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TRACKING CONTROL OF A INDUCTION MOTOR: AN OUTPUT FEEDBACK APPROACH

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Abstract: In this work the output feedback tracking problem for the angular position of a induction motor driving a mechanical load is addressed. It is assumed that the two rotor flux variables are not measured whilst the two stator current variables and also the angular position and velocity are available for feedback. The employed approach consists of the asymptotic reconstruction of a full state feedback stabilizing control law, without using a Lyapunov function and by applying a separation principle. Firstly, a Backstepping tracking controller is considered, then, a dynamic output feedback is synthesized for the two voltage control inputs, which asymptotically recovers the Backstepping control law.

Keywords: Design Methods, Non-linear control systems, Tracking, Output feedback control.

1. INTRODUCTION

The problem of tracking the angular position of induction motors has been addressed by a number of researchers (Marino and Spong, 1986). Both the exact feedback linearization (Slotine and Li, 1991) and the *Backstepping* (Dawson *et al.*, 1998) techniques have shown to be efficient for regulating this system. These approaches stabilize induction motors by designing a full information control law, after assuming that all state variables are available for feedback. This latter assumption restricts the practical application of the designed controller. In order to achieve a low cost system, by reducing the number of sensors, the output feedback control design problem is addressed in this work. The output feedback stabilization problem of nonlinear systems has been widely studied recently. Special attention has been paid to nonlinear systems that are linear in the unmeasured states, see for instance the classical approach proposed in Freeman and Kokotović (1996), and references therein.

A novel design technique to solve the output feedback stabilization problem has been developed by Karagiannis *et al.* (2003), which is based on ideas borrowed from the theory of the nonlinear regulator problem and the notions of immersion and invariance (Astolfi and Ortega, 2003). This control design approach consists of the application of a separation principle, which establishes that it is possible to solve the output feedback stabilization problem provided that two subproblems are solvable. The first subproblem refers to find a robust state feedback full information control law. The second problem conducts to design a robust stabilization control law by output injection. Thus, a dynamic globally stabilizing output feedback control law is obtained, whose dynamics asymptotically recover the full information control law. An important feature of this approach is that the stabilization mechanism does not rely on the construction of a Lyapunov function for the closed loop system. By closely following this method, a load position tracking controller is developed in this work for the electromechanical model of a induction motor, under the assumptions that the two rotor flux variables are not measured, whilst the two stator current variables and also the angular position and velocity are available for feedback. Firstly, a state feedback backstepping controller is considered, as the full information control law and, then, an output feedback controller is synthesized by employing the control design proposed in Karagiannis *et al.* (2003).

2. OUTPUT FEEDBACK STABILIZATION OF A CLASS OF NONLINEAR SYSTEMS

The design procedure proposed by Karagiannis *et al.* (2003) is revisited in this section. Consider systems described by equations of the form

$$\begin{aligned} \dot{\eta} &= A(y,u)\eta + B(y,u) \\ \dot{y} &= \psi_0(y,u) + \psi_1(y,u)\eta \end{aligned} \tag{1}$$

with state $(\eta, y) \in \Re^n \times \Re^p$, output y and control input $u \in \Re^m$. It is assumed that only the output y is available for feedback. Along with the system (1), a performance output ρ is considered, which is defined as

$$\rho = h(y,\eta) \tag{2}$$

for some mapping $h(\cdot)$. The output feedback regulation problem can be formulated as follows: *Consider the system (1) and the performance variable ρ defined in (2). Find a dynamic output feedback control law described by equations of the form*

$$\begin{aligned} \dot{\hat{\eta}} &= \pi(y,\hat{\eta}) \\ u &= \alpha(y,\hat{\eta}) \end{aligned} \tag{3}$$

such that all trajectories of the closed loop system (1)-(3) are bounded and

$$\lim_{t\to\infty} \rho(t) = 0. \tag{4}$$

The solution proposed in Karagiannis *et al.* (2003) is summarized in the following proposition.

