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A Hidden Markov disturbance model for Offset-free linear model predictive control

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Abstract: Model predictive controllers are often designed with integral action to impart robustness. For this, disturbance models are usually employed. It is customary to append integrated white-noises to either the input or output channels. However, neither by themselves may be adequate representations in the presence of switching disturbance patterns that are typically witnessed in process industries. In order to handle such scenarios, we first propose a differenced state-space formulation that can incorporate both input and output disturbances while retaining detectability. Then, we couple it with Hidden Markov Model (HMM) to express the switching characteristics of the disturbances. This bypasses the need to add artificial noises into state variables to consider both the input and output disturbances, as previously suggested. Simulation examples are provided to highlight closed-loop performance improvement as a result of the proposed formulation.

Keywords: Model predictive and optimization-based control; nonlinear process control

1. INTRODUCTION

Supported by constraint-handling capabilities and a relatively well-established body of theory, linear Model Predictive Control (MPC) is *de facto* the advanced process control method employed in process industries. Historical accounts and surveys on industrial MPC applications are given by Lee and Cooley (1997), Morari and Lee (1999), and Qin and Badgwell (2003) and the references therein.

MPC algorithms usually adhere to the following steps (Muske and Rawlings, 1993*b*), (Muske and Rawlings, 1993*a*). At each sample time k, steady-state targets for the states and inputs are determined. Then, a finite-horizon, open-loop optimization problem is solved yielding a finite sequence of actions. The first action is implemented, one unit-sample of time elapses, and the process repeats in a receding-horizon manner. Direct state feedback is usually absent in practice, and therefore an observer (*e.g.*, a Kalman filter) is used to construct state estimates. These are also supplied to the optimizer. The objective function reflects the trade-off between predicted deviations of states and inputs from their respective steady-state targets. Zero deviations correspond to the system operating at the desired setpoint.

An important feature of MPC is integral action, which gives offset-free control despite plant-model mismatch and step disturbances. This is normally achieved by using an appropriate disturbance model, typically step, or equivalently, integrated white noise signals into input or output channels (see Section 1.1). Although zero tracking error is guaranteed asymptotically irrespective of which disturbance model is used (assuming enough of them are added), transient closed-loop performance is affected by how closely the model employed matches the reality. Recently, Wong and Lee (2007) suggested using Hidden Markov Models to extend the disturbance identification to more realistic patterns observed in the process industries.

Given that the design of disturbance models is a promising area for academic research (Qin and Badgwell, 2003), this work provides a formal framework for more realistic disturbance modeling (i.e., switching dynamics are accounted for) that meets the requirement of integral action. Furthermore, we explore the flexibility of the proposed model in providing acceptable tracking performance, as compared to existing methods, in the case where there exists significant differences between the disturbance model and the actual noise dynamics.

As an illustration of time-varying disturbance characteristics that may occur in process industries, consider that depicted in Fig. 1, a time-series plot of disturbances entering the input and output channels of an arbitrary dynamical system. Specifically, Fig. 1 indicates four regime permutations where input and/or output disturbances are dominant. For example, ‘low:low’(regime 1) indicates that both input and output disturbances enter as white-noises. ‘Low:high’ (regime 2) suggests that significant non-stationary, random-walk type disturbances are entering the output channel whereas input disturbances are comparatively quiescent, and so on. For the purpose of modeling, it is postulated that these regimes switch among themselves in a probabilistic manner. This hypothesized scenario can be considered to be an approximation of

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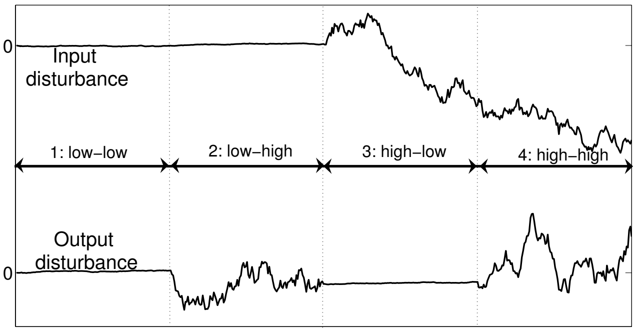

Fig. 1. Cross-correlated input and output disturbances, each exhibiting intermittent drifts

the case where there exists significant intermittent non-stationary disturbance patterns in each channel. Nevertheless, such time-varying behavior is oftentimes neglected during system identification, disturbance modeling and controller design, due to the lack of a suitable framework. This is usually to the detriment of the resulting closed loop performance, as will be shown in a later example.

