrix} C_1 & 0_{1\times1} \\ 0_{1\times3} & C_2 \end{bmatrix} \hat{\zeta} \right) \end{aligned} \quad (16)$$

with $\Lambda^{-1} = \left[\begin{array}{ccc|c} T_1^{-1} & 0 & 0 & 0 \\ 0 & T_1^{-2} & 0 & 0 \\ 0 & 0 & T_1^{-3} & 0 \\ \hline 0 & 0 & 0 & T_2^{-1} \end{array} \right]$, $T_1 > 0$ and $T_2 > 0$.

5. EXTENDED KALMAN FILTER

5.1 Preliminaries

The extended Kalman filter has been broadly used in the areas of control as a state estimator for nonlinear stochastic systems Gelb [1984]. It is essentially a set of mathematical equations that allow to implement a estimator that is optimal in the sense that it minimizes the estimated error covariance. It is noteworthy that the conditions necessary for optimality rarely exist, and yet the filter apparently works well for many applications in spite of this situation. Consider the non-linear system:

$$\begin{array}{l} \dot{x} = f(x,u) + w \\ z = h(x,v) \end{array} \tag{17}$$

Here $x \in I\!R^n$ and $z \in I\!R^m$ respectively represent the state and measurement vector. u stands for the control and $f(x,u)$ denotes the nonlinear function describing the evolution of the system. The variables w and v represent the process and measurement noise, which are usually unknown in the practical application. The extended Kalman filter for the system (17) is given by:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x},u) + K_{EKF}(z - C\hat{x}) \\ K_{EKF} = PC^T R^{-1} \\ \dot{P} = JP + PJ^T + Q - PC^T R^{-1} CP \end{cases} \tag{18}$$

with appropriate initial values $\hat{x}_0$ and P_0 (initial value vector and error covariance matrix) as well as adequate matrices Q (process noise covariance matrix), C (measurement matrix) and R (measurement noise covariance matrix) one can show the Riccati equation admits a solution. K_{EKF} stands for the Kalman gain of the EKF. The matrix J denotes the jacobian matrix:

$$J = \frac{\partial}{\partial x} f(x,u) \tag{19}$$

The matrices P_0, Q and R represent the tuning parameters for the extended Kalman filter. As there is no general rule for the choice of these tuning parameters besides some restrictions as positive definiteness and an adequate dimension, they have to be adjusted experimentally. It is worth noting that in literature it is common to choose them as diagonal matrices.

5.2 Application to the electropneumatic system

The non-linear function $f(x,u)$ is given by:

$$\begin{cases} \dot{p}_P = \dfrac{krT}{V_p(y)}\Big[q_m(u_P,p_P) - \dfrac{S_P}{rT}P_P v\Big] \\ \dot{a} \;= \dfrac{1}{M}\Big[\dfrac{S_P krT}{V_P(y)}\Big(q_m(u_P,p_P) - \dfrac{S_P p_P v}{rT}\Big) - \dots \\ \dots \dfrac{S_N krT}{V_N(y)}\Big(q_m(u_N,p_N) + \dfrac{S_N}{rT}p_N v\Big) - ba\Big] \\ \dot{v} \;= a \\ \dot{y} \;= v \end{cases} \tag{20}$$

and the measurement used for the correction stage of Kalman's equation consists of position of the piston and the pressure in the gas chamber P:

$$z = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} p_P \\ y \end{bmatrix} \tag{21}$$

The jacobian matrix is given by:

$$J = \begin{bmatrix} J_{11} & 0 & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{22}$$