Proposition 1. *Consider a system described by equations of the form (1) and the performance variable ρ defined as in (2). Suppose the following assumptions hold.*
(A1) There exists a full information control law

$$u^* = \alpha(y,\eta) \tag{5}$$

such that all trajectories of the closed loop system (1)-(3) are bounded and are such that condition (4) holds. Moreover, the system (1) with $u = \alpha(y, \eta + d(t))$ is globally bounded-input bounded-state stable with respect to the input $d(t)$.
(A2) There exists a mapping $\beta(y)$ such that the system

$$\dot{z} = \left(A(y,u) - \frac{\partial\beta}{\partial y}\psi_1(y,u) \right) z \tag{6}$$

is uniformly globally stable for any y, and u; and $z(t)$ is such that, for any fixed y and η,

$$\lim_{t\to\infty} [\alpha(y, \eta + z(t))] = \alpha(y,\eta). \tag{7}$$

Then there exists a dynamic output feedback control law, described by equations of the form (3), solving the output feedback regulation problem.

By defining the auxiliary vector

$$z = M\hat{\eta} - \eta + \beta(y), \tag{8}$$

where M is an invertible matrix and $\hat{\eta}$ is an estimate of the unmeasured states η, and by following the constructive proof of Proposition 1 in Karagiannis *et al.* (2003), it can be synthesized the dynamic output feedback control law

$$\begin{aligned} \dot{\hat{\eta}} = M^{-1} \Bigg[& A(y, \alpha(y, M\hat{\eta} + \beta(y)))(M\hat{\eta} + \beta(y)) \\ & + B(y, \alpha(y, M\hat{\eta} + \beta(y))) \\ & - \frac{\partial\beta}{\partial y}\psi_0(y, \alpha(y, M\hat{\eta} + \beta(y))) \\ & - \frac{\partial\beta}{\partial y}\psi_1(y, \alpha(y, M\hat{\eta} + \beta(y)))(M\hat{\eta} + \beta(y)) \Bigg] \\ u = \alpha(y, & M\hat{\eta} + \beta(y)), \end{aligned} \tag{9}$$

obtaining the closed loop system in the η, y and z coordinates

$$\begin{aligned} \dot{\eta} &= A(y, \alpha(y, M\hat{\eta} + \beta(y)))\eta \\ &\quad + B(y, \alpha(y, M\hat{\eta} + \beta(y))) \\ \dot{y} &= \psi_0(y, \alpha(y, M\hat{\eta} + \beta(y))) \\ &\quad + \psi_1(y, \alpha(y, M\hat{\eta} + \beta(y)))\eta \\ \dot{z} &= \Bigg[A(y, \alpha(y, M\hat{\eta} + \beta(y))) \\ &\quad - \frac{\partial\beta}{\partial y}\psi_1(y, \alpha(y, M\hat{\eta} + \beta(y))) \Bigg] z \end{aligned} \tag{10}$$

As a result, by Assumption *(A2)*, the variable z remains bounded for all t, and it is such that equation (7) holds. Furthermore, by Assumption *(A1)*, y and η are bounded for all t and condition (4) holds. Roughly speaking, this design method relies on the asymptotic reconstruction of a stabilizing full information state feedback control law, by synthesizing a dynamic output feedback controller, including a nonlinear observer for the unmeasured states.

3. BACKSTEPPING CONTROL OF A INDUCTION MOTOR

Consider a triphasic induction motor driving a mechanical load giving by the following differential equations (Dawson *et al.*, 1998)

$$M\ddot{q} + B\dot{q} + Nsin(q) = \psi_a I_b - \psi_b I_a \quad (11)$$
$$L_I \dot{I}_a = -R_I I_a + \alpha_1 \psi_a + \alpha_2 \psi_b \dot{q} + V_a \quad (12)$$
$$L_I \dot{I}_b = -R_I I_b + \alpha_1 \psi_b - \alpha_2 \psi_a \dot{q} + V_b \quad (13)$$
$$L_r \dot{\psi}_a = -R_r \psi_a - \alpha_3 \dot{q} \psi_b + K_I I_a \quad (14)$$
$$L_r \dot{\psi}_b = -R_r \psi_b + \alpha_3 \dot{q} \psi_a + K_I I_b \quad (15)$$

where $q(t)$, $\dot{q}(t)$ and $\ddot{q}(t)$, are the angular position, velocity and acceleration of the mechanical load; $I_a(t)$, $I_b(t)$ are the stator currents; $\psi_a(t)$, $\psi_b(t)$ are the rotor fluxes; and $V_a(t)$, $V_b(t)$ are the stator input voltage. The positive constants L_I, R_I, K_I, α_1, α_2 and α_3 are the electrical parameters of the motor related in the following way