1.1 Typical Offset-free Control Approaches

In MPC applications, offset-free control is usually achieved by augmenting the process model with integrators (these represent step input/ state ($d_k \in \mathbb{R}^{n_{d_k}}$) and output ($p_k \in \mathbb{R}^{n_{p_k}}$) disturbances) (Muske and Badgwell, 2002):

$$\begin{pmatrix} x_{k+1} \\ d_{k+1} \\ p_{k+1} \end{pmatrix} = \begin{pmatrix} A & G_d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} x_k \\ d_k \\ p_k \end{pmatrix} + \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} u_k$$

$$y_k = \left(C \;\; 0 \;\; G_p \right) \begin{pmatrix} x_k \\ d_k \\ p_k \end{pmatrix} \qquad (1)$$

The effect of this augmentation is to shift the steady-state targets to compensate for plant-model mismatch or step disturbances. Here, $x_k \in \mathbb{R}^{n_x}$ represents the system state at time k, $y_k \in \mathbb{R}^{n_y}$, the controlled (and presumably measured) outputs and $u_k \in \mathbb{R}^{n_u}$, the inputs. Furthermore, the augmentation is such that $n_{d_k} + n_{p_k} \leq n_y$ and other constraints on G_d and G_p are obeyed, to obey the requirement of detectability. This means that whilst the (A, B, C)-tuple represents the process model used for the regulator calculation, (1) is the model actually employed for calculating steady-state targets and obtaining state estimates $\hat{x}_{k|k}, \hat{d}_{k|k}, \hat{p}_{k|k}$.

It is noted that this practice of adding fictitious states has little bearing to disturbance identification. In response to this, researchers have developed techniques to estimate in an on-line fashion, noise parameters required by the Kalman filter. Nevertheless, this 'design' approach (Pannocchia and Rawlings, 2003) implicitly assumes that step disturbances enter either in the input or output channels ($G_d = B, G_p = I$ is a typical choice) and as such may be limiting in the face of a wrong assumption or more complicated dynamics, as suggested in this work. Moreover, as pointed out by Shinskey (1994), the popular choice of $G_p = I_{n_y}, G_d = 0$ in earlier versions of MPC (*e.g.*, Dynamic Matrix Control (Cutler and Ramaker, 1979)) can lead to sluggish behaviour when the system dynamics contains "slow" poles. In principle, one can add state noises to compensate for this, but this can result in a large number of parameters to learn online for the case of high-dimensional systems (Odelson and Rawlings, 2003; Odelson *et al.*, 2006).

The proposed formulation is achieved by extending the detectable velocity form used by Prett and Garcia (1988) for integral action, to the scenario where there exists the additional complexity of switching noise dynamics.

2. A DETECTABLE FORMULATION FOR MPC VIA A HIDDEN MARKOV FRAMEWORK

The example above reveals a need for addressing abruptly switching dynamics. Hidden Markov Models (HMMs) is a potential mathematical framework for this. A finite-state Markov chain whose realization at time k, denoted by $r_k \in \mathcal{J} \triangleq \{1, 2, \ldots, J\}, J \in \mathbb{Z}^+$, is assumed to govern the dynamics of noise regime changes. In the example above, $\mathcal{J}$ corresponds to the numeric set $\{1, 2, 3, 4\}$, with '1' denoting 'low:low', '2', 'low:high', and so on. The term 'Hidden' indicates that r_k is not known with probability one and must be inferred from available measurements, which are viewed as realizations of the corresponding underlying distribution.

The parameters of this Markov chain defines the probabilistic nature of the regime transitions. Specifically, the Markovian jumps are dictated by a transition matrix $\Pi = \Big(\mathrm{pr}(r_k = j | r_{k-1} = i) \triangleq p_{ij}\Big)$, such that the each row of Π sums to one. An implicit assumption is that the transitions depends only on the immediate past. All Markov chains under consideration are ergodic. For simplicity, the Markov chain is assumed to be at steady state, satisfying $\pi = \Pi'\pi$, where π is a column vector containing the unconditional probabilities of each regime. In other words, π gives the initial probability distribution of r.

