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Adaptive Observer and Kalman Filtering

Khadidja Benzemrane * **Gilney Damm** **
Giovanni L. Santosuosso ***

* *IBISC - CNRS/Université d'Evry Val d'Essonne, Evry, France (e-mail: khadidja.benzemrane@ibisc.fr)*
** *IBISC - CNRS/Université d'Evry Val d'Essonne, Evry, France (e-mail: gilney.damm@ibisc.fr)*
*** *Dipartimento di Ingegneria Elettronica, Universitá di Roma Tor Vergata, Via del Politecnico 1, Rome, Italy (e-mail: santosuosso@ing.uniroma2.it)*

Abstract: In this paper the problem of the speed estimation of an Unmanned Aerial Vehicle is addressed, when only the standard outputs (acceleration, angles and angular speeds) are available for measurement. We focus our analysis on a prototype drone - a 4 rotors helicopter robot- which is not equipped with GPS related devices and relies on the Inertial Measurement Unit (IMU) only. Two different approaches have been compared. The first one uses a classical method based on Kalman Filtering while the second solution is provided in the frame of adaptive observation theory. These estimators have been tested in two situations : when exact measurements are available and in the more realistic case of noisy acceleration measurements.

1. INTRODUCTION

The problem of speed estimation plays an important role in the context of vehicles control. For land moving robots, the odometer time derivative has a satisfactory performance while for large flying airships (manned or unmanned) velocity estimations can be obtained via approximate derivation of the successive measurements from GPS sensors, motivated by the small resulting errors compared to the measured variables. For fast aircrafts the standard procedure is integrating the acceleration and coupling this result to the derivative of GPS measurements.

Two critical issues arise in an "open loop" strategy like direct acceleration integration: an unknown constant estimation error is produced even when exact acceleration measurements are available while a random drift is induced by noisy acceleration estimations. In practice, numerical integration along with measurement noise induces a very fast growing velocity measurement error. Thus the information obtained from the GPS is used to bound this error in the framework of sensor fusion technique where initial conditions for acceleration integration are provided by GPS devices. This technique provides bounded errors that are related to the GPS order of precision. The resulting errors are usually small compared to the size of airships and the distance with respect to obstacles. In the same way, the control systems usually applied on these airships are robust enough to accept the residual disturbance on the speed estimation provided by these methods.

Unfortunately, the above estimation approach cannot be implemented on small drones less than 1 meter wide and flying at low speed that we consider in this note. This issue is even more relevant in indoors or simply urban applications. As a matter of fact, errors induced by a GPS system may reach many meters; a technical solution would be the use of a D-GPS system. These systems are known as centimetric GPS and have a precision of some centimetres, but they also are very expensive and low-range operating equipment. Another scheme is to use Doppler measurements coupled with GPS, that can actually increase velocity estimation accuracy.

In this work we focus our attention on the case when no GPS is available while Inertial Measurement Units (IMU) can be used. IMU actually represent a feasible choice: they are easily available, low cost devices. More important, they have a low size, limited weight, and provide very good airship measurements of accelerations, angles and angular speeds. However, the information provided by these devices needs to be processed to yield reliable velocity estimates. The latter quantity is necessary for an efficient drone control design, since most literature about small flying robots control assumes that at least the speeds are available (see Castillo et al. [2004], Guenardt et al. [2005], Pounds et al [2002]). This is motivated by the fact that is hardly possible for a human pilot (inboard or in tele-operation) to regulate a drone by giving references and trajectories as inputs based on the sole acceleration measurement. Notice that in the general case, *the reliable estimation of the speed vector is still an open problem.* Recently, control system and robotics communities have regained interest for the development of observers applied to UAVs due to the important developments of embedded electronics and micro-controllers. This technological improvement has motivated the testing of more sophisticated algorithms implemented in real time. This recent interest may be illustrated by recent publications as Guenardt et al. [2007], Bonnabel et al. [2006], Zhao et al. [2005]and Vik et al. [2001].