$$L_I = L_s - M_e^2/L_r \qquad K_I = R_r M_e$$
$$R_I = (M_e^2 R_r + L_r^2 R_s)/Lr^2$$
$$\alpha_1 = M_e R_r / L_r^2 \quad \alpha_2 = n_p M_e / L_r \quad \alpha_3 = n_p L_r$$

where R_s, R_r, n_p, L_s, L_r and M_e, are the stator resistance, the rotor resistance, the number of pole pairs, the stator inductance, the rotor inductance and the mutual inductance, respectively. Whilst in the differential equation modelling the mechanical part (11), M is the mechanical inertia of all system, N is a constant relating the load mass and the gravitational constant,and B is the coefficient of viscous friction. Assuming that all state variables are available for feedback, a stabilizing controller has been synthesized in Dawson *et al.* (1998), by following the *integrator backstepping* methodology. In the seek of clarity, the steps followed there are revisited in this section.

3.1 Position/Velocity tracking control

We define the load position tracking error $e(t)$ as

$$e = q_d - q \quad (16)$$

where $q_d(t)$ represents a smooth and bounded desired load position. In addition, we define the filtered tracking error $r(t)$ as

$$r = \dot{e} + \alpha e \quad (17)$$

where α is a positive scalar constant. The overall tracking control objective will be met if $r(t)$, and hence $e(t)$, is driven to zero. After obtaining the open-loop filtered tracking error dynamics, it is multiplied by M and substituted in the mechanical system dynamics in (11), to obtain

$$M\dot{r} = W_\tau \theta_\tau - (\psi_a I_b - \psi_b I_a) \quad (18)$$

where the known regression matrix W_τ and the parameter vector θ_τ are given by

$$W_\tau = [\,\ddot{q}_d + \alpha\dot{e} \;\; \dot{q} \;\; sin(q)\,],\; \theta_\tau = [\,M \;\; B \;\; N\,]^T$$

Due to the structure of the electromechanical system (11)-(15), the mechanical subsystem error dynamics (18) lack a true torque level control input. We add and subtract a desired torque signal $\tau_d(t)$ in (18) to yield

$$M\dot{r} = W_\tau \theta_\tau - \tau_d + \eta_\tau \quad (19)$$

where $\eta_\tau(t)$ represents the torque tracking error defined by

$$\eta_\tau = \tau_d - (\psi_a I_b - \psi_b I_a). \quad (20)$$

The voltage control inputs must be designed in order to compensate for the effects of $\eta_\tau(t)$. To accomplish this additional control objective the dynamics of the torque tracking error are needed. By taking time derivative of $\eta_\tau(t)$, multiplying the result by L_I and substituting the right-hand sides of (12)-(15) results

$$\begin{aligned} L_I \dot{\eta}_\tau = {} & L_I \dot{\tau}_d - L_I L_r^{-1} I_b(-R_r \psi_a - \alpha_3 \dot{q} \psi_b) \\ & + L_I L_r^{-1} I_a(-R_r \psi_b - \alpha_3 \dot{q} \psi_a) \\ & - \psi_a(-R_I I_b - \alpha_2 \psi_a \dot{q}) \\ & + \psi_b(-R_I I_a + \alpha_2 \psi_b \dot{q}) + \psi_b V_a - \psi_a V_b \end{aligned} \quad (21)$$

Notice that the voltage control inputs V_a and V_b have appeared on the right-hand side of (21).

3.2 Flux tracking objective

In order to avoid unbounded signals, the rotor fluxes are forced to track a bounded signal. We define the flux tracking error $\eta_\psi(t)$, as

$$\eta_\psi = \psi_d - \frac{1}{2}\gamma = \psi_d - \frac{1}{2}(\psi_a^2 + \psi_b^2) \quad (22)$$

where $\psi_d(t)$ is the desired flux. Differentiating (22) with respect to time and multiplying by L_r and, finally, by substituting $L_r \dot{\psi}_a$ and $L_r \dot{\psi}_b$ from (14) and (15), the dynamics of the flux tracking error are given by

$$L_r\dot{\eta}_\psi = L_r\dot{\psi}_d + R_r\gamma - K_I(\psi_a I_a + \psi_b I_b) \quad (23)$$