With this, it is postulated that the dynamical system of concern evolves according to the following:

$$\begin{aligned} x_{k+1} &= Ax_k + Bd_{k+1} + Bu_k \\ d_{k+1} &= d_k + \omega^d_{r_{k+1}} \\ p_{k+1} &= p_k + \omega^p_{r_{k+1}} \\ y_k &= Cx_k + p_k + v_k \end{aligned} \qquad (2)$$

where as before, d_k and p_k are the augmented input and output disturbances respectively. In turn, these integrators are driven by zero-mean, uncorrelated, white Gaussian signals $\omega^d_r \sim \mathcal{N}(0, Q^d_r)$, and $\omega^p_r \sim \mathcal{N}(0, Q^p_r)$, whose covariances at each k are associated with r_k, as is suggested by the notation employed. $v_k \sim \mathcal{N}(0, R)$ is normally-distributed measurement noise.

Having established this, and provided that the original system of concern is detectable, a detectable differenced formulation (3) that will be used by the receding-horizon regulator as well as the state-estimator is proposed next.

2.1 A Detectable Formulation for the Regulator and State-Estimator

By differencing (2), we get

$$\underbrace{\begin{pmatrix} \Delta x_{k+1} \\ z_{k+1} \end{pmatrix}}_{\eta_{k+1}} = \underbrace{\begin{pmatrix} A & 0 \\ CA & I \end{pmatrix}}_{\tilde{A}} \underbrace{\begin{pmatrix} \Delta x_k \\ z_k \end{pmatrix}}_{\eta_k} + \underbrace{\begin{pmatrix} B \\ CB \end{pmatrix}}_{\tilde{B}} \left(\Delta u_k + \omega^d_{r_{k+1}} \right) + \begin{pmatrix} 0 \\ I \end{pmatrix} \omega^p_{r_{k+1}}$$

$$= \tilde{A}\eta_k + \tilde{B}\Delta u_k + \underbrace{\tilde{B}\omega^d_{r_{k+1}} + \begin{pmatrix} 0 \\ I \end{pmatrix} \omega^p_{r_{k+1}}}_{\xi_{r_{k+1}}}$$

$$y_k = \underbrace{\begin{pmatrix} 0 & I \end{pmatrix}}_{\tilde{C}} \begin{pmatrix} \Delta x_k \\ z_k \end{pmatrix} + y^* + v_k \quad (3)$$

Here, $z_k \triangleq (y_k - v_k - y^*)$, with y^* denoting the desired output setpoint. Also, Δ represents a time-differencing operator such that, $\Delta u_k \triangleq u_k - u_{k-1}$ and so on. $\xi_{r_{k+1}}$, the covariance of which depends on the regime r, represents the effective state noise of (3). $\omega^d_{r_{k+1}}$, and $\omega^p_{r_{k+1}}$ refer to zero-mean white Gaussian signals that drive the input and output integrators, respectively. The subscript r_{k+1} emphasizes the fact that the realizations of ω^d and ω^p depend (through their respective covariances) on the Markov state. Since it is assumed that $\omega^d_{r_{k+1}}$, and $\omega^p_{r_{k+1}}$ are uncorrelated, then we have (4)

$$\mathbb{E}(\xi_{r_{k+1}}) = \tilde{B}\mathbb{E}(\omega^d_{r_{k+1}})(\omega^d_{r_{k+1}})'\tilde{B}' + \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{E}(\omega^p_{r_{k+1}})(\omega^p_{r_{k+1}})' \end{pmatrix} \quad (4)$$

The benefits of using such a formulation is that detectability is ensured under rather mild conditions. This is favorable as compared to approaches based on (1), where certain choices of G_d, G_p (Muske and Badgwell, 2002) do not guarantee offset-free control. In this formulation, it is clear that the steady-state targets are zero for both Δx and z.

Due to the dependence of noise statistics on r_k, (3) is a special case of a 'Markov Jump Linear System' (MJLS) (Costa *et al.*, 2004). For such systems, the optimal filter involves exponentially growing number of linear filters and hence the computation becomes intractable. A popular sub-optimal filtering technique, the n-th order Generalized Pseudo-Bayesian (GPBn) algorithm (Bar-Shalom and Li, 1993) is briefly introduced since any model predictive algorithm requires an estimator to work in tandem with the regulator.