Motivated by previous arguments, this note compares an observation strategy that solves the problem of the speed estimation of an Unmanned Aerial Vehicle, when the linear acceleration, the angles and the angular speeds are

available for measurement with a more classical estimator based on Kalman filtering (see Astrom et al. [1997]). The considered observer is based on robust nonlinear adaptive techniques as Marino et al. [1995] and Damm et al. [2004]. This strategy yields a satisfactory performance in a number of similar cases and for this reason has been chosen as first method. The authors believe that a strongly nonlinear and fast system as a drone must be treated by nonlinear techniques, while the number of disturbances and noisy measurements, as well as possibly different and time varying equilibrium points indicate the use of robust and adaptive techniques. Other techniques as variable structure observers and other high gain schemes should be considered in latter works, though the presence of noisy measurements may be a problem for those strategies.

In the present work, we focus our analysis on a prototype drone - a 4 rotors helicopter robot shown in Fig. 1 - produced to operate in an urban environment at the IBISC Laboratory - CNRS, University of Evry, which is not equipped with GPS related devices and relies on the Inertial Measurement Unit only (see Pradel et al. [2007]). The observation strategy final goal is to provide reliable estimations in order to design the robot's stabilizing control. The simulation results obtained on the model of the real drone in the presence of noise allows us to compare performances and the robustness of the observers.

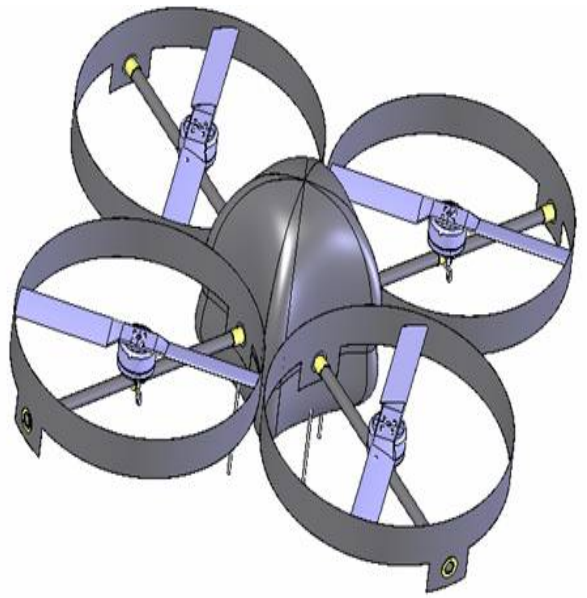

Fig. 1. Representation of the UAV

2. PROBLEM FORMULATION

The motion of a four rotor aerial robot can be described by the following equations (see Azouz et al. [2007]).

$$\begin{aligned}
\dot{x} &= cos\theta\ cos\psi\ u \\
&\quad + (sin\phi\ sin\theta\ cos\psi\ - cos\phi\ sin\psi)\ v \\
&\quad + (cos\phi\ sin\theta\ cos\psi\ + sin\phi\ sin\psi)\ w \\
\dot{y} &= cos\theta\ sin\psi\ u \\
&\quad + (sin\phi\ sin\theta\ sin\psi\ + cos\phi\ cos\psi)\ v \\
&\quad + (cos\phi\ sin\theta\ sin\psi\ - sin\phi\ cos\psi)\ w \\
\dot{z} &= -sin\theta\ u + sin\phi\ cos\theta\ v + cos\phi\ cos\theta\ w
\end{aligned} \tag{1}$$

$$\begin{aligned}
\dot{\phi} &= p + (sin(\phi)\ q + cos(\phi)\ r)\ tan(\theta) \\
\dot{\theta} &= cos(\phi)\ q - sin(\phi)\ r \\
\dot{\psi} &= (sin(\phi)\ q + cos(\phi)\ r)\ cos(\theta)^{-1}
\end{aligned} \tag{2}$$