By dividing the latter equation by K_I, we obtain

$$\overline{L}_r\dot{\eta}_\psi = Y_\psi\theta_\psi - (\psi_a I_a + \psi_b I_b) \quad (24)$$

where Y_ψ and θ_ψ are given by

$$Y_\psi = \left[\, \dot{\psi}_d \;\; \gamma \,\right], \qquad \theta_\psi = \left[\, \overline{L}_r \;\; \overline{R}_r \,\right]^T \quad (25)$$

with $\overline{L}_r = \frac{L_r}{K_I}$ and $\overline{R}_r = \frac{R_r}{K_I}$. Similar to the position tracking objective, it shall be used the *integrator backstepping* approach to add and subtract a fictitious flux controller $u_I(t)$, in (24) to obtain

$$\overline{L}_r\dot{\eta}_\psi = Y_\psi\theta_\psi - u_I + \eta_I \quad (26)$$

where $\eta_I(t)$ is defined by

$$\eta_I = u_I - (\psi_a I_a + \psi_b I_b). \quad (27)$$

From (26), we can see that if the auxiliary variable $\eta_I(t)$ were zero, the fictitious flux controller $u_I(t)$, could be easily designed to force $\eta_\psi(t)$ to zero. To this end, we construct the open loop dynamics for $\eta_I(t)$ and follow a similar procedure to the one used in the torque tracking error dynamics, obtaining

$$\begin{aligned} L_I\dot{\eta}_I = {} & L_I\dot{u}_I - \psi_a V_a - \psi_b V_b \\ & -L_I L_r^{-1} I_a(-R_r\psi_a - \alpha_3\dot{q}\psi_b + K_I I_a) \\ & -L_I L_r^{-1} I_b(-R_r\psi_b - \alpha_3\dot{q}\psi_a + K_I I_b) \\ & -\psi_a(-R_I I_a + \alpha_1\psi_a) \\ & -\psi_b(-R_I I_b + \alpha_1\psi_b). \end{aligned} \quad (28)$$

Notice that the voltage control inputs V_a and V_b have also appeared on the right-hand side of (28).Based on the structure of (19) and, in order to force the load along the desired position trajectory, τ_d is specified as

$$\tau_d = W_\tau\theta_\tau + k_s r \quad (29)$$

where k_s is a positive constant control gain. The resulting closed-loop filtered tracking error dynamics is given by

$$M\dot{r} = -k_s r + \eta_\tau \quad (30)$$

To complete the open-loop system description for the dynamics of η_τ, we firstly compute

$$\begin{aligned} \dot{\tau}_d &= \dot{W}_\tau\theta_\tau + k_s\dot{r} \\ &= M(q_d^{'''} + \alpha(\ddot{q}_d - \ddot{q})) + B\ddot{q} + N\dot{q}\cos(q) \\ &\quad + k_s(\ddot{q}_d - \ddot{q} + \alpha\dot{e}) \end{aligned} \quad (31)$$

where $\ddot{q}$ can be obtained from (11)

$$\ddot{q} = -\frac{B}{M}\dot{q} - \frac{N}{M}sin(q) + \frac{1}{M}(\psi_a I_b - \psi_b I_a). \quad (32)$$

Notice that (31) can be rewritten as

$$\begin{aligned} \dot{\tau}_d = {} & M(q_d^{'''} + \alpha\ddot{q}_d) + N\dot{q}cos(q) + k_s(\ddot{q}_d + \alpha\dot{e}) \\ & +(B - M\alpha - k_s)\ddot{q}. \end{aligned} \quad (33)$$

Then, after substituting (33) in (21), the torque tracking error has the form

$$L_I\dot{\eta}_\tau = w_a - (\psi_a V_b - \psi_b V_a) \quad (34)$$

where the auxiliary variable w_a is given by

$$\begin{aligned} w_a = {} & L_I\big[M(q_d^{'''} + \alpha\ddot{q}_d) + N\dot{q}cos(q) \\ & +k_s(\ddot{q}_d + \alpha\dot{e})\big] + \psi_b(-R_I I_a + \alpha_2\psi_b\dot{q}) \\ & -L_I L_r^{-1} I_b(-R_r\psi_a - \alpha_3\dot{q}\psi_b) \\ & +L_I L_r^{-1} I_a(-R_r\psi_b + \alpha_3\dot{q}\psi_a) \\ & -\psi_a(-R_I I_b - \alpha_2\psi_a\dot{q}) + L_I(B - M\alpha - k_s)\ddot{q} \end{aligned} \quad (35)$$