Where:

$$\begin{cases} J_{11} = \dfrac{krT}{V_P(y)}\Big[\dfrac{\partial q_m(u_P,p_P)}{\partial p_P} - \dfrac{S_P}{rT}v\Big] \\ J_{13} = -\dfrac{S_P k}{V_P(y)}p_P \\ J_{14} = -\dfrac{S_P krT}{(V_P(y))^2}\Big[q_m(u_P,p_P) - \dfrac{S_P}{rT}p_P v\Big] \\ J_{21} = \dfrac{S_P}{M}J_{11} - \dfrac{S_P}{M}\dfrac{krT}{V_N(y)}\Big[\dfrac{\partial q_m(u_N,p_N)}{\partial p_N} + \dfrac{v}{rT}\Big] \\ J_{22} = -\dfrac{krT}{V_N(y)}\Big[\dfrac{\partial q_m(u_N,p_N)}{\partial p_N} - \dfrac{S_N}{rT}v\Big] - \dfrac{b}{M} \\ J_{23} = -\dfrac{S_p^2 k}{MV_P(y)}p_P - \dfrac{S_N^2 k}{MV_N(y)}p_N + \dots \\ \dots \dfrac{S_N krT}{MV_N(y)}\Big[\dfrac{\partial q_m(u_N,p_N)}{\partial p_N} + bv\Big] \\ J_{24} = \dfrac{-S_N^2 krT}{M\,(V_N(y))^2}\Big[q_m(u_N,p_N) - \dfrac{S_N}{rT}p_N v\Big] - \dots \\ \dots \dfrac{S_P^2 krT}{M\,(V_P(y))^2}\Big[q_m(u_P,p_P) - \dfrac{S_P}{rT}p_P v\Big] \end{cases} \tag{23}$$

6. RESULTS

6.1 Simulation results

The objectives of this section are to underline the different parameters synthesis of both observers, the choice of the numerical solver and the difficulty to obtain good performances for an electropneumatic system. The High Gain Observer and EKF synthesis have been performed with Matlab and compared in the same Simulink model for a direct comparison of both observers.

High Gain Observer: specific tuning
The High Gain Observer needs to choose the matrix gain K:

- Choice of K_1. Dynamics connected to h_1 block is tuned by K_1,
$$K_1 = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{12} & 0 \\ 0 & 0 & K_{13} \end{bmatrix}$$
such that $(A_1 - K_1 C_1)$ is Hurwitz.
- Choice of K_2. Dynamic connected to h_2 block is fixed by K_2 such as $(A_2 - K_2 C_2)$ is Hurwitz. As $A_2 = 0$ and $C_2 = 1$, this condition is fulfilled if $K_2 > 0$.

With these choices for K_1 and K_2, and using the state transformation $\hat{x} = \Phi^{-1}(\hat{\zeta})$, an exponential observer for (5) reads as:

$$\begin{cases} \dot{\hat{x}}_1 \;= \dfrac{[k_1 + k_2 u_P + k_3\hat{x}_1\hat{x}_3]}{V_P(\hat{x}_4)} + T_2^{-1}K_2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 \;= \dfrac{[k_4 + k_5 u_N + k_6\hat{x}_2\hat{x}_3]}{V_N(\hat{x}_4)} + (T_1^{-3}K_{13}/k_8 \\ \qquad -K_{12}\dfrac{k_9}{k_8}T_1^{-2})(x_4 - \hat{x}_4) - K_2\dfrac{k_7}{k_8}T_2^{-1}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 \;= k_7\hat{x}_1 + k_8\hat{x}_2 + k_9\hat{x}_3 + k_{10} + T_1^{-2}K_{12}(x_4 - \hat{x}_4) \\ \dot{\hat{x}}_4 \;= \hat{x}_3 + T_1^{-1}K_{11}(x_4 - \hat{x}_4) \end{cases} \tag{24}$$