$$\begin{aligned}
I_{xx}\dot{p} &= -l_b k_T(\omega_1^2\ cos\beta_1 - \omega_3^2\ cos\beta_3) - (I_{zz} - I_{yy})rq \\
&\quad -qI_r(\omega_1\ cos\beta_1 + \omega_2 + \omega_3\ cos\beta_3 + \omega_4) \\
I_{yy}\dot{q} &= l_b k_T(\omega_2^2 - \omega_4^2) - (I_{xx} - I_{zz})rp \\
&\quad -rI_r(\omega_1 sin\beta_1 + \omega_3 sin\beta_3) \\
&\quad +pI_r(\omega_1 cos\beta_1 + \omega_2 + \omega_3 cos\beta_3 + \omega_4) \\
&\quad +k_M(\omega_3^2 sin\beta_3 - \omega_1^2 sin\beta_1) \\
I_{zz}\dot{r} &= -l_b k_T(\omega_1^2 sin\beta_1 - \omega_3^2 sin\beta_3) - (I_{yy} - I_{xx})pq \\
&\quad +qI_r(\omega_1 sin\beta_1 + \omega_3 sin\beta_3) \\
&\quad +k_M(\omega_3^2 sin\beta_3 + \omega_4^2 - \omega_1^2 sin\beta_1 - \omega_2^2)
\end{aligned} \tag{3}$$

$$\begin{aligned}
\dot{u} &= (-qw + rv - g\ sin\theta) - \frac{k_T}{m}(\omega_1^2\ sin\beta_1 + \omega_3^2\ sin\beta_3) \\
\dot{v} &= (-ru + pw + g\ sin\phi\ cos\theta) \\
\dot{w} &= (-pv + qu + g\ cos\phi\ cos\theta) \\
&\quad -\frac{k_T}{m}(\omega_1^2\ cos\beta_1 + \omega_2^2 + \omega_3^2\ cos\beta_3 + \omega_4^2)
\end{aligned} \tag{4}$$

where $\eta_1 \triangleq [x\ \ y\ \ z]^T$ is the position vector represented in the global reference frame, $\eta_2 \triangleq [\phi\ \ \theta\ \ \psi]^T$ is the Euler angles vector represented in the global reference frame (roll pitch and yaw respectively), $\nu_1 \triangleq [u\ \ v\ \ w]^T$ is the speed vector represented in the local reference frame (surge, sway and heave respectively) and $\nu_2 \triangleq [p\ \ q\ \ r]^T$ is the angular speed vector represented in the local reference frame. In this model, the control input vector is $\Omega = [\omega_1\ \ \omega_2\ \ \omega_3\ \ \omega_4\ \ \beta_1\ \ \beta_3]^T$ where $\omega_i, \quad i = 1 \ldots 4$, are the angular speed of the 4 rotors, and β_1 and β_3 represent the orientation of the rotors 1 and 3.

Constants k_T and k_M, respectively, relate rotor speeds along with resulting thrust and torque; l_B is the lenght of each drone's arm and I_r is the rotor's inertia moment constant.

The paper task is to compare the performances of two different observers designed to provide estimation for the unmeasurable state variables ν_1, based on the measurable variables η_2, $\dot{\nu}_1$ and ν_2 given by the standard sensors embedded in the drone.

The two considered observers are respectively an adaptive observer designed to be robust with respect to measurement noise and a more classical estimation strategy based on Kalman filtering.

The interest in introducing a tuning parameter of the adaptive observer, to enhance the quality of the estimation, will be illustrated by simulation results.

3. ROBUST ADAPTIVE OBSERVER

In this section, we describe the estimation strategy presented in Benzemrane et al. [2007], following the techniques presented in Marino et al. [2004], as well as an outline of its stability proof.

First, the system (4) can be re-written as :

$$\begin{aligned} \dot{\nu}_1(t) &= A(t)\nu_1(t) + b(t) \\ y &= \dot{\nu}_1 \end{aligned} \tag{5}$$

where A, b and y are defined as

$$A(t) = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}$$

$$b(t) = \begin{bmatrix} -g\, sin\theta - \frac{k_T}{m}(\omega_1^2\, sin\beta_1 + \omega_3^2\, sin\beta_3) \\ g\, sin\phi cos\theta \\ g\, cos\phi\, cos\theta - \frac{k_T}{m}(\omega_1^2\, cos\beta_1 + \omega_2^2 + \omega_3^2\, cos\beta_3 + \omega_4^2) \end{bmatrix}$$