2.2 GPB2: Obtaining State Estimates $\hat{\eta}$

The main idea is to have trajectories whose last n terms differ be merged (via moment-matching) into a single Gaussian, parameterized by $\{\hat{\eta}_{k|k}, P_{k|k} \triangleq \mathbb{E}(\eta_k - \hat{\eta}_{k|k})(\cdot)'\}$. Let $\mathcal{R}^k_{k-n+1} \triangleq (r_{k-n+1}, \ldots, r_k)$ be a sequence of the n most recent Markov-state realizations and Y^k_0, the measurement sequence $(y_0, \ldots, y_k)$. Let $\hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+1})$ denote the estimate of η_k that accounts for the n most recent Markov-state realizations. Similarly, we let the corresponding estimation error be represented as $P_{k|k}(\mathcal{R}^k_{k-n+1}) \triangleq \mathbb{E}(\eta_k - \hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+1}))(\cdot)'$.

Computing $\hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+1})$ is achieved by employing a time-varying Kalman filter initialized with $\hat{\eta}_{k-n+1|k-n+1}(r_{k-n+1})$ and $P_{k-n+1|k-n+1}(r_{k-n+1})$, which can be obtained from the following recursive equations for computing the filtered state-estimates $\hat{\eta}_{k|k}(r_k)$ (5) and error covariance, $P_{k|k}(r_k)$(6).

$$\hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+2}) = \sum_{r_{k-n+1}\in\mathcal{J}} \hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+1})\mathrm{pr}(r_{k-n+1}|\mathcal{R}^k_{k-n+2}, Y^k_0) \quad (5)$$

$$P_{k|k}(\mathcal{R}^k_{k-n+2}) = \sum_{r_{k-n+1}\in\mathcal{J}} [\{\hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+2}) - \hat{\eta}_{k|k}(\mathcal{R}^k_{k-n+1})\}\{\cdot\}' + P_{k|k}(\mathcal{R}^k_{k-n+1})] \cdot \mathrm{pr}(r_{k-n+1}|\mathcal{R}^k_{k-n+2}, Y^k_0) \quad (6)$$

Obtaining the pair $(\hat{\eta}_{k|k}, P_{k|k})$ is done by combining $\{\hat{\eta}_{k|k}(r_k), P_{k|k}(r_k)\}$ via $\mathrm{pr}(r_k|Y^k_0)$ (8). The merging probabilities in (6), and (5) are obtained recursively (Bar-Shalom and Li, 1993) via Bayes rule. The computations for $n = 2$, which is used in this work, are shown:

$$\mathrm{pr}(r_{k-1}|r_k, Y^k_0) = \frac{\mathrm{pr}(y_k|r_k, r_{k-1}, Y^{k-1}_0)\mathrm{pr}(r_k|r_{k-1})\mathrm{pr}(r_{k-1}|Y^{k-1}_0)}{\sum_{r_{k-1}\in\mathcal{J}} \mathrm{pr}(y_k|r_k, r_{k-1}, Y^{k-1}_0)\mathrm{pr}(r_k|r_{k-1})\mathrm{pr}(r_{k-1}|Y^{k-1}_0)} \quad (7)$$

$$\mathrm{pr}(r_k|Y^k_0) = \frac{\sum_{r_{k-1}\in\mathcal{J}} \mathrm{pr}(y_k|r_k, r_{k-1}, Y^{k-1}_0)\mathrm{pr}(r_k|r_{k-1})\mathrm{pr}(r_{k-1}|Y^{k-1}_0)}{\sum_{r_k}\sum_{r_{k-1}} \mathrm{pr}(y_k|r_k, r_{k-1}, Y^{k-1}_0)\mathrm{pr}(r_k|r_{k-1})\mathrm{pr}(r_{k-1}|Y^{k-1}_0)} \quad (8)$$

2.3 Remarks on System Identification of Markov Jump Linear Systems (MJLS)

Identification of linear jump systems, an area of interest in its own right, is not the focus of this paper. Readers are referred to (Murphy, 1998), (Pavlovic *et al.*, 1999), (Juloski *et al.*, 2005) for approaches that are established in the literature and (Wong and Lee, 2007) for a recent application to the specific problem of disturbance identification. For the purpose of this paper, it is assumed that the system and Markov parameters are available, either through identification or based on process insights, for estimator/ controller design.