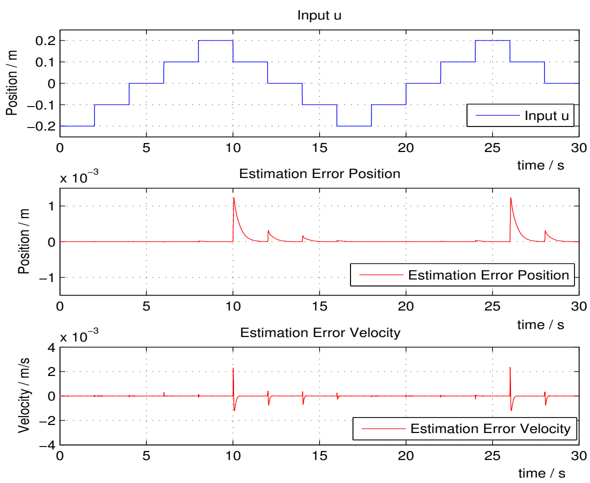

Fig. 2. EKF: Desired position, position and velocity estimation errors

EKF: specific tuning

For the EKF the matrices Q, R^{-1} and P_0 were chosen in order to ensure their positive definition (diagonal matrices). Another aspect of the EKF is that the jacobian matrix J has to be calculated in real-time. Because of the complexity of the model given by (20) and (4) and in order to ensure the correctness of the derivation of the vector field $f(x, u)$ with respect to the state vector x, the calculation of J (23) has been done and checked with the mathematic software *Maple*.

High Gain Observer and EKF: other parameters tuning

The initial value for the state vector is the same in both cases. Another aspect concerns the numerical solving. For the electropneumatic system the state vector contains the position measured in m and pressures measured in Pa, which corresponds to a difference roughly of eight power of ten. The numerical gap between these variables leads in practice to an ill-conditioning matrix J which causes numerical problems in the resolution of the Riccati equation in (18). To overcome this problem, variable step-size solvers like "ode 23s(stiff/Rosenbrock)" either "ode 15s (stiff/NDF)" either "ode 23t (Mod.stiff/Trapezoidal)" or "ode 23tb (stiff/TR-BDF22)" were used to simulate the EKF. Tests apply only on the High Gain observer simulation revealed that its performances do not depend very much on the solver chosen and the same simulation results were obtained using a more simple solver algortihm such as "ode 1(Euler)" with a fixed step size.

Discussion of the simulation results

The simulation results of the EKF and High Gain Observer are given on Figures 2, 3, 4 and 5. Simulation results show the High Gain observer improves the position estimation by almost two powers of ten with respect to the EKF, but it is worth the other state variable (Figures 2 and 4). However the pressure, velocity and acceleration estimations given by the EKF are better than those obtained with the High Gain observer. Whatever the method used to estimate the state variables, the performances depend on the choices of tuning parameters such as initial values, weighting matrices, the matrix gain K, etc.. At this stage,