$$y(t) = \dot{\nu}_1 = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix}$$

In order to quantify the observer performance in case of measurement noise on the acceleration signals, we propose to take into account an additional measurement noise. We assume that a disturbed acceleration a_μ is available; $a_\mu \triangleq \dot{\nu}_1 + \mu(t)$, where the vector $\mu \in R^3$ represents the measurement noise such that (defining $\mu_M \in R^+$):

$$\|\mu\| \leq \mu_M$$

Consider the cascaded filters with matrix states $M_\mu \in R^3 \times R^3$, $Q_\mu \in R^3 \times R^3$, with arbitrary initial conditions (with $|Q_\mu(0)| > 0$) defined as :

$$\dot{M}_\mu = -\bar{\alpha} M_\mu + A(t) \tag{6}$$

$$\dot{Q}_\mu = -\beta Q_\mu - \frac{k}{4} Q_\mu Q_\mu^T Q_\mu + M_\mu^T M_\mu \tag{7}$$

where $\bar{\alpha} \in R^+$ is such that $\bar{\alpha} = \alpha + \frac{k}{4}$, $\beta, k \in R^+$ are tuning parameters chosen by the designer, along with the two vector systems with state $\rho_\mu \in R^3$, $\delta_\mu \in R^3$

$$\begin{aligned} \dot{\rho}_\mu &= -\bar{\alpha}\, \rho_\mu + A(t)\nu_1 + \mu + M_\mu a_\mu \\ &= -\bar{\alpha}\, \rho_\mu + a_\mu - b(t) + M_\mu a_\mu \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{\delta}_\mu &= -\beta\delta_\mu + \rho_\mu \\ &\triangleq -\beta\delta_\mu + \tilde{z}_\mu + M_\mu \nu_1 \end{aligned} \tag{9}$$

with arbitrary initial conditions.

In fact, by setting $\tilde{z}_\mu = \rho_\mu - M_\mu \nu_1$, we now have :

$$\dot{\tilde{z}}_\mu = -\bar{\alpha}\, \tilde{z}_\mu + (M_\mu + I)\mu \tag{10}$$

We introduce an estimate of the linear velocity

$$\hat{\nu}_1 = \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix}$$

along with a filter vector state $\xi_\mu \in R^3$ that satisfy the differential equations :

$$\begin{aligned} \dot{\xi}_\mu = &- \beta\xi_\mu - \frac{k}{4} Q_\mu Q_\mu^T \xi + Q_\mu a_\mu - \dot{M}_\mu{}^T \delta_\mu \\ &- \frac{k}{4} Q_\mu Q_\mu^T M_\mu^T \delta_\mu \end{aligned} \tag{11}$$

$$\dot{\hat{\nu}}_1 = \gamma(-Q_\mu \hat{\nu}_1 + M_\mu^T \delta_\mu + \xi_\mu) + a_\mu \tag{12}$$

with $\gamma \in R^3$ tuning parameter to be chosen by the designer

By setting the estimation error variables $\tilde{\nu}_1 \triangleq \nu_1 - \hat{\nu}_1$ and $\chi_\mu \triangleq Q\nu_1 - M_\mu^T \delta_\mu - \xi_\mu$, we obtain :

$$\dot{\chi}_\mu = -(\beta + \frac{k}{4} Q_\mu Q_\mu^T)\chi_\mu - M_\mu^T \tilde{z}_\mu - Q_\mu \mu \tag{13}$$

$$\dot{\tilde{\nu}}_1 = \gamma(-Q_\mu \tilde{\nu}_1 + \chi_\mu) - \mu \tag{14}$$

From (14) along with (10) and (13), and if matrix Q is positive definite, we may guarantee the exponential convergence of estimation error. This result is based on the assumption:

Hypothesis 1. There are positive integers T, K_1^* and K_2^* such that

$$K_1^* I \geq \int_t^{t+T} A(\tau)^T A(\tau) d\tau \geq K_2^* I \qquad \text{for all } t \in \mathbf{R}$$

Remark 1. Hypothesis 1 is a mathematical formulation of the properties that A(t) is bounded and persistently exciting (PE). The first is easily obtained for a physical system as the drone, while the second is obtained if the UAV tracks a trajectory which is in practice a small periodic orbit. Besides, airships dynamics provides vibrations in real applications, that may already give persistency of excitation.