We now give a summary of the MPC algorithm to be employed.

2.4 The MPC Algorithm

At each time k, the GPB2 estimator yields $\hat{\eta}_{k|k}$. This quantity is then fed into the following finite-horizon optimization problem. N denotes a user-specified control-horizon and $S_\eta \geq 0$, $S_\eta^N \geq 0$, $S_u > 0$ are weighting matrices reflecting a trade-off between aggressive setpoint tracking and the amount of actuator movement.

$$\{\Delta\nu_j^*\}_{j=0}^{N-1} \triangleq \text{argmin} \left[\tilde{\eta}_N' S_\eta^N \tilde{\eta}_N + \sum_{j=0}^{N-1} \left(\tilde{\eta}_j' S_\eta \tilde{\eta}_j + \Delta\nu_j' S_u \Delta\nu_j \right) \right]$$

$$S_\eta \triangleq \begin{pmatrix} 0 & 0 \\ 0 & S_y \end{pmatrix} \quad S_\eta^N \triangleq \begin{pmatrix} 0 & 0 \\ 0 & S_y^N \end{pmatrix} \tag{9}$$

subject to:

$$\begin{aligned} \tilde{\eta}_0 &= \hat{\eta}_{k|k} \\ \tilde{\eta}_{j+1} &= \tilde{A}\tilde{\eta}_j + \tilde{B}\Delta\nu_j, \quad j = 0, 1, \ldots, N-1 \\ \Delta\nu_j &\in \mathcal{X}, \quad j = 0, 1, \ldots, N-1 \\ \tilde{\eta}_j &\in \mathcal{U}, \quad j = 0, 1, \ldots, N-1 \end{aligned} \tag{10}$$

Then, one implements $u_k := \Delta\nu_0 + u_{k-1}$, and the whole process repeats. $\mathcal{X}$ and $\mathcal{U}$ are polyhedrons defining the feasible regions satisfying the constraints. It is noted that the above formulation subsumes well-known cases such as unconstrained Linear Quadratic Regulation (LQR).

We now show the flexibility of the proposed formulation in the face of several simulated disturbance scenarios.

3. EXAMPLE

For simplicity, consider the triple $(A = 0.9, B = 1, C = 1.5)$ parameterizing a nominal Single-Input-Single-Output (SISO) system

3.1 Plant Simulation

Generally speaking, it is not certain in advance if input or output disturbances will dominate. Furthermore, there may exist probabilistic switches between regimes, as postulated in this work. To evaluate the performance of the proposed controller under these possible situations, four simulation scenarios are considered. As special instances of (2), these (and their simulation parameters) correspond to

- I. Levels of input noise << levels of output noise. This can be thought of the case where $d_k = 0, \forall k$. $\mathbb{E}(\omega^d \omega^{d'})$ corresponds to the 'low:high' regime as reported in Table (1). Note that in this scenario, the subscript r is dropped from ω^d since no switching occurs.

- II. Levels of input noise >> levels of output noise. Similarly, in this case, $p_k = 0$, $\forall k$ and $\mathbb{E}(\omega^p \omega^{p'})$ corresponds to the 'high:low' regime.

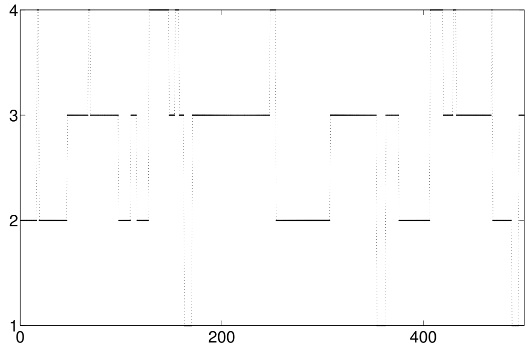

Fig. 2. Time series plot of regime: a typical realization for scenario IV.

- III. Levels of input noise are comparable to levels of output noise. Here, the noise parameters are the same as those in 'high:high' regime of Table (1).