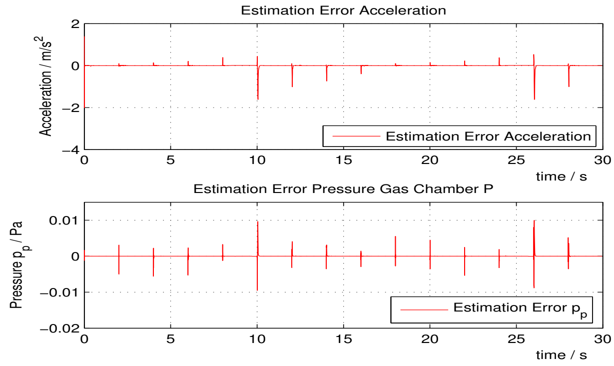

Fig. 3. EKF: Acceleration and pressure estimation errors

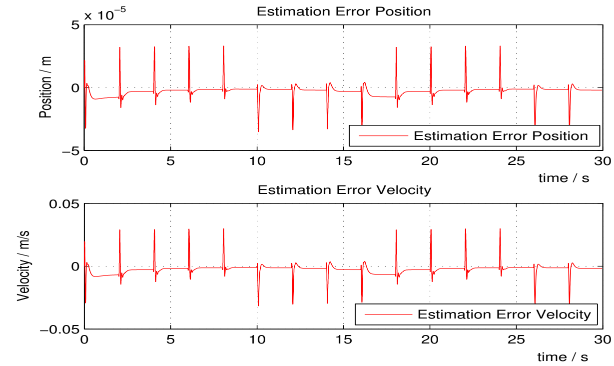

Fig. 4. HG Observer: Position and velocity estimation errors

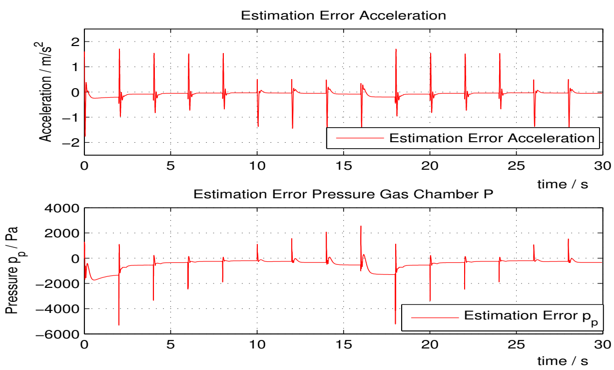

Fig. 5. HG Observer: Acceleration and pressure estimation errors

different choices could be made by the user and the time spent on tuning the observers must be of course stand in proportion to the expected performances. However, one can underline the difficulty to correctly tune the EKF, especially the well choice of the weighting matrices, to achieve desired performance.

6.2 *Practical results*

Because of the dSPACE card used in practice does not support variable step size solvers, the implementation of the extended Kalman filter failed in spite of a lot of choices of tuning parameters. Concerning the implementation of the High Gain observer, the same tuning parameters were chosen. Practical results are shown in Figures 6 and 7.

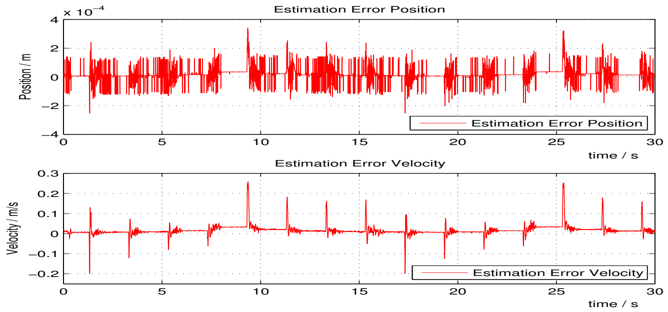

Fig. 6. HG Observer: Practical position and velocity estimation errors

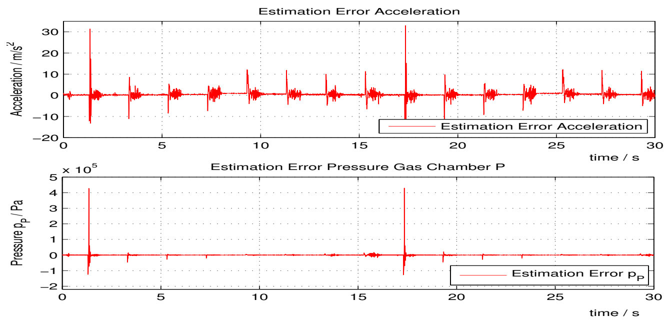

Fig. 7. HG Observer: Practical acceleration and pressure estimation errors

The convergence time of the high-gain observer is difficult to determine exactly, as the estimation error curves, especially the position error plot, are quite noisy. From the velocity, acceleration and pressure plots we can estimate the convergence time around $0.6\,s$. This result is greater compared to the simulation result, where it only took about $0.4\,s$. Experimental results show that the estimation errors of the state variables are suitable (Figures 6 and 7). The estimation errors obtained in practice are slightly more important than those obtained in simulation. The differences can come from modelling errors, the optimal choice of the tuning parameters and the presence of noise.

7. CONCLUSION

The objective of this paper was to evaluate and compare the estimation performance of two non-linear observers in simulation as well as in a practical application. In the simulation the EKF and the High Gain Observer have quite similar results. Whatever the method used to estimate the state variables, the performances depend on the choices of tuning parameters such as initial values, weighting matrices, the matrix gain K, etc.. At the stage of the simulation, different choices could be made by the user and the time spent on tuning the observers must be of course stand in proportion to the expected performances. However, one can underline the difficulty to correctly tune the EKF, especially the well choice of the weighting matrices, to achieve desired performance. In practice, some other difficulties rise and the simulation results should be viewed with some degree of reservation because the EKF could not be implemented in real-time on the actual system. The implementation of the High Gain Observer revealed satisfactory results. Nevertheless one may get an idea of the real performance of the EKF when examining these comparison results because some other simulations were made with additive measurement noise on one of the measured variables. Here the good behavior of the EKF compared to the High Gain Observer is clear. More investigations should be adressed in order to check the robustness of both observers with respect to this aspect.

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