Proposition 1. Consider the UAV model described by (1)-(4). Under Hypothesis 1, if the variables η_2, ν_2, a_μ are available for measurement, the dynamic observer described by (6), (7), (8), (9) with state $M_\mu(t) \in R^3 \times R^3$, $Q_\mu(t) \in R^3 \times R^3$, $\rho_\mu(t) \in R^3$, $\delta_\mu(t) \in R^3$, $\hat{\nu}_1(t) \in R^3$, is such that for any given $\epsilon > 0$, there exist a suitable combination of the tuning parameters yielding $\mathcal{L}_2$ and $\mathcal{L}_\infty$ gains from the measurement error μ to the estimations error $(\nu_1 - \hat{\nu}_1)$ smaller than ϵ, for any initial conditions $M_\mu(0) \in \mathrm{R}^3 \times \mathrm{R}^3$, $Q_\mu(0) \in \mathrm{R}^3 \times \mathrm{R}^3$, $\rho_\mu(0) \in \mathrm{R}^3$, $\delta_\mu(0) \in \mathrm{R}^3$, $\hat{\nu}_1(0) \in \mathrm{R}^3$ with $|Q_\mu(0)| > 0$.

Proof 1. Stability can be shown by following standard Lyapunov arguments.

$$V = \frac{1}{2}\kappa_1\, \tilde{\nu}_1^T \tilde{\nu}_1 + \frac{1}{2}\kappa_2\, \chi_\mu^T \chi_\mu + \frac{1}{2}\kappa_3\, \tilde{z}_\mu^T \tilde{z}_\mu \tag{15}$$

By computing the time derivative of V, we have :

$$\begin{aligned}\dot{V} = &-\kappa_1\gamma\tilde{\nu}_1^T Q_\mu \tilde{\nu}_1 - \kappa_2\beta\chi_\mu^T\chi_\mu - \kappa_2\frac{k}{4}\chi_\mu^T Q_\mu Q_\mu^T\, \chi_\mu \\ &- \kappa_3(\alpha + \frac{k}{4})\tilde{z}_\mu^T\tilde{z}_\mu + \kappa_1\gamma\tilde{\nu}_1^T\chi_\mu - \kappa_2\chi_\mu^T M_\mu^T\tilde{z}_\mu \\ &- \kappa_1\tilde{\nu}_1^T\mu - \kappa_2\chi_\mu^T Q_\mu\mu + \kappa_3\tilde{z}_\mu^T(M_\mu + I)\mu\end{aligned}$$

As stated in Hypothesis 1, matrix Q_μ must be positive definite. Defining now the constants $|\lambda_{max}(M_\mu)| = c_1$, $\lambda_{min}(Q_\mu) = c_2$, we obtain :

$$\begin{aligned}\dot{V} \leq &-\kappa_1\gamma c_2 \left\|\tilde{\nu}_1\right\|^2 - \kappa_2\beta\left\|\chi_\mu\right\|^2 - \kappa_3\alpha\left\|\tilde{z}_\mu\right\|^2 \\ &+\kappa_1\gamma\frac{c_2}{2}\left\|\tilde{\nu}_1\right\|^2 + \kappa_1 k\left\|\tilde{\nu}_1\right\|^2 + \frac{\kappa_1\gamma}{2c_2}\left\|\chi_\mu\right\|^2 \\ &+\frac{\kappa_2\beta}{4}\left\|\chi_\mu\right\|^2 + \frac{c_1^2\kappa_2}{\beta}\left\|\tilde{z}_\mu\right\|^2 + \frac{\kappa_3}{k}(c_1+1)^2\left\|\mu\right\|^2 \\ &+\frac{\kappa_2}{k}\left\|\mu\right\|^2 + \frac{\kappa_1}{k}\left\|\mu\right\|^2\end{aligned} \tag{16}$$