- IV. Relative levels of input and output noise switch in a probabilistic manner (see Fig. 1). The transition probability matrix Π, used in the simulated studies, is given in (11) and reflects the situation where either input or output (but not both) disturbances dominate. In accordance with intuition, it can also be seen that relative to regimes 2 and 3, the system tends to spend less time, on average, in the 'high-high' and 'low-low' regimes. For instance, the expected duration the system spends in regime 1 or 4 is $\frac{1}{1-0.8} = 5$ time units whereas that spent in regime 2 or 3 is $\frac{1}{1-0.97} \approx 33$ units. Furthermore, drastic 'low-low' to 'high-high' transitions (and vice versa) are forbidden. The noise parameters used for simulation can be found in Table (1). These effects are captured in Fig. 3.1.

$$\Pi = \begin{pmatrix} 0.800 & 0.100 & 0.100 & 0.000 \\ 0.010 & 0.970 & 0.010 & 0.010 \\ 0.010 & 0.010 & 0.970 & 0.010 \\ 0.000 & 0.100 & 0.100 & 0.800 \end{pmatrix} \tag{11}$$

Table 1. Noise Parameters used in Simulation Studies

Input Noise: Output Noise	Regime $r \in 1, 2, 3, 4$	Q_r^d	Q_r^p
'Low:Low'	$r = 1$	10^{-10}	10^{-10}
'Low:High'	$r = 2$	10^{-10}	50
'High:Low'	$r = 3$	10	10^{-10}
'High:High'	$r = 4$	10	50

For simplicity, all simulations were run with negligible measurement noise, that is $R \approx 0$.

3.2 Parameters Used by the Estimator & Control Objective

In order to investigate the effect of disturbance model vs. plant simulation mismatch, four model-predictive controllers were constructed, all based on the proposed velocity form (3). These differ only in the estimators employed. Namely, these estimator/ controllers assume:

- I. Output disturbance only. In this case, $\mathbb{E}(\xi\xi')$ can be computed using data from Table (1) corresponding to $r = 2$, and (4). As noted by Muske and Rawlings

(1993*b*), the resulting steady-state Kalman gain is $\left[\begin{array}{cc} 0 & I \end{array}\right]$ parameterizing an open-loop observer for Δx and a deadbeat observer for z. Again, the subscript r is dropped for the same reasons as before.

- II. Input disturbance only. Similarly one uses $r = 3$ data from Table (1), for computing $\mathbb{E}(\xi\xi')$. A deadbeat observer is obtained in this case.
- III.Output and input disturbances. Here, one uses $r = 4$ data from Table (1), for computing $\mathbb{E}(\xi\xi')$.
- IV. Switching behavior, i.e., a GPB2 estimator, with Π given in (11) is employed.

Although the velocity form is employed, estimator/ controllers I, II and III can be regarded as special cases of (1), and as such are considered as standard MPC formulations for imparting integral action.

For all the (simulation scenario, controller) pairs considered, the objective function, with $y^* = 0$ is as follows (12):

$$\min \frac{1}{N}\sum_{k=0}^{N}\left[\eta_k' \left(\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right)\eta_k\right]_{N\to\infty} \tag{12}$$

Constraints have been removed for clarity of exposition and this objective function results in a deadbeat controller (13):

$$\Delta u_k = -\left(\begin{array}{cc} 0.90 & 0.67 \end{array}\right)\hat{\eta}_{k|k} \tag{13}$$

3.3 Results & Discussion

For each (simulation scenario, controller) combination, $S = 500$ realizations, each of duration $K = 500$, were run. Furthermore, for each realization s, of each simulation scenario, a controller coupled with a time-varying Kalman filter with knowledge of the true simulation dynamics. Although this assumption is impractical, such a controller was employed for benchmarking purposes. For example, in scenario IV corresponding to switching disturbances, such a time-varying Kalman filter would have had access to the actual Markov regime. As mentioned in Section (2.1), such ideality is in contrast to the GPB2 estimator which relies on moment matching to bound computational costs. The performance index $\mathcal{P}$ [1] is the average ratio of squared-tracking error for each controller to that corresponding to the ideal estimator/ controller. The results are reported in Table (2).

Figs. 3, 4, 5 6 are time-series plots of typical realizations of y corresponding to the four simulation scenarios.

As can be seen from Table 2 and Figs. 3, 4, 5 6, the proposed model predictive controller yields the best performance amongst all the estimator/ controllers other than that which coincides with the actual simulation scenario.