From (34), a voltage control input can be designed to force the torque tracking error $\eta_\tau(t)$ to zero

$$\psi_a V_b - \psi_b V_a = w_a + k_1\eta_\tau + r \quad (36)$$

where k_1 is a positive control gain. Substituting (36) in (34), the closed-loop torque tracking error system yields

$$L_I\dot{\eta}_\tau = -k_1\eta_\tau - r \quad (37)$$

To achieve the second control objective of forcing the flux tracking error $\eta_\psi(t)$ to zero, a fictitious flux controller $u_I(t)$ is designed as

$$u_I = Y_\psi\theta_\psi + k_2\eta_\psi \quad (38)$$

where k_2 is a positive control gain. Substituting (38) in (26), the closed-loop dynamics for the flux tracking error yield

$$\overline{L}_r\dot{\eta}_\psi = -k_2\eta_\psi + \eta_I. \quad (39)$$

To ensure $\eta_I(t)$ goes to zero, a voltage controller must be designed. Taking the time derivative of (38), multiplying by L_I, substituting (14) and (15), and then substituting in (28), the open loop dynamics for $\eta_I(t)$ results

$$L_I\dot{\eta}_I = w_b - (\psi_a V_a + \psi_b V_b) \quad (40)$$

where the auxiliary scalar variable w_b is given by

$$\begin{aligned} w_b = {} & \overline{L}_r L_I\ddot{\psi}_d \\ & +L_r^{-1}(2L_I\overline{R}_r\psi_a - L_I I_a) \\ & \quad \times(-R_r\psi_a - \alpha_3\dot{q}\psi_b + K_I I_a) \\ & +k_2 L_I\overline{L}_r^{-1}(Y_\psi\theta_\psi - (\psi_a I_a + \psi_b I_b)) \\ & +L_r^{-1}(2L_I\overline{R}_r\psi_b - L_I I_b) \\ & \quad \times(-R_r\psi_b + \alpha_3\dot{q}\psi_a + K_I I_b) \\ & -\psi_a(-R_I I_a + \alpha_1\psi_a) - \psi_b(-R_I I_b + \alpha_1\psi_b) \end{aligned} \quad (41)$$

From the structure of (40), we propose the following voltage input control V_a and V_b, to drive $\eta_I(t)$ to zero

$$\psi_a V_a + \psi_b V_b = w_b + k_3\eta_I + \eta_\psi \tag{42}$$

where k_3 is a positive control gain. After substituting the right-hand side of (42) in (40), we obtain the closed-loop description for $\eta_I(t)$ as

$$L_I\dot{\eta}_I = -k_3\eta_I - \eta_\psi \tag{43}$$

Based on (36) and (42), we can find V_a and V_b as the following error dynamics

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \frac{1}{-\gamma}\begin{bmatrix} \psi_b & -\psi_a \\ -\psi_a & -\psi_b \end{bmatrix}\begin{bmatrix} w_a + k_1\eta_\tau + r \\ w_b + k_3\eta_I + \eta_\psi \end{bmatrix} \tag{44}$$

The stability analysis in Dawson *et al.* (1998) shows that the filtered tracking error goes to zero exponentially fast.

4. OUTPUT FEEDBACK CONTROL OF AN INDUCTION MOTOR

By assuming that the magnetic fluxes ψ_a and ψ_b are non measurable state variables and considering system (1), it can be identified

$$A(\cdot) = \begin{pmatrix} -\frac{R_r}{L_r} & -\frac{\alpha_3}{L_r}\omega \\ \frac{\alpha_3}{Lr}\omega & -\frac{R_r}{L_r} \end{pmatrix} \quad B(\cdot) = \begin{pmatrix} \frac{K_I}{L_r}I_a \\ \frac{K_I}{L_r}I_b \end{pmatrix} \tag{45}$$

and the functions

$$\psi_0(\cdot) = \begin{pmatrix} \omega \\ -\frac{N}{M}Sen(\theta) - \frac{B}{M}\omega \\ -\frac{R_I}{L_I}I_a + \frac{1}{L_I}V_a \\ -\frac{R_I}{L_I}I_b + \frac{1}{L_I}V_b \end{pmatrix} \tag{46}$$

$$\psi_1(\cdot) = \begin{pmatrix} 0 & 0 \\ \frac{I_b}{M} & -\frac{I_a}{M} \\ \frac{\alpha_1}{L_I} & \frac{\alpha_2}{L_I}\omega \\ -\frac{\alpha_2}{L_I}\omega & \frac{\alpha_1}{L_I} \end{pmatrix} \tag{47}$$

with $\eta = [\psi_a, \psi_b]^T$ and $y = [\theta, \omega, I_a, I_b]^T$.