By choosing :

$$\begin{aligned}\kappa_1 &= \frac{1}{\gamma} \\ \kappa_2 &= \frac{2}{\beta c_2} \\ \kappa_3 &= \frac{2\kappa_2 c_1^2}{\beta\alpha} = \frac{4c_1^2}{\beta^2\alpha c_2}\end{aligned}$$

and by setting the design condition $\gamma > \frac{2k}{c_2}$, we may finally state that for a suitable $\alpha_i \in \mathrm{R}^+$, equation (16) can be expressed as:

$$\dot{V} \leq -\alpha_1\left\|\tilde{\nu}_1\right\|^2 - \alpha_2\left\|\chi_\mu\right\|^2 - \alpha_3\left\|\tilde{z}_\mu\right\|^2 + \frac{\alpha_4}{k}\left\|\mu\right\|^2 \tag{17}$$

which guarantees arbitrary $\mathcal{L}_\infty$ and $\mathcal{L}_2$ robustness from the measurement error μ to the estimation errors $\tilde{\nu}_1$, χ_μ and $\tilde{z}_\mu$.

Remark 2. An interesting issue arises from previous arguments. The problem of estimating linear velocities from accelerations is known as very difficult in terms of the existence of observability. As a matter of fact, the linearization of this system around the equilibrium point is non-observable.

The adaptive observer requires the classical persistence of excitation condition to obtain correct estimations. It is interesting to remark that to fulfil this condition automatically implies in the existence of observability of the linearized system. This is the reason that we may apply other observers like the Extended Kalman Filter (see Makhloufi [2007]) for this system since we use a trajectory that satisfies the same condition. The persistence of excitation condition reflects then the characteristics of observability of the system.

4. SIMULATION RESULTS

In this section we illustrate the behaviour of the presented observer designed to estimate the linear velocity of an UAV based on the measurable angles, angular velocities and linear accelerations. These simulations also compare it to an Extended Kalman Filter considered here as a benchmark.

In all simulations, the observer estimations have initial conditions set to zero, while the desired states are time varying (and different from zero at $t = 0$). In all simulations, both observers track the desired time varying states, with different performances and noise rejections. We have used the following parameters values:

$m = 2.500Kg$	$k_T = 10^{-5}N.s^2$
$l_b = 23cm$	$I_R = 10010^{-7}Kgm^2$
$I_{xx} = 22493110^{-7}Kgm^2$	$I_{yy} = 22261110^{-7}Kgm^2$
$I_{zz} = 32513010^{-7}Kgm^2$	$k_M = 910^{-5}Ns^2m$
$u_G = 0.032m$	

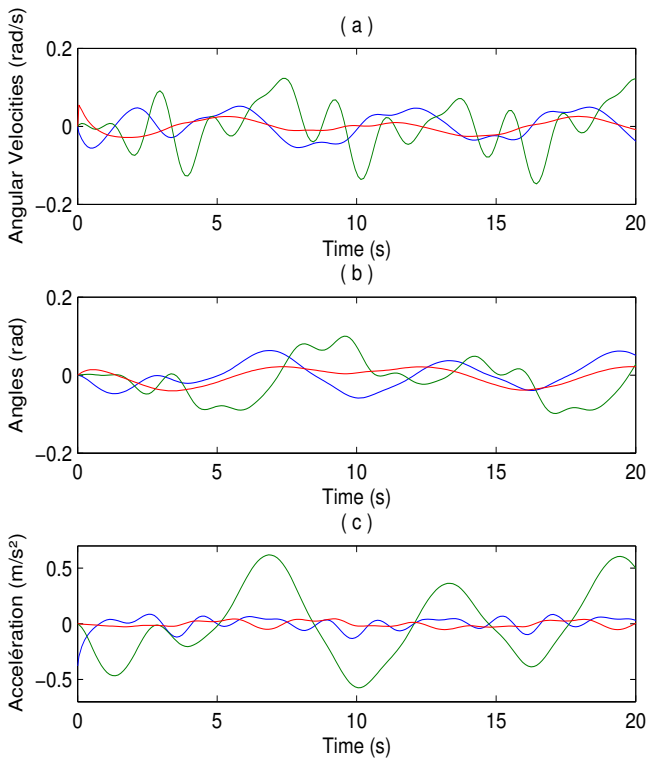

Fig. 2. (a)Angular Speed, (b)Angles, (c) Acceleration

In Fig. 2 the angular speeds, the angles, and the accelerations are plotted. As shown above, the persistency of excitation condition is needed to guarantee the observability of the system. Small periodic orbits were considered in order to satisfy the persistency of excitation condition.