[1] $\mathcal{P} \triangleq \frac{1}{S}\sum_{s=1}^{S}\left(\frac{\sum_{k=1}^{K} y_k^2}{\sum_{k=1}^{K}(y_b)_k^2}\right)$; y_b refers to the output corresponding to the benchmarking estimator/ controller

Table 2. Mean of Relative-Squared Error over 500 Realizations

	Estimator			
Scenario	Output	Input	Output&Input	GPB2
Output	1.00	1.62	1.21	1.03
Input	5.15	1.00	1.33	1.02
Output&Input	1.84	1.24	1.00	1.05
Switching	3.21	1.52	1.22	1.08

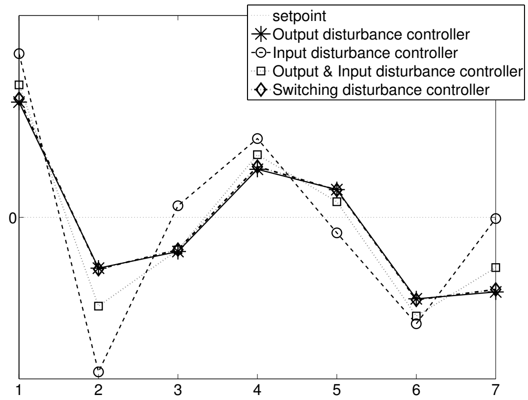

Fig. 3. Output disturbance: y vs. time

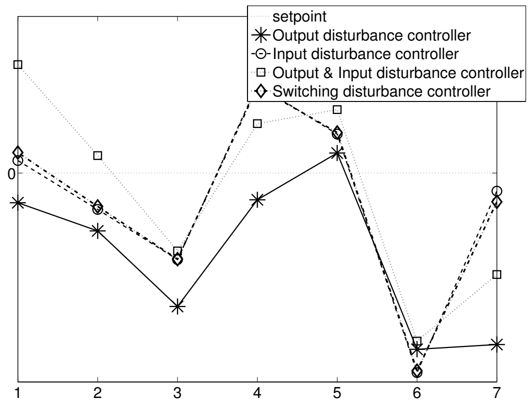

Fig. 4. Input disturbance: y vs. time

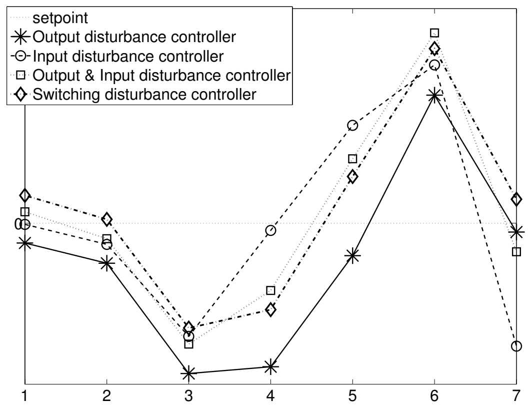

Fig. 5. Output and Input disturbances: y vs. time

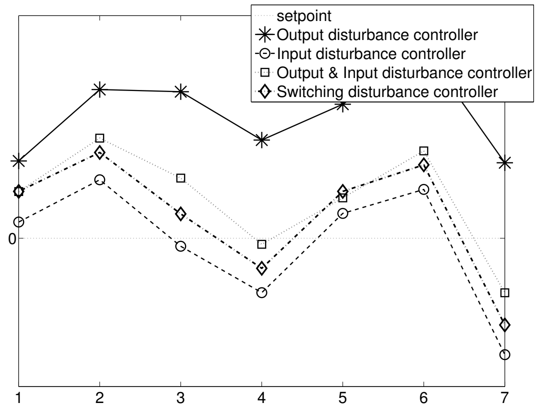

Fig. 6. Switching disturbance: y vs. time

This suggests that the proposed formulation is generally more robust than the standard controllers employed. Due to the relatively large time constant of the system ($A = 0.9$), the output estimator/ regulator expectedly (Shinskey, 1994) gave the poorest performance for the scenarios it was not designed for.