By following the design method described in Karagiannis *et al.* (2003), it can be synthesized a dynamic feedback control law of the form

$$\begin{aligned} \dot{\hat{\eta}} &= \omega \\ u &= \alpha(y, \hat{\eta} + \beta(y)). \end{aligned} \tag{48}$$

Then, by defining the auxiliary vector

$$z = \hat{\eta} + \beta(y) - \eta, \tag{49}$$

and considering (6), $\beta(y)$ should be appropriately chosen to guarantee stability of the auxiliary dynamics $\dot{z} = \overline{A}z$. In this case we propose the following mapping

$$\beta(y) = \begin{bmatrix} \beta_a(I_a) \\ \beta_b(I_b) \end{bmatrix} = \begin{bmatrix} \frac{L_i R_r}{\alpha_1 L_r}k_a I_a \\ -\frac{\alpha_3 L_i}{\alpha_2 L_r}I_b \end{bmatrix} \tag{50}$$

to obtain the auxiliary dynamics

$$\dot{z} = \overline{A}z = \begin{bmatrix} -\frac{R_r}{L_r}(1+k_a) & -\omega(1+k_a)n_p \\ 0 & 0 \end{bmatrix} z \tag{51}$$

The $\overline{A}$ matrix has an eigenvalue at zero and the other one at $-\frac{R_r}{L_r}(1+k_a)$, with k_a a design parameter. In other words, the auxiliary dynamics are stable and thus the rotor fluxes are bounded. The output feedback control law is obtained by replacing the state variables of η by the corresponding variables $\hat{\eta} + \beta(y)$ in the full information control designed in the previous section, i.e.

$$\begin{bmatrix} \psi_a \\ \psi_b \end{bmatrix} \mapsto \begin{bmatrix} \widehat{\psi_a} + \beta_a(I_a) \\ \widehat{\psi_b} + \beta_b(I_b) \end{bmatrix} \tag{52}$$

Finally, the designed dynamic output feedback tracking control law is as follows:

$$\begin{aligned} \dot{\hat{\eta}} &= A\begin{bmatrix} \widehat{\psi_a} + \beta_a(I_a) \\ \widehat{\psi_b} + \beta_b(I_b) \end{bmatrix} + \begin{bmatrix} \frac{K_I}{L_r}I_a \\ \frac{K_I}{L_r}I_b \end{bmatrix} \\ &\quad - \begin{bmatrix} \frac{\partial\beta_a}{\partial y} \\ -\frac{\partial\beta_b}{\partial y} \end{bmatrix}\left(\psi_0 + \psi_1\begin{bmatrix} \widehat{\psi_a} + \beta_a(I_a) \\ \widehat{\psi_b} + \beta_b(I_b) \end{bmatrix}\right) \\ u &= \frac{1}{\widehat{\gamma}}\begin{bmatrix} \widehat{\psi_b} + \beta_b(I_b) & -\widehat{\psi_a} - \beta_a(I_a) \\ -\widehat{\psi_a} - \beta_a(I_a) & -(\widehat{\psi_b} - \beta_b(I_b)) \end{bmatrix} \\ &\quad \times \begin{bmatrix} \widehat{w}_a + k_1\eta_\tau + r \\ \widehat{w}_b + k_3\eta_I + \eta_\psi \end{bmatrix} \end{aligned} \tag{53}$$

with

$$\widehat{\gamma} = -\left((\widehat{\psi_a} + \beta_a(I_a))^2 + (\widehat{\psi_b} + \beta_b(I_b))^2 \right)$$

and $\widehat{w}_a$, $\widehat{w}_b$ are the corresponding w_a and w_b functions obtained after applying (52).