We have then considered additive measurement noise. The measured acceleration is presented in Fig. 3, while Fig.

4 shows the velocity that has to be estimated by the observer.

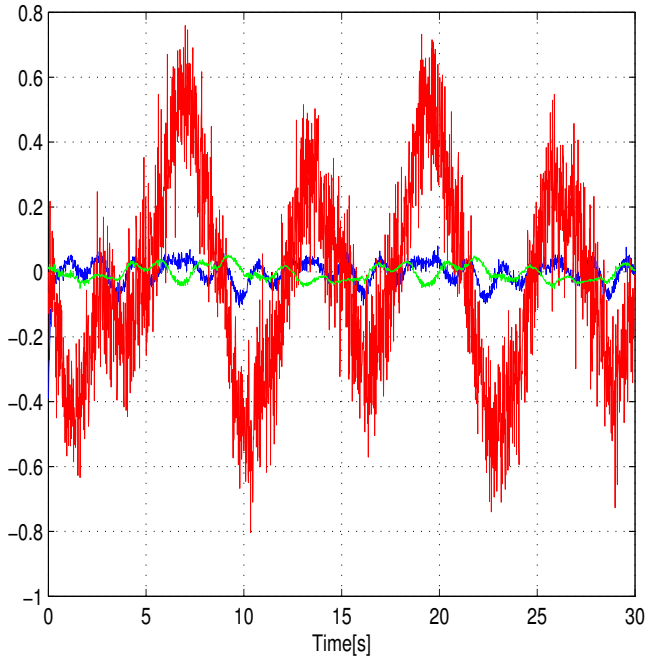

Fig. 3. Acceleration

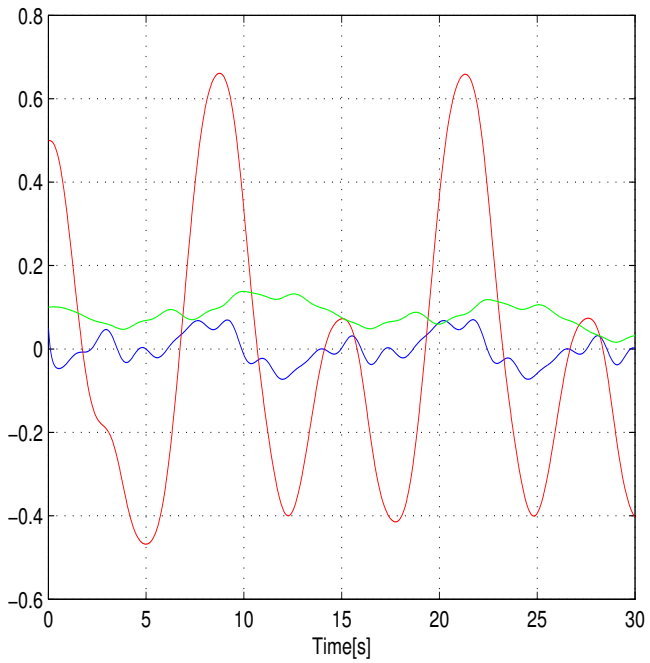

Fig. 4. Linear velocity to estimate

The first simulations are undertaken using the adaptiv observer, where the disturbance is attenuated followin the design parameter k. To describe the effect of thi parameter, we present two simulations using different values of k. In Fig. 5, we show the results of the simulations using $k = 0$ and $k = 50$. The main effects of larger values of k are a greater attenuation, in trend of a slower convergence rate. These results are in accordance with our claims, and illustrate the mechanisms of the tuning procedure.