Furthermore, using a formulation with includes both input and output disturbances often means using an observer gain that 'averages' the input and output disturbance effects. How this translates to final closed-loop performance is not clear. Using the time-varying GPB2 estimator results in a dynamic observer gain that is a function of current measurements. Despite the mismatch in terms of Π, the control performance is still acceptable.

4. CONCLUSION & FUTURE WORK

The contributions of this work are three-fold. First, a formulation that remains detectable regardless of number of step disturbances is proposed. Second, the proposed formulation is capable of accommodating switching behavior in the disturbance patterns. Such an approach can be viewed as a first step towards describing abruptly switching, or more generally, non-stationary disturbances common in process industries. Compared to the previously proposed approach for accommodating both input and output disturbances, the proposed approach is more straightforward, circumventing the need to add artificial state noise, which can be computationally demanding for high-dimensional state space systems.

4.1 Future Work

As a special type of hybrid system, Markov Jump Linear Systems represent a descriptive class of models that can capture a wide-range of dynamics (Costa *et al.*, 2004). It is clear that advances in system identification and disturbance modeling are to proceed together so as to achieve more realistic modeling and eventually better control performance. As such, this is an ongoing research project.

REFERENCES

Bar-Shalom, Yaakov and Xiao-Rong Li (1993). *Estimation and Tracking: Principles, Techniques and Software.* Artech House.

Costa, O.L.V., M.D. Fragoso and R.P. Marques (2004). *Discrete-Time Markov Jump Linear Systems.* Probability and Its Applications.

Cutler, C.R. and B.L. Ramaker (1979). Dynamic matrix control - a computer control algorithm. In: *AIChE Meeting.* Houston, TX.

Juloski, A., W.P.M.H. Heemels, G. Ferrari-Trecate, R. Vidal, S. Paoletti and J.H.G. Niessen (2005). Comparison of four procedures for the identification of hybrid systems. In: *8th International Workshop on Hybrid Systems: Computation and Control* (M. Morari and L. Thiele, Eds.). Vol. 3414. pp. 354–369.

Lee, Jay H. and Brian Cooley (1997). Recent advances in model predictive control and other related areas. In: *5th International Conference on Chemical Process Control* (J. Kantor, C. Garcia and B. Carnahan, Eds.). Vol. 91. pp. 201–216.

Morari, Manfred and Jay H. Lee (1999). Model predictive control: Past, present and future. *Computers and Chemical Engineering* **23**, 667–682.

Murphy, K. P. (1998). Switching kalman filters. Technical report. DEC/ Compaq Cambridge Research Labs.

Muske, Kenneth R. and Thomas A Badgwell (2002). Disturbance modeling for offset-free linear model predictive control. *Journal of Process Control* **12**, 617–632.

Muske, K.R. and J.B. Rawlings (1993*a*). Linear model predictive control of unstable processes. *Journal of Process Control* **3**(2), 85–96.

Muske, K.R. and J.B. Rawlings (1993*b*). Model predictive control with linear models. *AIChE Journal* **39**(2), 262–287.

Odelson, B. J. and James B. Rawlings (2003). Online monitoring of mpc disturbance models using closed-loop data. In: *American Control Conference.* Denver, Colorado.

Odelson, B. J., M. R. Rajamani and J. B. Rawlings (2006). A new autocovariance least-squares method for estimating noise covariances. *Automatica* **2006**, 303–308.

Pannocchia, Gabriele and James B. Rawlings (2003). Disturbance models for offset-free model predictive control. *AIChE Journal* **49**(2), 426–437.

Pavlovic, V., J. Rehg, T. Cham and K.P. Murphy (1999). A dynamic bayesian network approach to figure tracking using learned dynamical models. In: *7th IEEE International Conference on Computer Vision.* Vol. 1. Kerkyra, Greece. pp. 94–101.

Prett, D.M. and C.E. Garcia (1988). *Fundamental Process Control.* Butterworth, Boston.

Qin, S. Joe and Thomas A. Badgwell (2003). A survey of industrial model predictive control technology. *Control Engineering Practice* **11**, 733–764.

Shinskey, F.G. (1994). *Feedback Controllers for the Process Industries.* McGraw-Hill, New York.

Wong, Wee Chin and Jay H. Lee (2007). Disturbance modeling for process control via hidden markov models. In: *8th International IFAC Symposium on Dynamics and Control of Process Systems.* Cancun, Mexico. pp. 235–240.