4.1 Digital Simulations

Digital simulations were carried out to evaluate the performance of the induction motor regulated by the output feedback dynamic controller. The system parameters were

$$R_s = 3.05\Omega,\ R_r = 2.12\Omega,\ L_s = 0.243H,\ n_p = 1,$$
$$L_r = 0.306H,\ M_e = 0.225H, G = 9.81Kg - m/s^2$$
$$J = 2.1x10^{-4}Kg - m^2, B_0 = 0.015Nm - s/rad,$$
$$L_0 = 0.305m,\ m = 0.401Kg.$$

The desired trajectories for the load position and magnetic flux were chosen, respectively,

$$q_d = \frac{\pi}{2} sin(5t)(1 - e^{-0.1t^3}) \quad \text{rad}$$
$$\psi_d = 2(1 - e^{-t^2}) + 1 \quad \text{Wb.Wb.}$$

The design parameters were set at $k_a = 1$, $k_s = k_1 = 0.82$, $K_2 = k_3 = 10$ and $\alpha = 1$. The closed-loop performance of the designed controlled is shown in the figures below. Figure 1 shows the asymptotic convergence of the controlled angular position to the desired trajectory, whilst figures 2 and 3 depict both real and estimate magnetic fluxes for phase a and phase b, respectively.

5. CONCLUSIONS

The output feedback tracking problem of a induction motor driving a mechanical load, when the rotor fluxes are not measured has been solved by applying a systematic design procedure based on a separation principle. It was shown that by applying this approach, a feedback backstepping full information control law can be recovered from a dynamic output feedback controller, including a nonlinear observer to estimate the unmeasured state variables. Digital simulations showed the asymptotic stability of the controlled angular position of the induction motor and boundedness of the remaining state variables.

REFERENCES

Astolfi, A. and R. Ortega (2003). Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems. *IEEE Transactions on Automatic Control* **48**(4), 590–606.

Dawson, D.M., J. Hu and T.C. Burg (1998). *Nonlinear Control of Electric Machinery.* Marcel Dekker. New York.

Freeman, Randy and Petar Kokotović (1996). Tracking controllers for systems linear in the unmeasured states. *Automatica* **32**, 735–746.

Karagiannis, D., A. Astolfi and R. Ortega (2003). Two results for adaptive output feedback stabilization of nonlinear systems. *Automatica* **39**, 858–866.

Marino, R. and M.W. Spong (1986). Nonlinear control techniques for flexible joint manipulators: A single link case study. *IEEE Int. Conference on Robotics and Automation* pp. 1026–1030.

Slotine, J.-J. E. and W. Li (1991). *Applied Nonlinear Control.* Prentice-Hall. Englewood Cliffs, N.J.

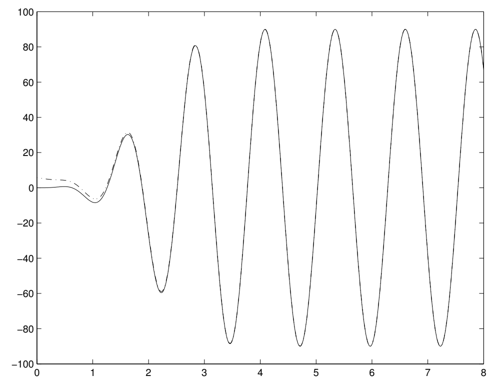

Fig. 1 Controlled angular position.

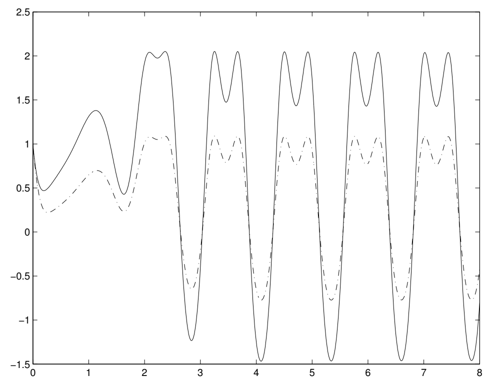

Fig. 2 Rotor magnetic flux, Phase a.

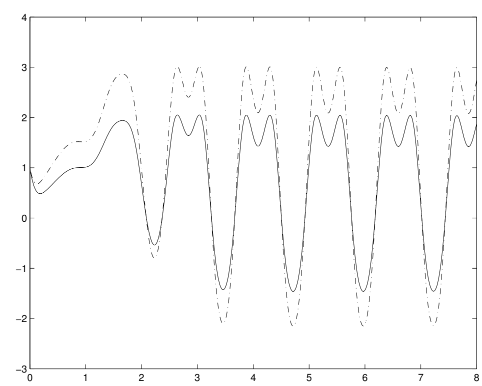

Fig. 3 Rotor magnetic flux, Phase b.