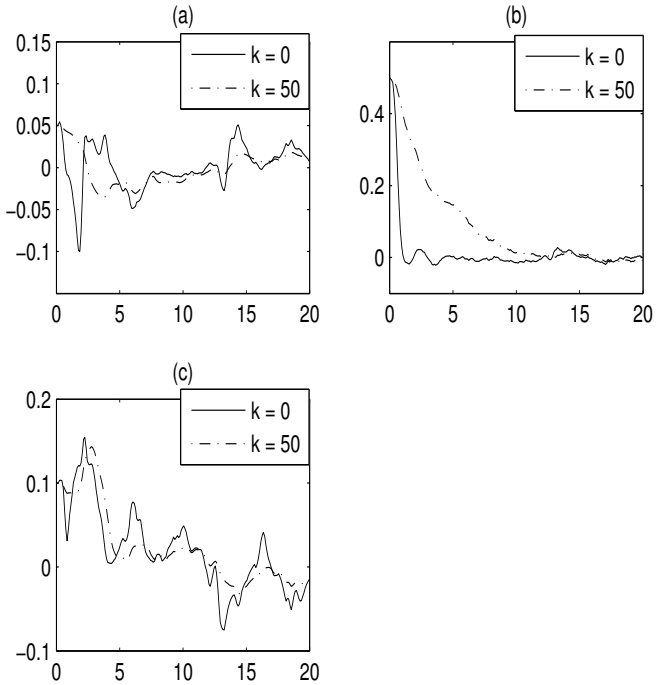

Fig. 5. Estimated values of u (a), v (b) and w (c) depending on k

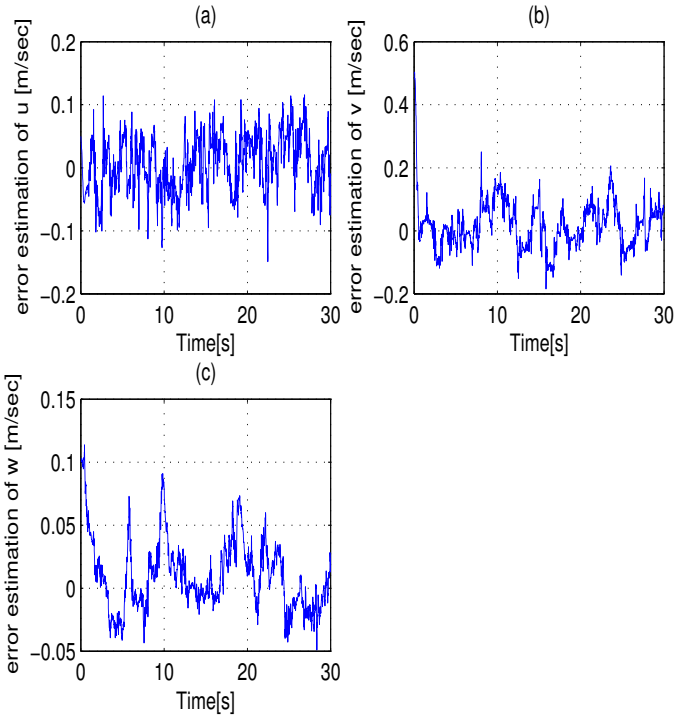

Fig. 6. Estimated values of u (a), v (b) and w (c) using the kalman filter

Comparing fig. 5 and 6, we can see that the estimation given by the adaptive observer is better than the Kalman filter estimation for the noisy acceleration considered in this section (see figure 3). The adaptive observer seems to be less sensitive to measurement noise than the Kalman filter. In fact, even when the value of the tuning parameter k is equal to zero (no parameter to k to guarantee the robustness to noise), the speed estimation is better than the one given by the Kalman filter.

5. CONCLUSION

In this paper, a solution to the classical problem of linear speed estimation has been presented when only linear acceleration and angular measurements are available. This corresponds to the case where no GPS is available in a UAV. A nonlinear robust adaptive observer, that guarantees arbitrary L_2 attenuation with respect to measurement noise, has been introduced This estimation strategy is compared to the classical Extended Kalman Filter approach. The theoretical results are confirmed via the simulations, in particular the noise attenuation by an arbitrary design parameter. An important issue concerns the persistence of excitation condition introduced by the adaptive algorithm that provides an observability condition to the linearized system. When this property is not satisfied, the linearized system becomes non-observable. The simulation results provide useful insight about the parameter tuning effect to achieve a satisfactory observer performance.

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